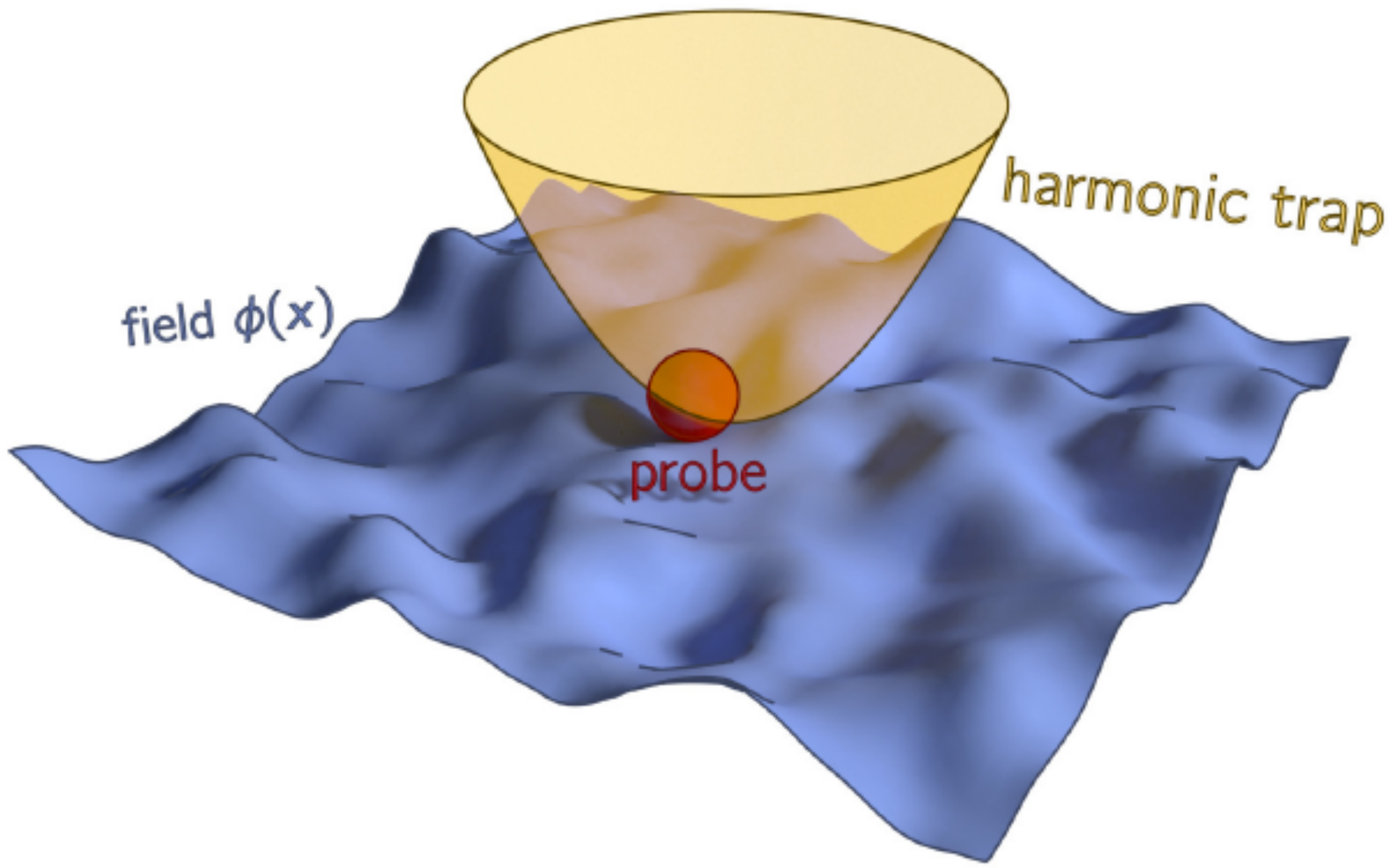
Dynamics of probe particles in near-critical fields

Davide Venturelli

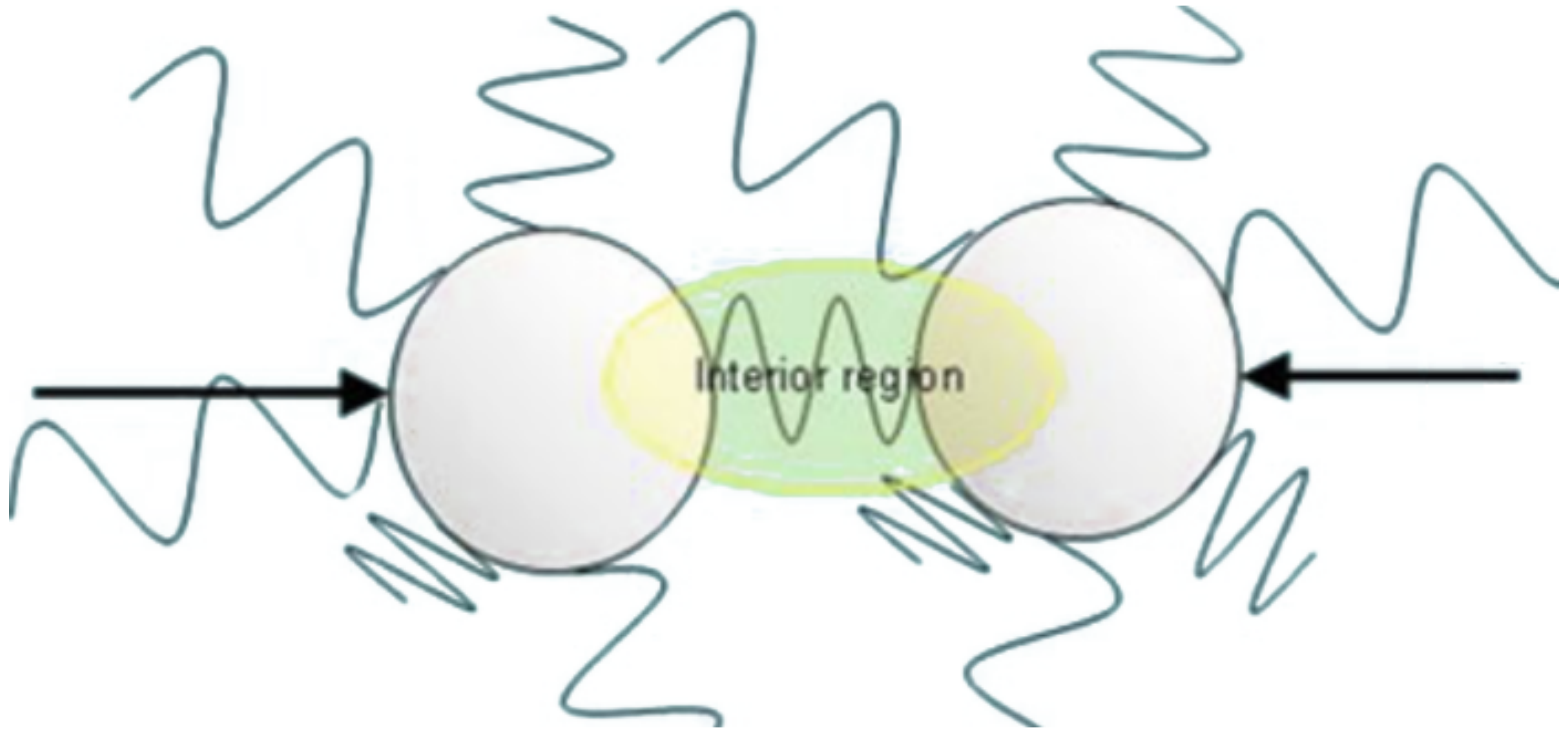
iSoDays, Bari, 30 September 2022

SISSA

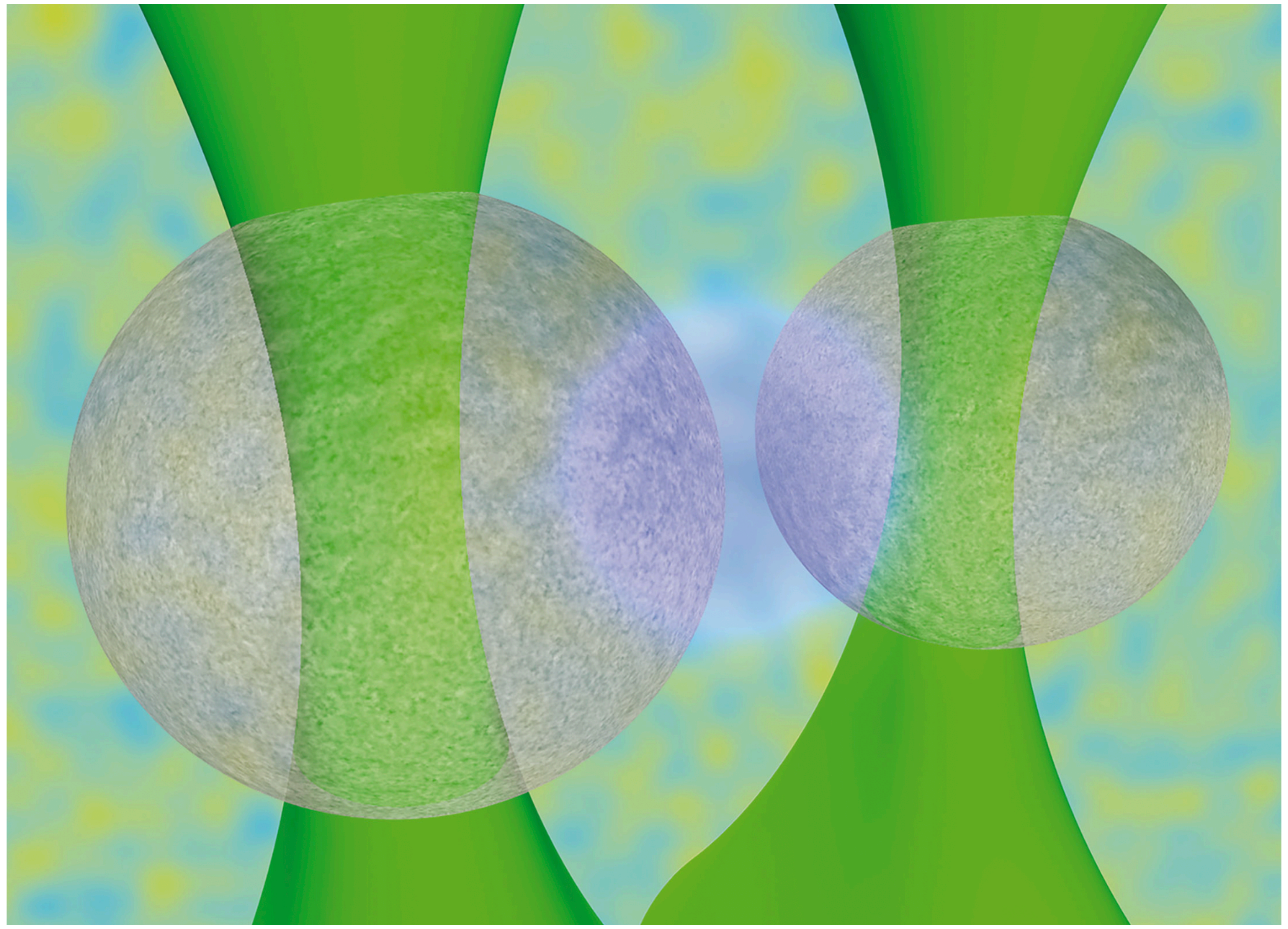


Particles in near-critical media

Why do we care?



[Magazzù et al. 2018]



- Fluctuation-induced (Casimir) forces
- Effective (attractive/repulsive) interactions
- Retardation effects on the particle dynamics

Particle in a complex medium

From Brownian motion to non-linear memory

- **Brownian motion**

$$m\,\ddot{x}(t) \;=\; -\gamma\,\dot{x}(t) \;+\; \zeta(t) - V'(x(t))$$

Separation of timescales…

$$\langle \zeta(t)\zeta(t')\rangle = 2\,k_{\mathrm{B}}T\,\gamma\,\delta(t-t')$$

- **GLE**

$$m\,\ddot{x}(t) \;=\; -\int^{t} \mathrm{d}t'\,\Gamma(t-t')\,\dot{x}(t') \;+\; \zeta(t) - V'(x(t))$$

$$\langle \zeta(t)\zeta(t')\rangle = k_{\mathrm{B}}T\,\Gamma(|t-t'|)$$

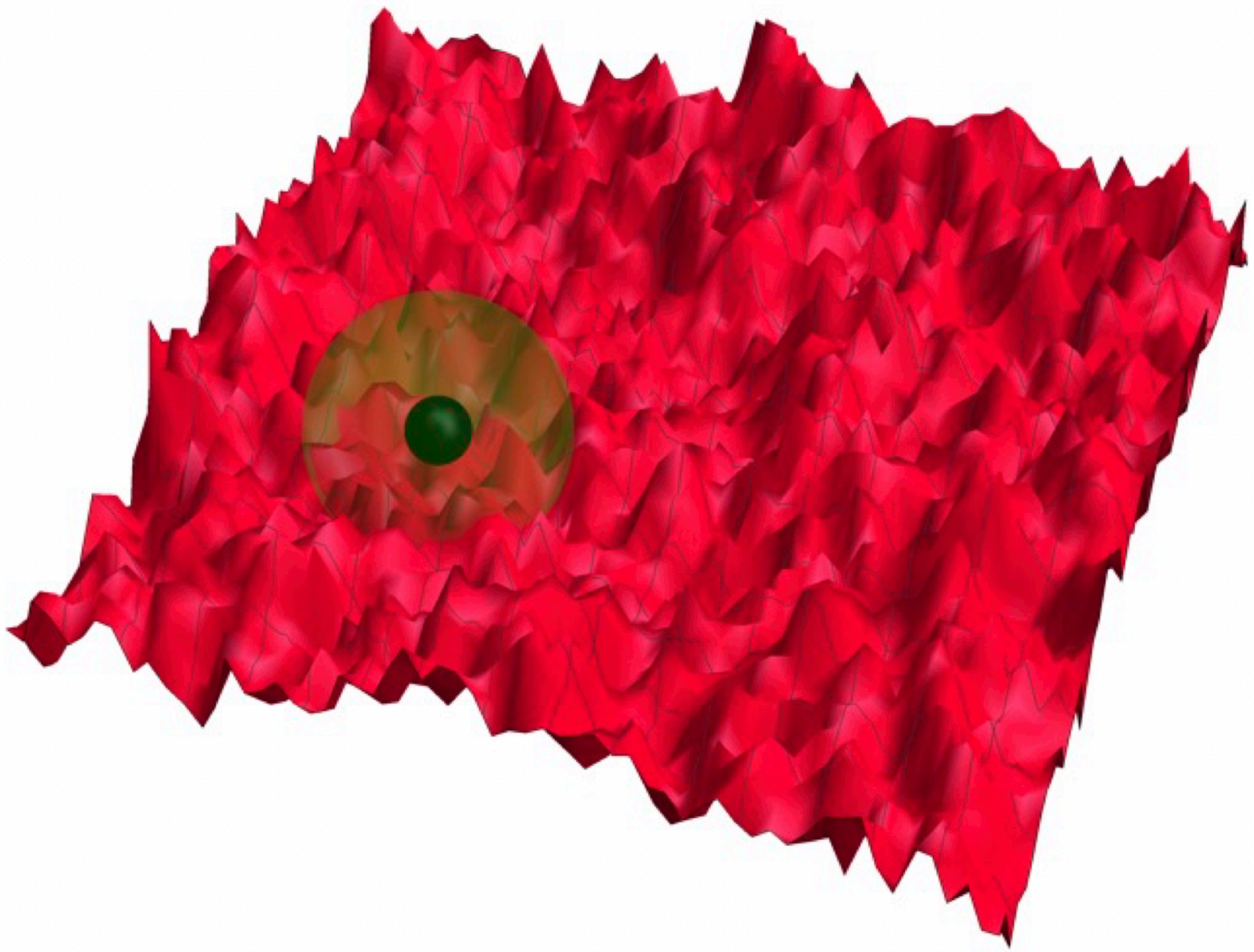
[e.g. Caldeira&Leggett '83]

Medium-induced forces? Energy flows?

Universality

and how it can help us

- Close to a **continuous PT**, different systems may exhibit same critical properties
- Trade a complex system for a simpler one within the same **universality class**
- Replace the medium by a suitable (dynamical) **field-theory**



The model

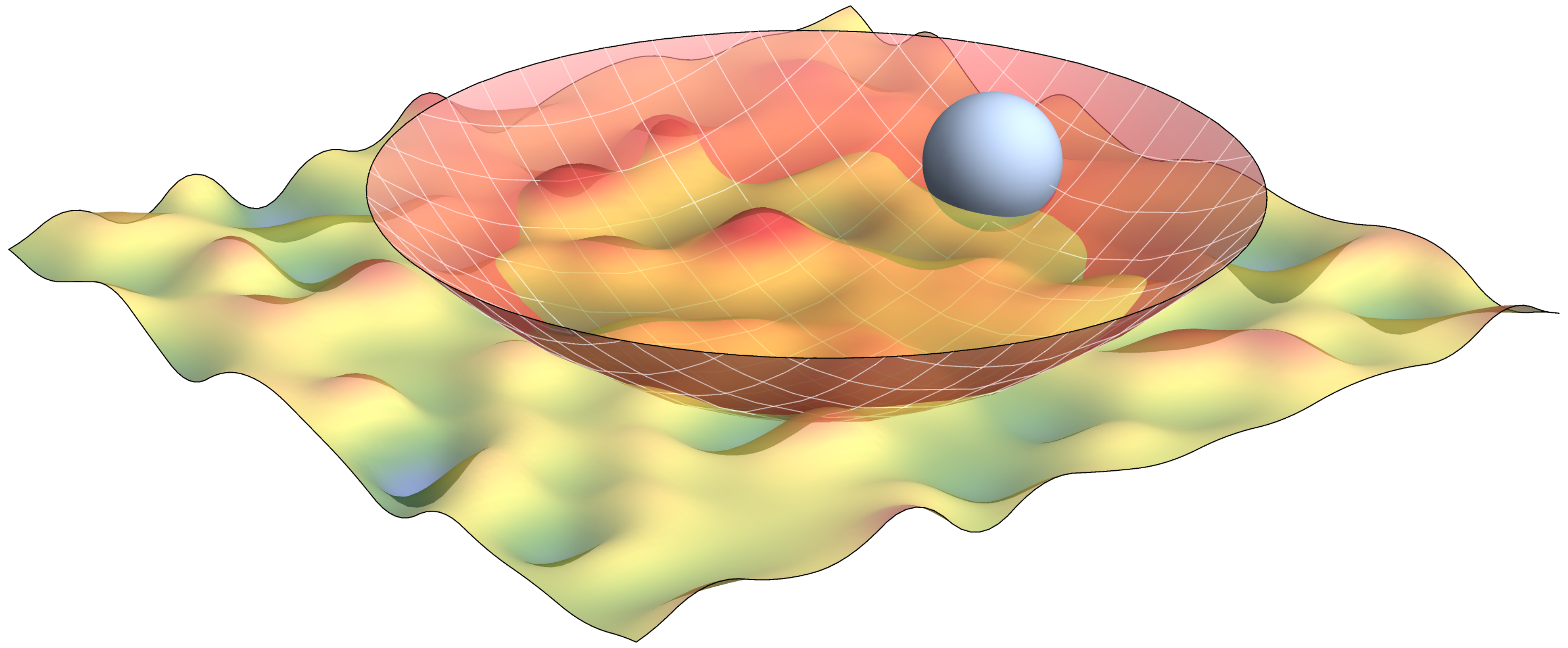
$\xi = r^{-1/2}$ **sets the range of spatial correlations of** $\phi(x,t)$

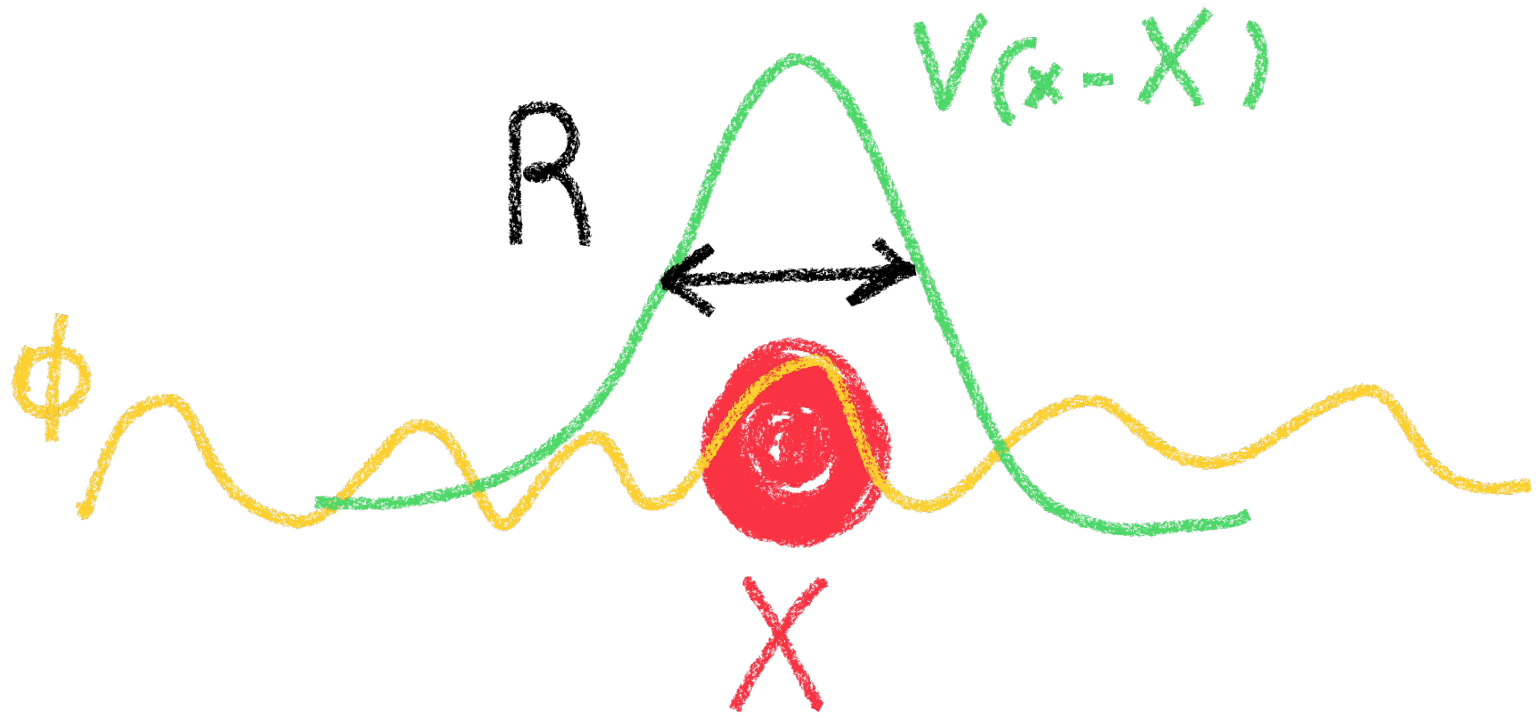
$$\mathcal{H}[\phi, \mathbf{X}] = \mathcal{H}_\phi + \mathcal{U}(\mathbf{X}) - \lambda \mathcal{H}_{\text{int}}$$

$$\mathcal{H}_\phi = \int \mathrm{d}^d\mathbf{x} \left[\frac{1}{2}(\nabla\phi)^2 + \frac{1}{2} r \phi^2 \right]$$

$$\mathcal{U}(\mathbf{X}) = \frac{\kappa}{2} X^2$$

$$\mathcal{H}_{\text{int}} = \int \mathrm{d}^d\mathbf{x}\, \phi(\mathbf{x}) V(\mathbf{x} - \mathbf{X})$$

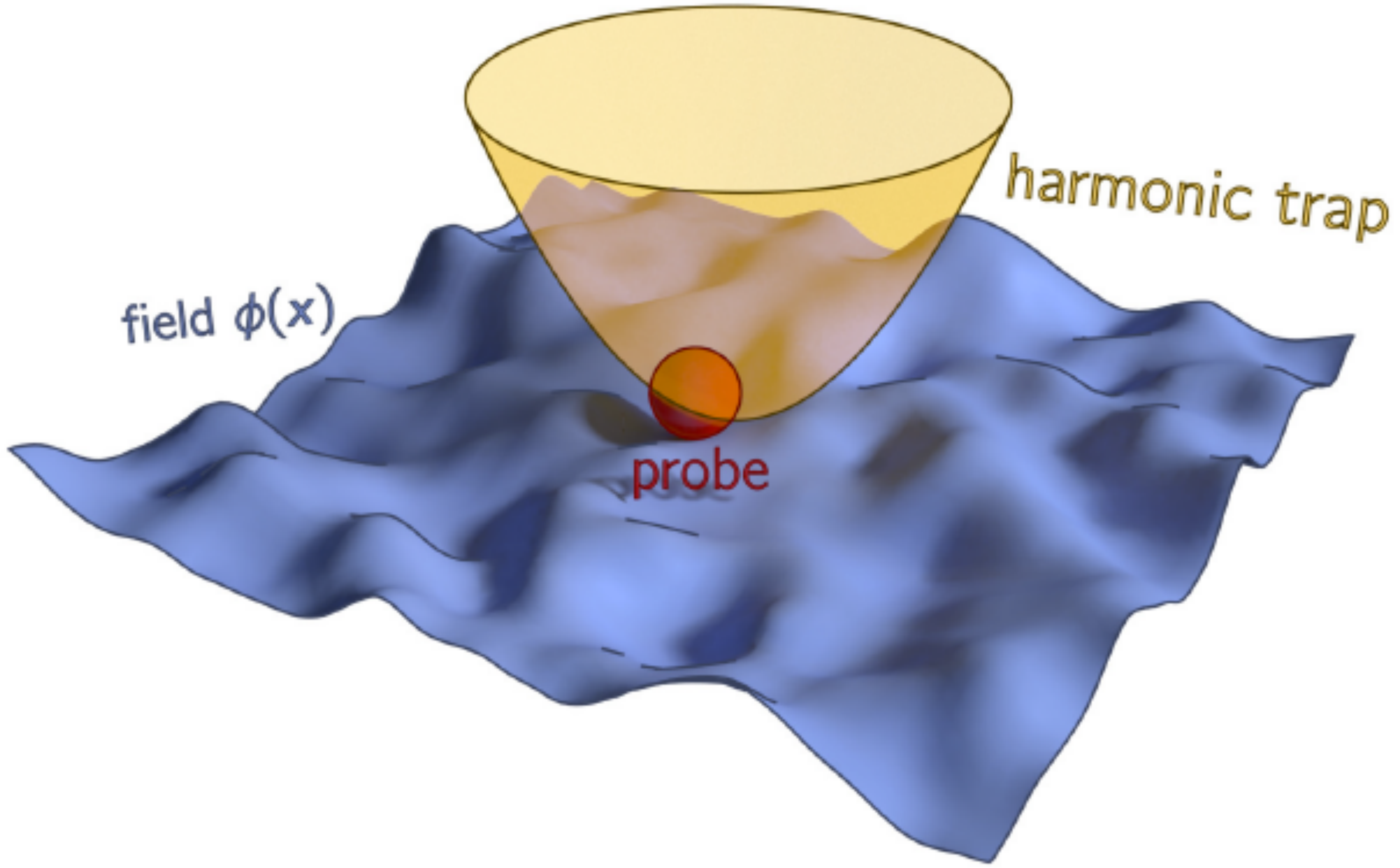




$V(x - X)$ **extends within the size R of the particle**

Dynamics

ϕ(x,t) and X(t) influence each other along their stochastic evolution,



$$\dot{\mathbf{X}}(t) = -\nu \boldsymbol{\nabla}_X \mathcal{H} + \boldsymbol{\xi}(t)$$

$$\partial_t \phi(\mathbf{x}, t) = -D(i\boldsymbol{\nabla})^\alpha \frac{\delta \mathcal{H}}{\delta \phi(\mathbf{x}, t)} + \zeta(\mathbf{x}, t)$$

in contact with a thermal bath @T,

$$\begin{cases} \langle \xi_i(t) \xi_j(t') \rangle = 2\nu T \delta_{ij} \delta(t - t') \\ \langle \zeta(\mathbf{x}, t) \zeta(\mathbf{x}', t') \rangle = 2DT(i\boldsymbol{\nabla})^\alpha \delta^d(\mathbf{x} - \mathbf{x}') \delta(t - t') \end{cases}$$

Effective particle dynamics

- Equilibrium is trivial
  (locality + translational invariance)
  → fun things happen out of equilibrium.

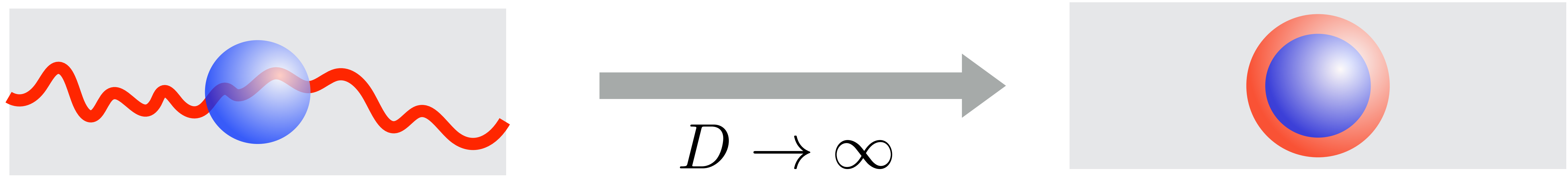
$$P_{\text{eq}}(\mathbf{X}) \propto \int \mathcal{D}\phi e^{-\beta\mathcal{H}[\phi,\mathbf{X}]} \propto e^{-\beta\mathcal{U}_X}$$

Two possible approximations:

1. Weak-coupling approximation
   (or MSR path integral + perturbation theory)

$$\begin{cases} \mathbf{X}(t) = \sum_n \lambda^n \mathbf{X}^{(n)}(t) \\ \phi(\mathbf{x},t) = \sum_n \lambda^n \phi^{(n)}(\mathbf{x},t) \end{cases}$$

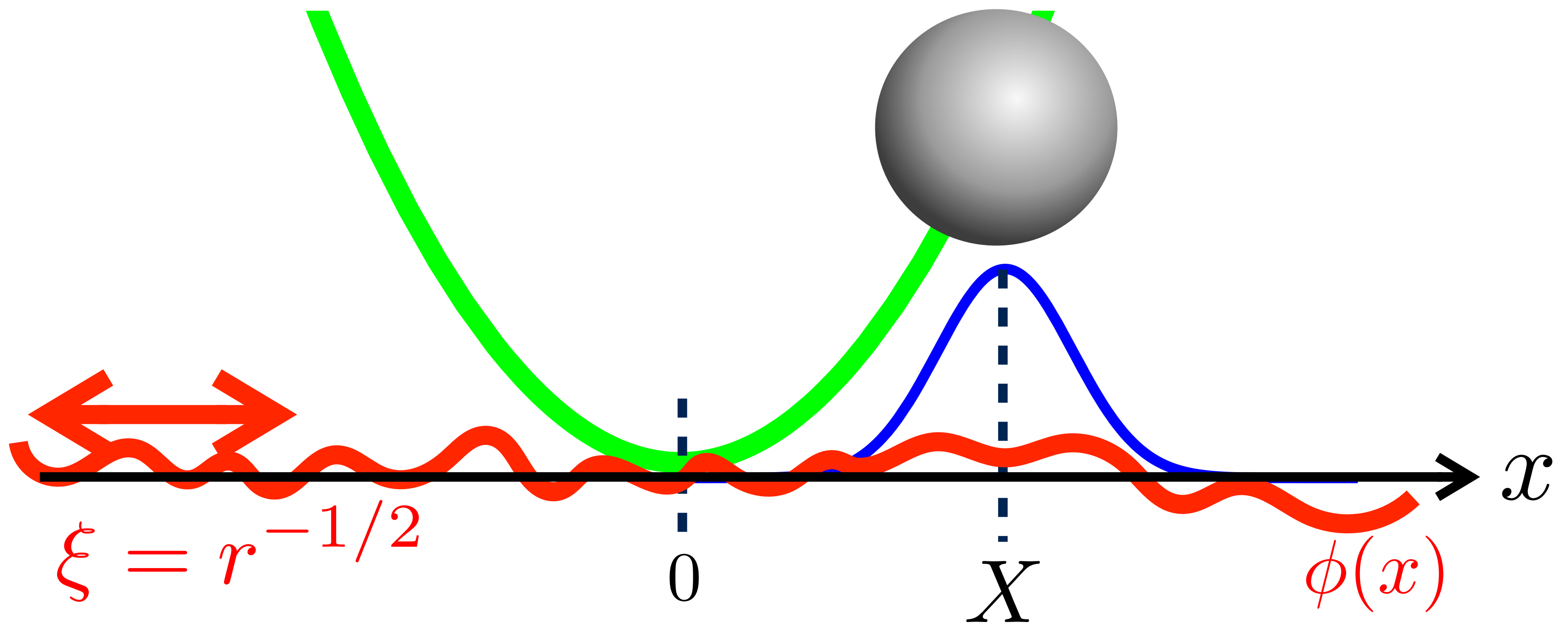
2. Adiabatic approximation



[Kaneko, '61; Theiss, Titulauer '85; …. Gross '21]

Relaxation towards equilibrium

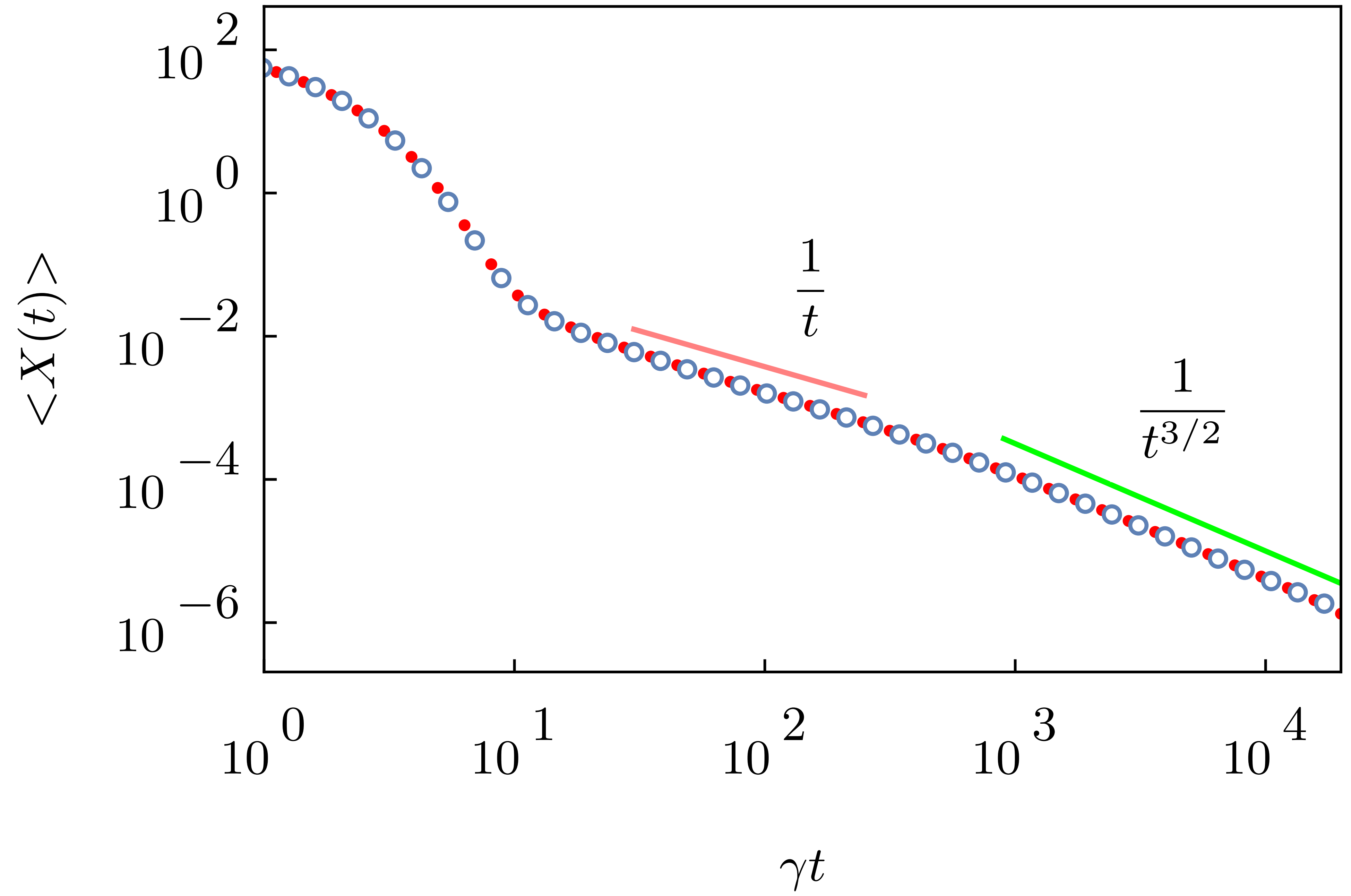
@ long times, $\langle X^{(2)}(t)\rangle \sim \begin{cases} t^{-\left(1+\frac{d}{2}\right)}, & \text{Model A, } r=0 \\ t^{-\left(1+\frac{d}{4}\right)}, & \text{Model B, } r=0 \\ t^{-\left(2+\frac{d}{2}\right)}, & \text{Model B, } r>0 \end{cases}$



$$\langle X(t)\rangle \simeq c_0 t^{-\alpha_0} f\left(t/t_c\right)$$

$$f(\tau) \sim \begin{cases} \tau^{-\beta_0} & \text{for } \tau \gg 1 \\ \text{const.} & \text{for } \tau \lesssim 1 \end{cases}$$

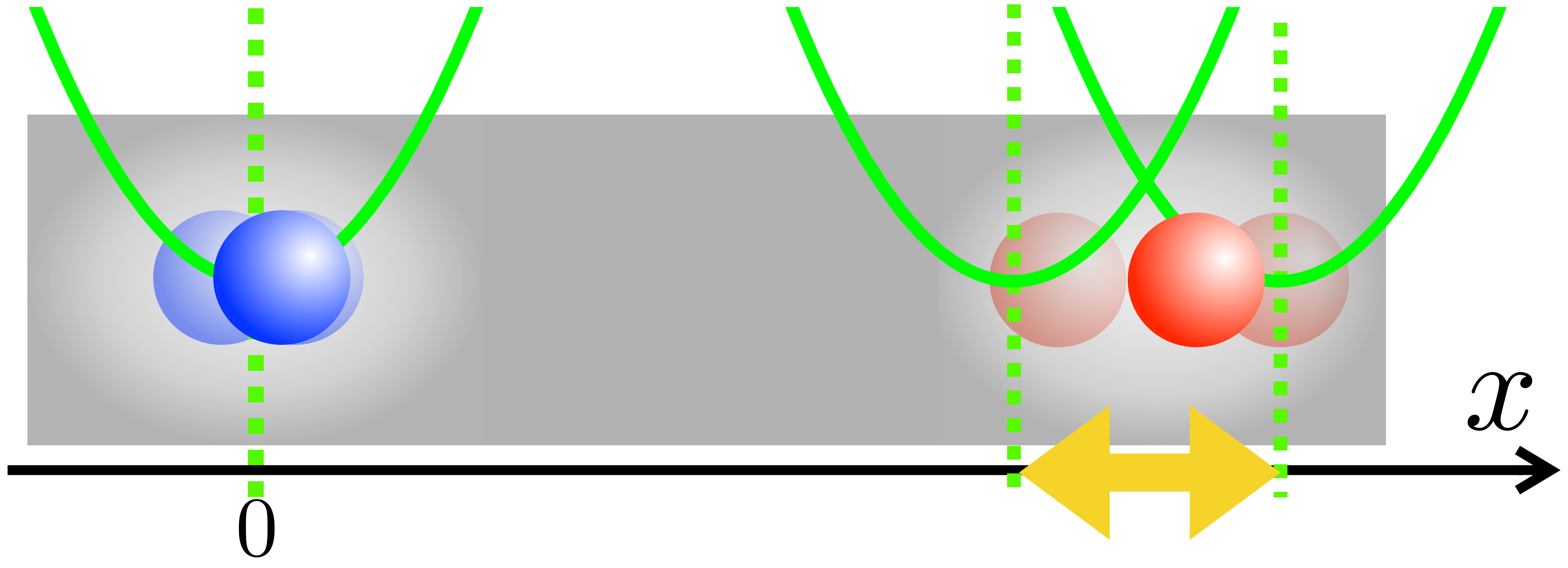
$$t_c = \tau_\phi^{-1}(q \sim 1/X_0)$$



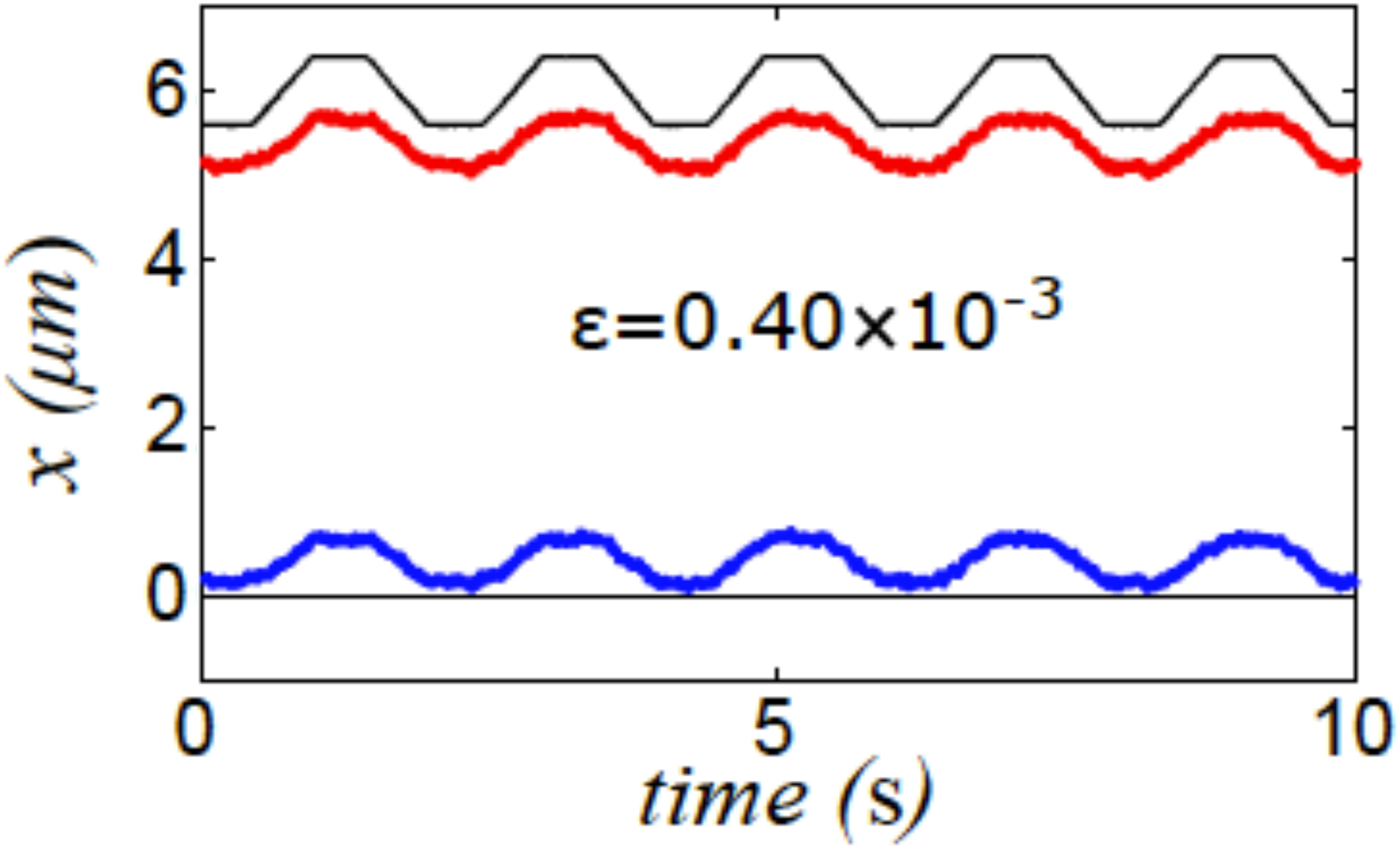
[DV, F. Ferraro, A. Gambassi, PRE 105 (5), 054125]

Energy Transfer between Colloids via Critical Interactions

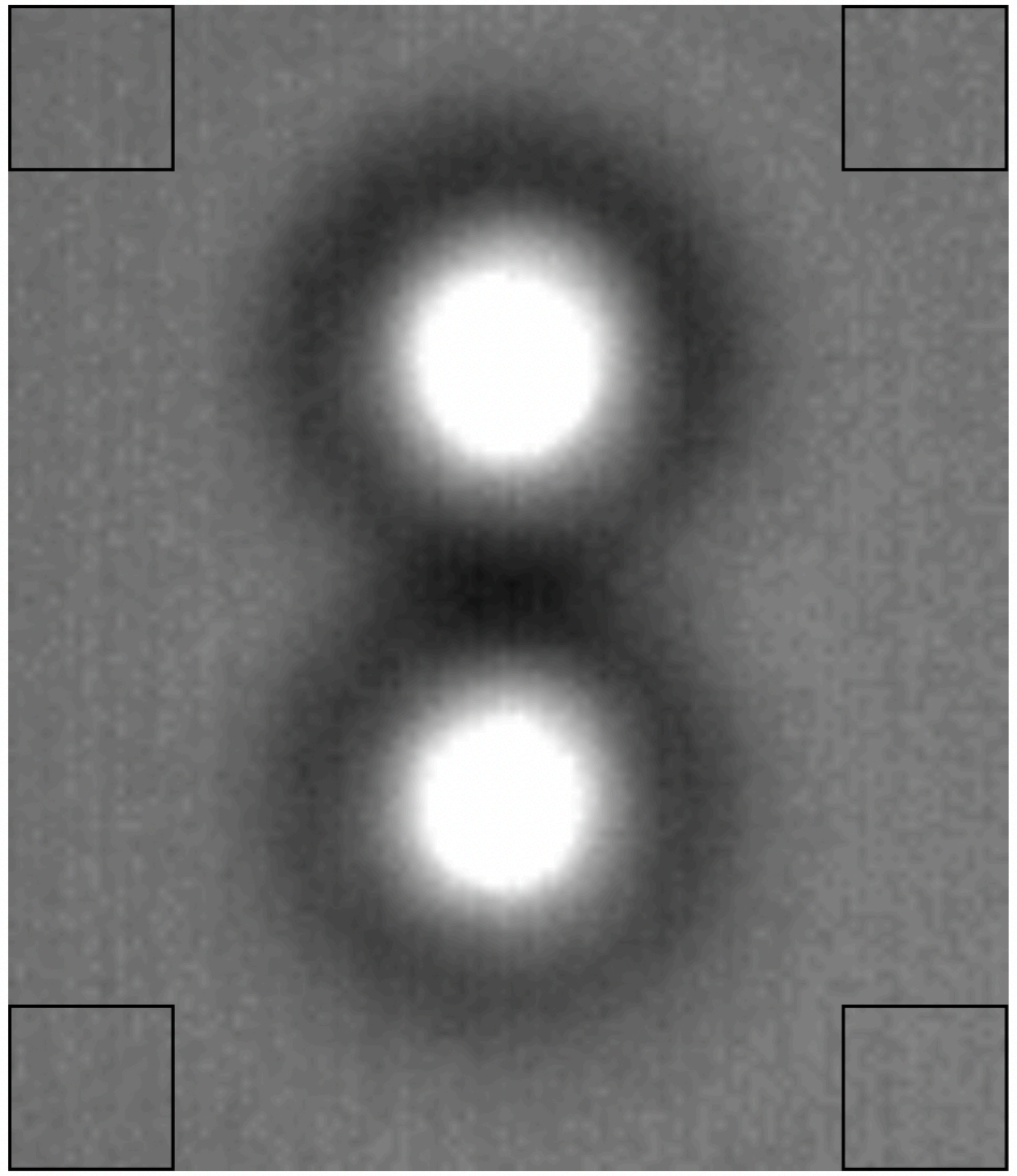
Ignacio A. Martínez [1,2,*], Clemence Devailly [1,3], Artyom Petrosyan [1] and Sergio Ciliberto [1,*]



cartoon
not in scale



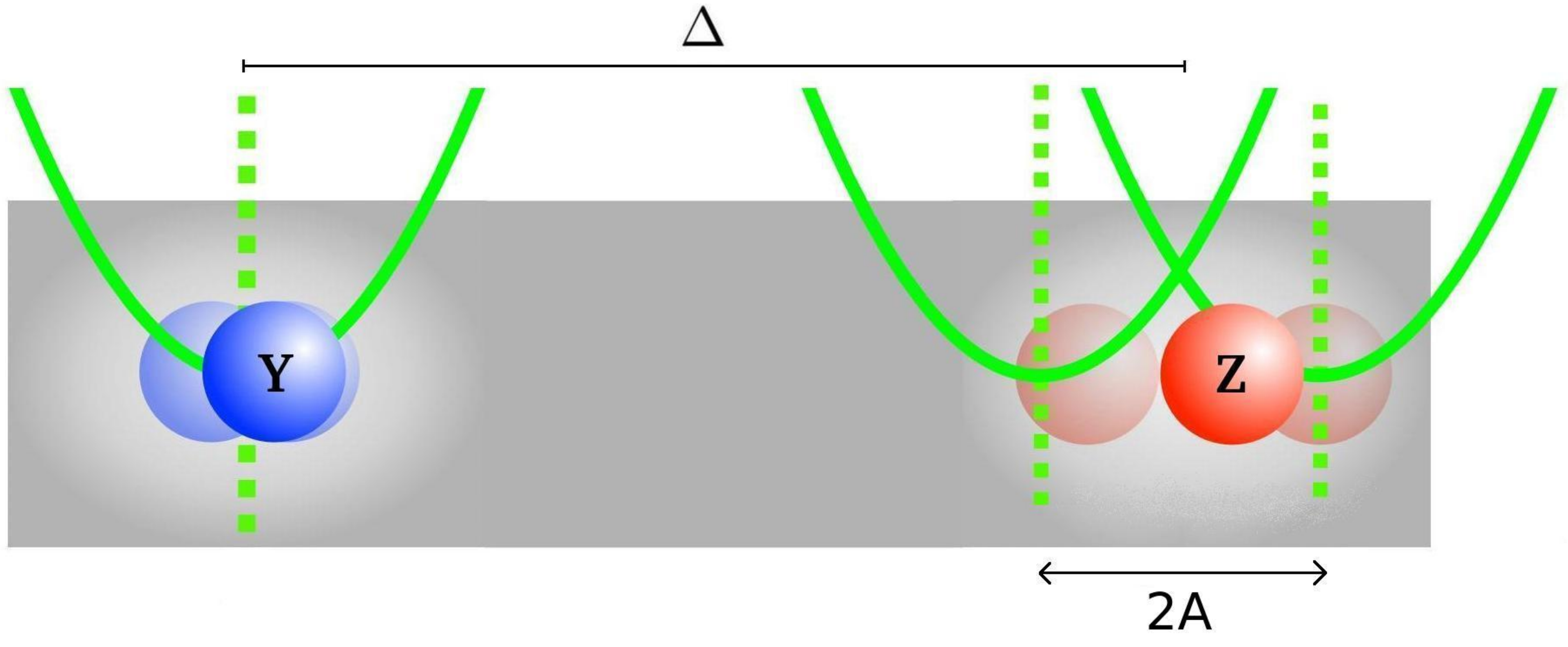
$$\varepsilon = (T_c - T)/T_c$$



[2017]

Two particles

Model



2 (independent) particles in a the field,

$$\mathcal{H} = \mathcal{H}_\phi + \mathcal{U}_Y + \mathcal{U}_Z - \lambda \left(\mathcal{H}_{\text{int}}^Y + \mathcal{H}_{\text{int}}^Z \right)$$

$$\int \mathrm{d}^d\mathbf{x}\, \phi(\mathbf{x}) \left[V^{(z)}(\mathbf{x} - \mathbf{Z}) + V^{(y)}(\mathbf{x} - \mathbf{Y}) \right]$$

One of them is **driven** periodically,

$$\mathcal{U}_Z = \frac{k_z}{2} \left[\mathbf{Z} - \mathbf{Z}_F(t) \right]^2$$

$$\mathbf{Z}_F(t) = \mathbf{\Delta} + \mathbf{A} \sin(\Omega t)$$

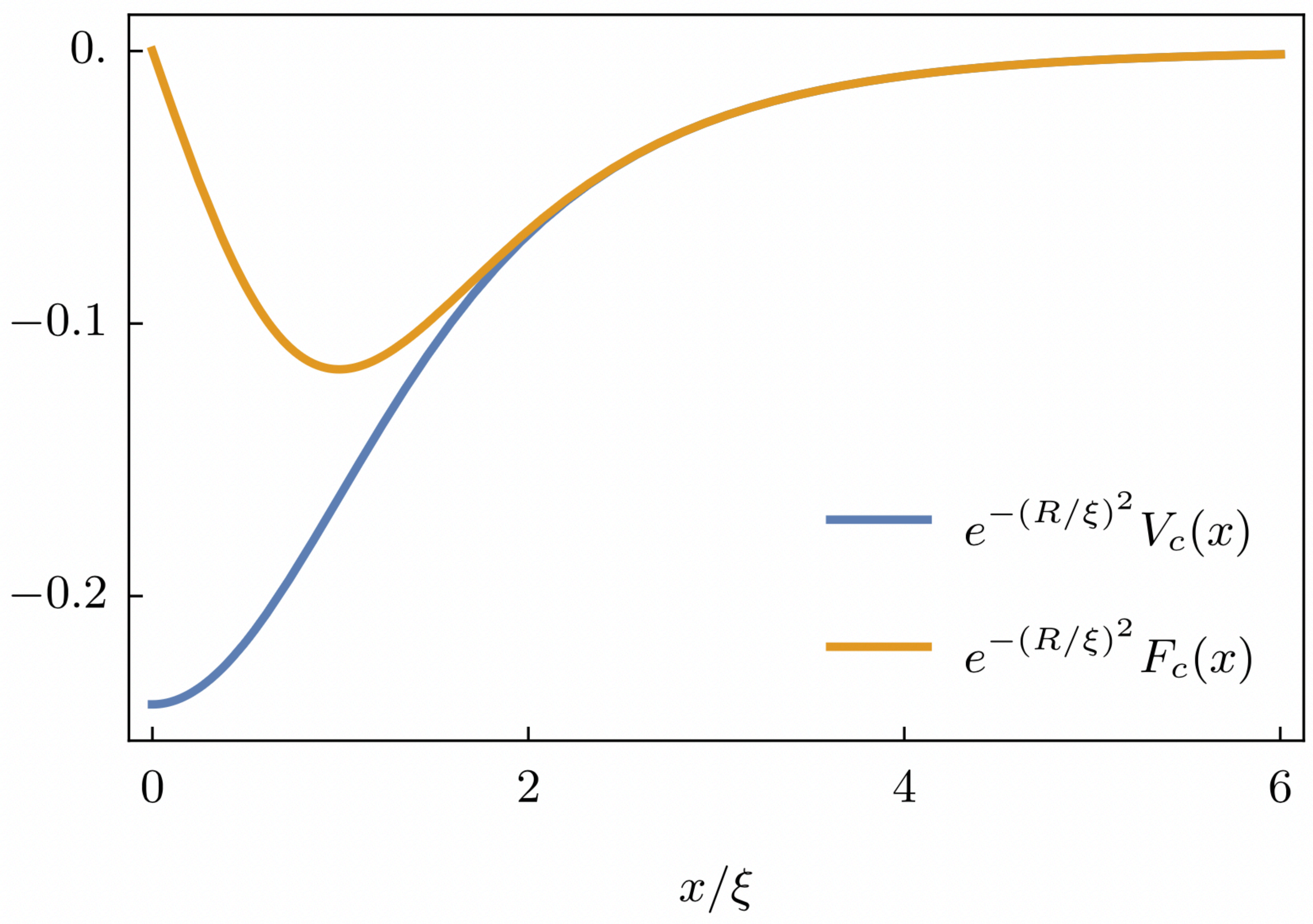
→ how does $Y(t)$ respond?

Two particles

Adiabatic approximation:

$$\mathcal{P}(\mathbf{Y},\mathbf{Z}) \propto e^{-\beta(\mathcal{U}_y+\mathcal{U}_z)} \int \mathcal{D}\phi\, e^{-\beta(\mathcal{H}_\phi - \lambda \mathcal{H}_{\text{int}})} \propto e^{-\beta[\mathcal{U}_Y+\mathcal{U}_Z+\lambda^2 V_c(\mathbf{Y},\mathbf{Z})]}$$

$$\dot{\mathbf{Y}}_{\text{ad}}(t) = -\nu_y \nabla_y \left[\mathcal{U}_y(\mathbf{Y}) + \lambda^2 V_c(\mathbf{Y},\mathbf{Z})\right] + \boldsymbol{\xi}(t)$$



Weak-coupling approximation:

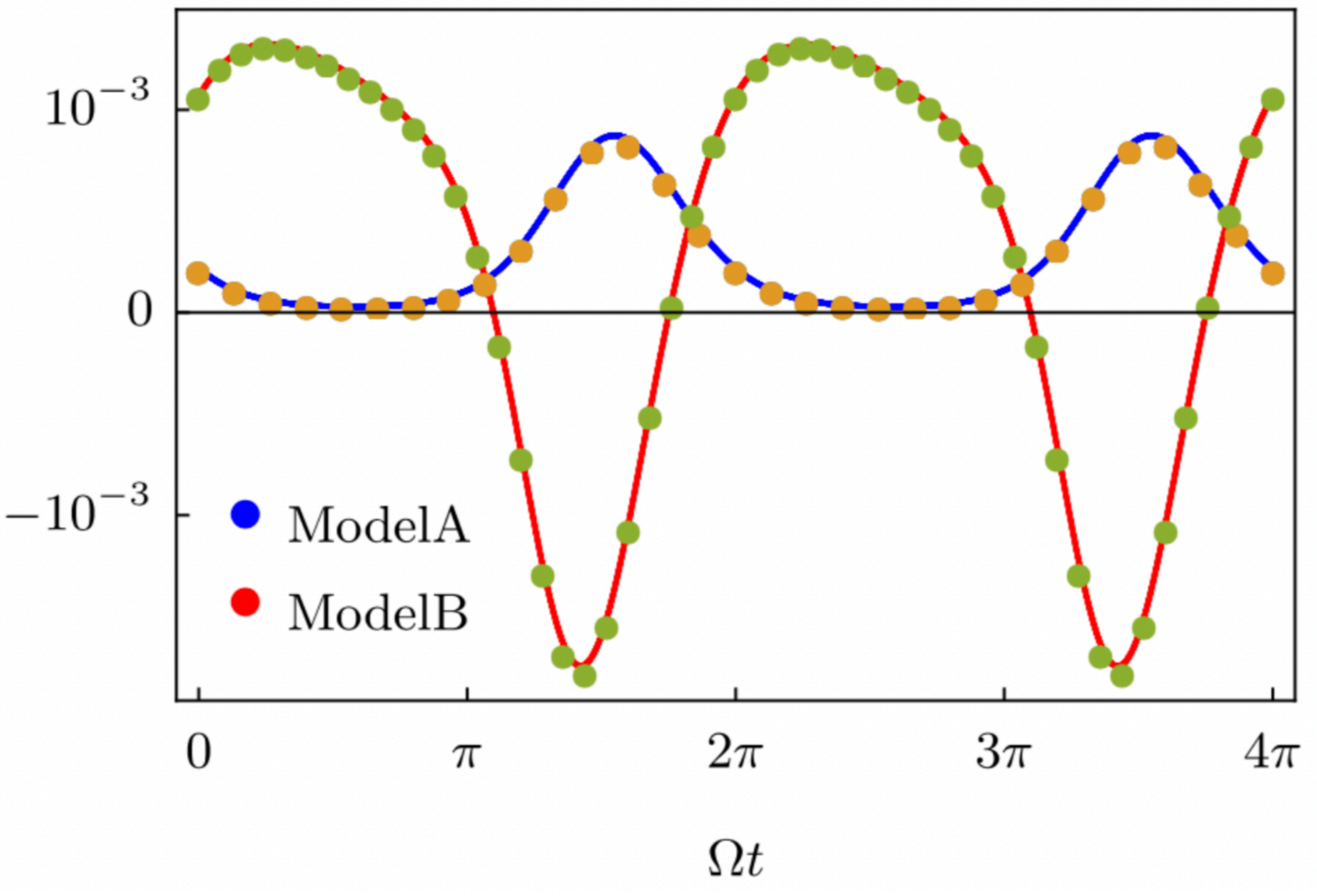
$$\partial_t P_1(\mathbf{y},t) = \mathcal{L}_0 P_1(\mathbf{y},t) + \lambda^2 \mathcal{L}_z(t) P_1(\mathbf{y},t) + \lambda^2 \int_{t_0}^{t} \mathrm{d}s \int \mathrm{d}\mathbf{x}\, \mathcal{L}(\mathbf{y}-\mathbf{x};t,s) P_2(\mathbf{y},t;\mathbf{x},s) + \mathcal{O}(\lambda^4)$$

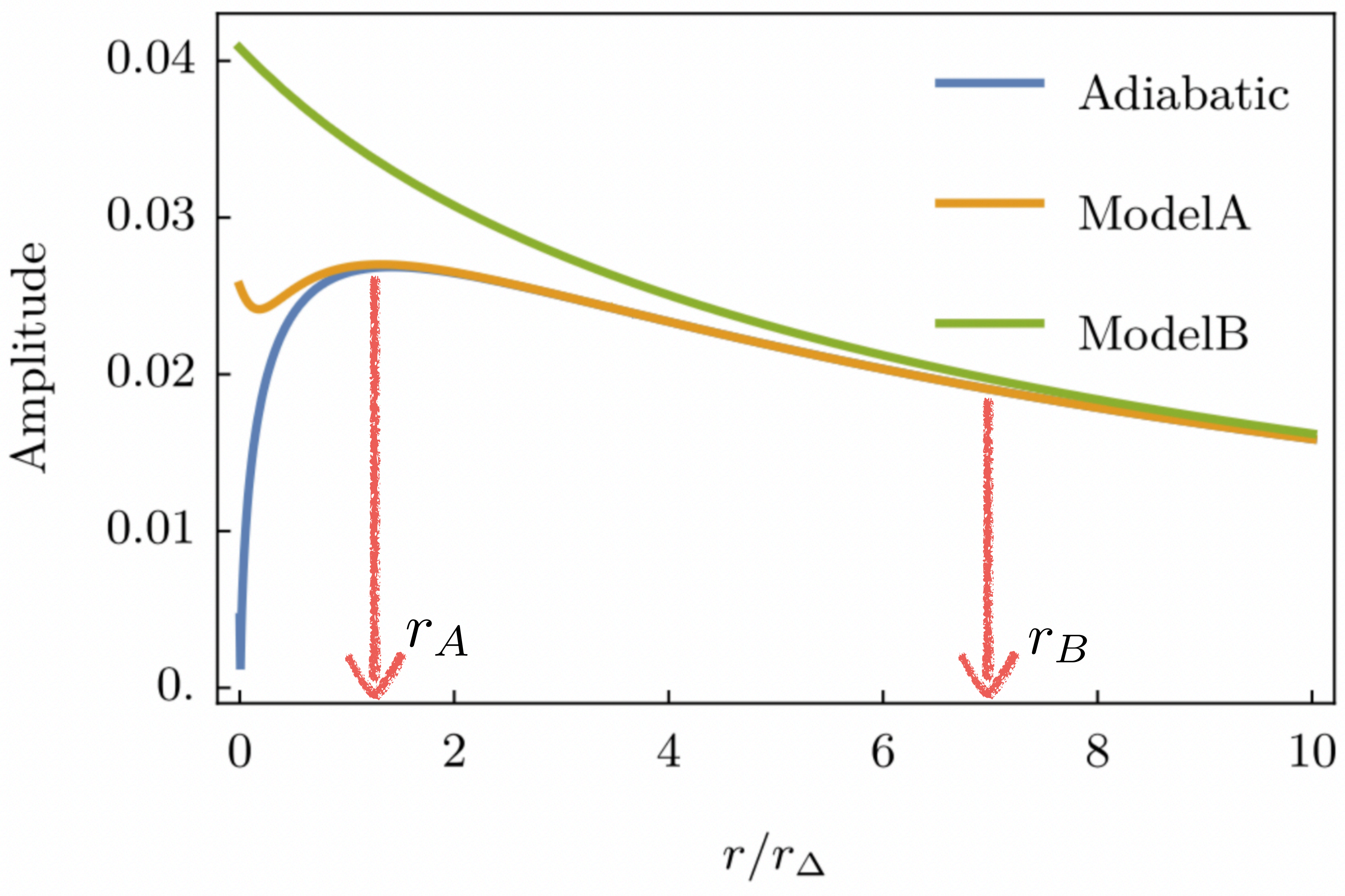
⟼ cumulant gen. func. of Y(t)

[DV, A. Gambassi, 2206.01664 (2022)]

Two particles

Non-adiabatic response





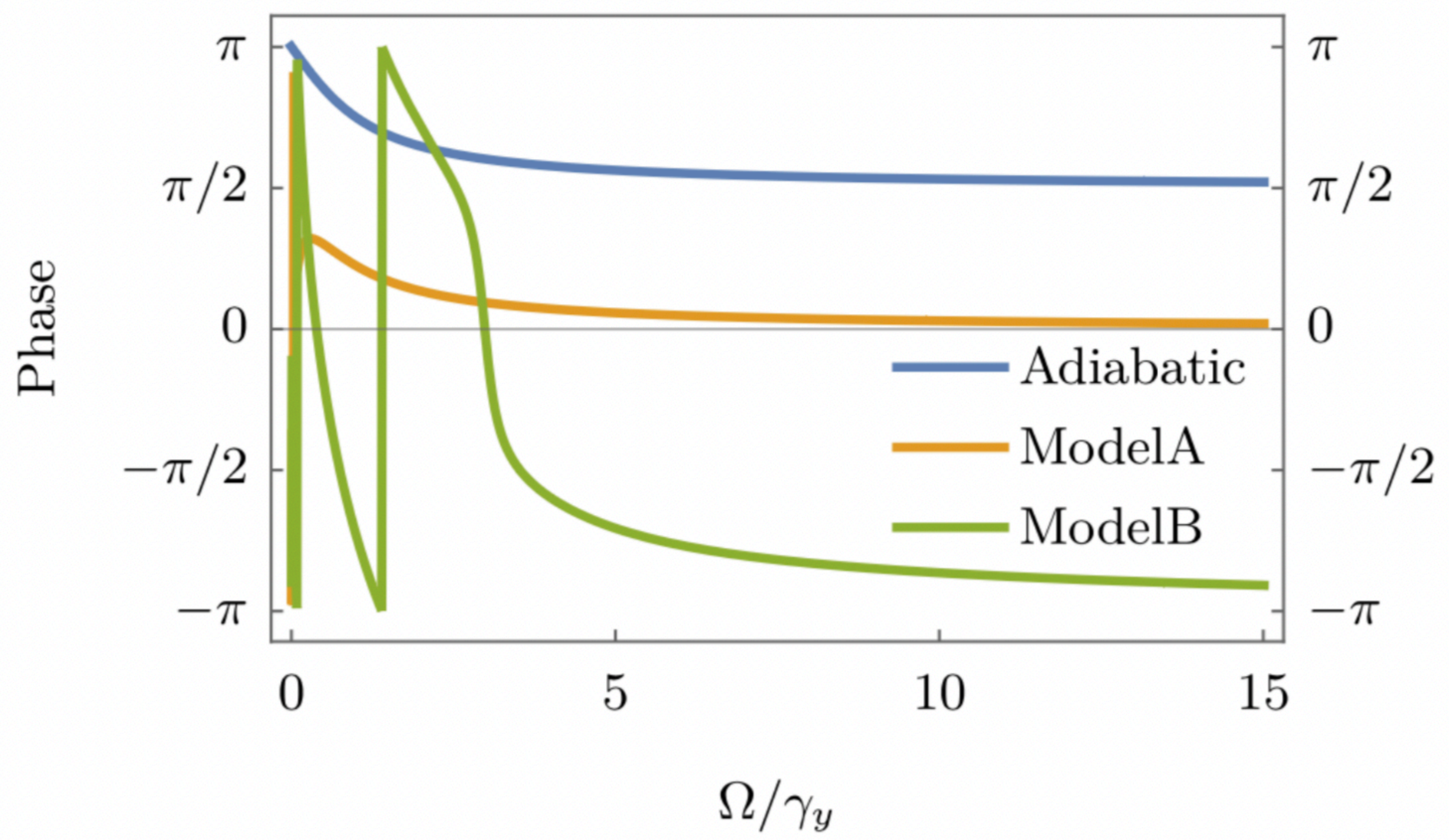
Competing timescales
$$\begin{cases} \tau_\phi^{-1} \sim Dq^\alpha(q^2 + r) \\ \tau_\Omega^{-1} \sim \Omega \end{cases}$$

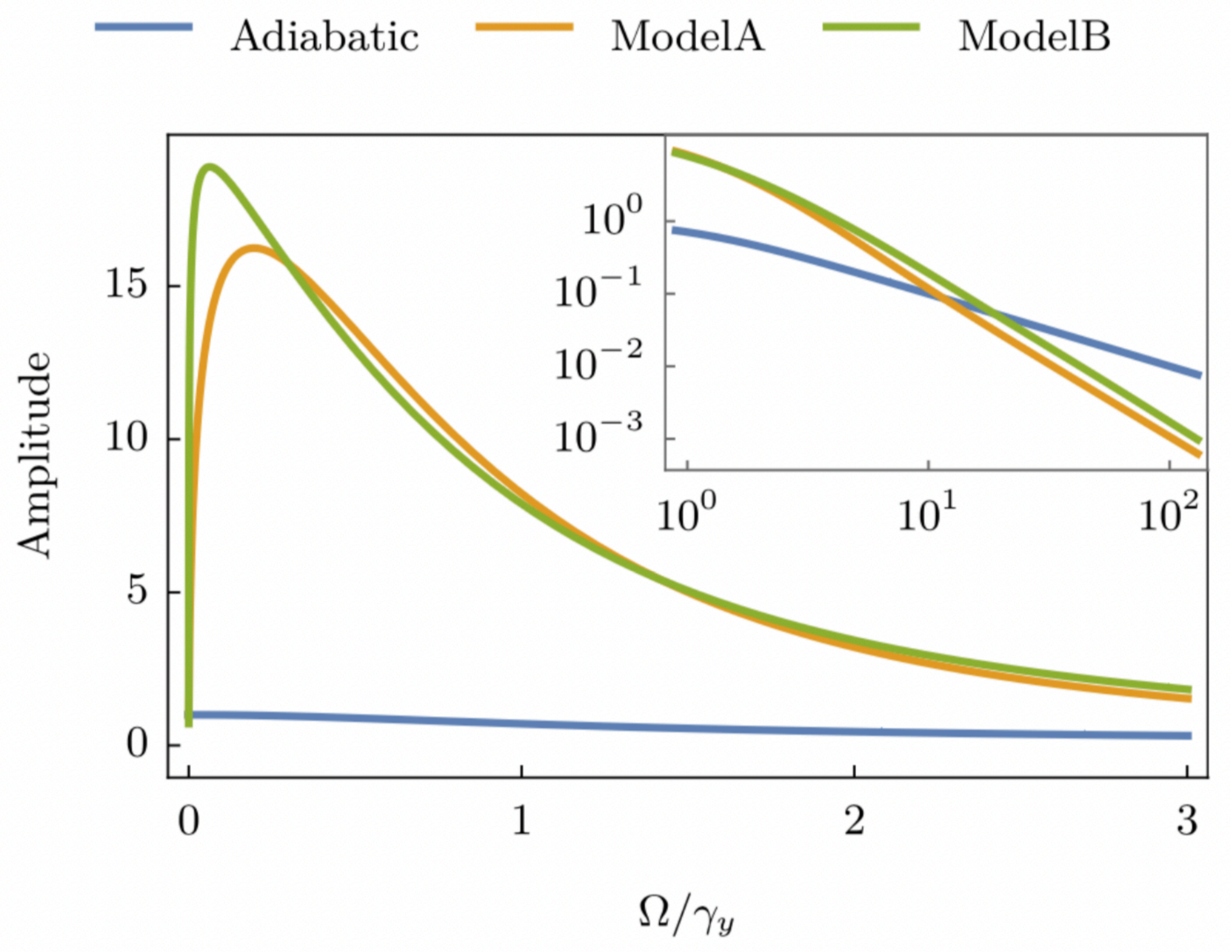
Choosing $q \sim r^{1/2} = 1/\xi \quad \Rightarrow \quad r_A \sim \Omega \,, \; r_B \sim \Omega^{1/2}$

Two particles

Frequency-dependent response

Retardation effects:



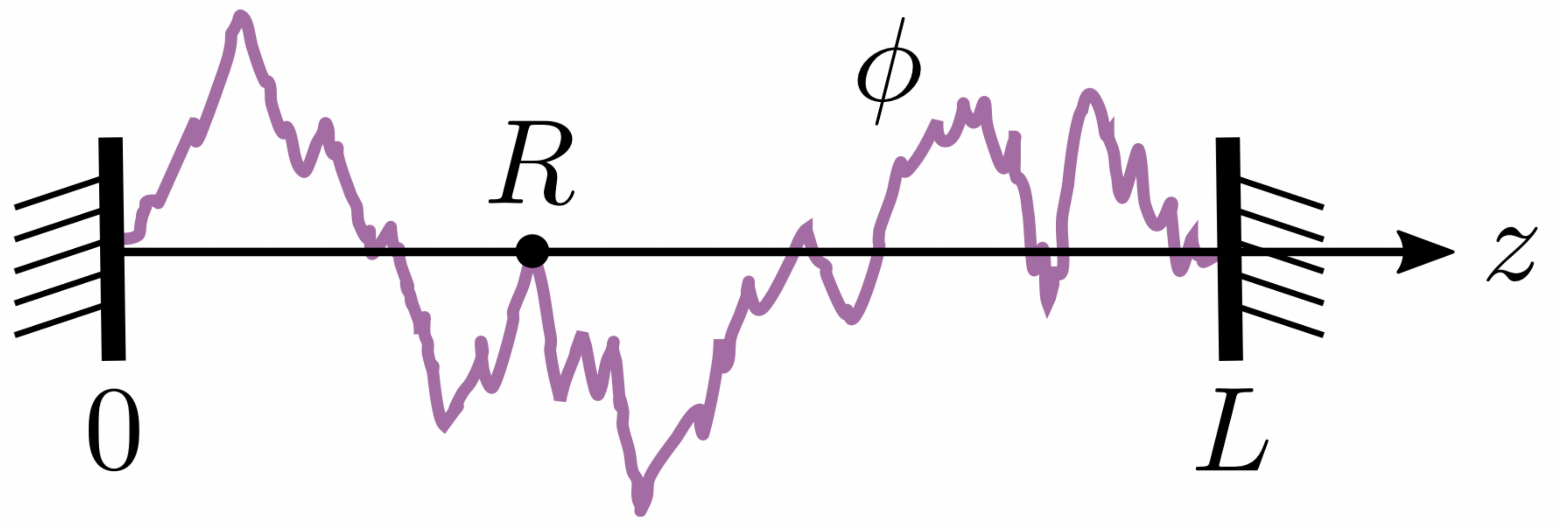


When $\xi \gg \Delta$, the non-equilibrium response is **peaked** around

$$\Omega_{\text{peak}} \sim \tau_\phi^{-1}(q \simeq 1/\Delta) \simeq D/\Delta^z$$

Particle + field in confinement

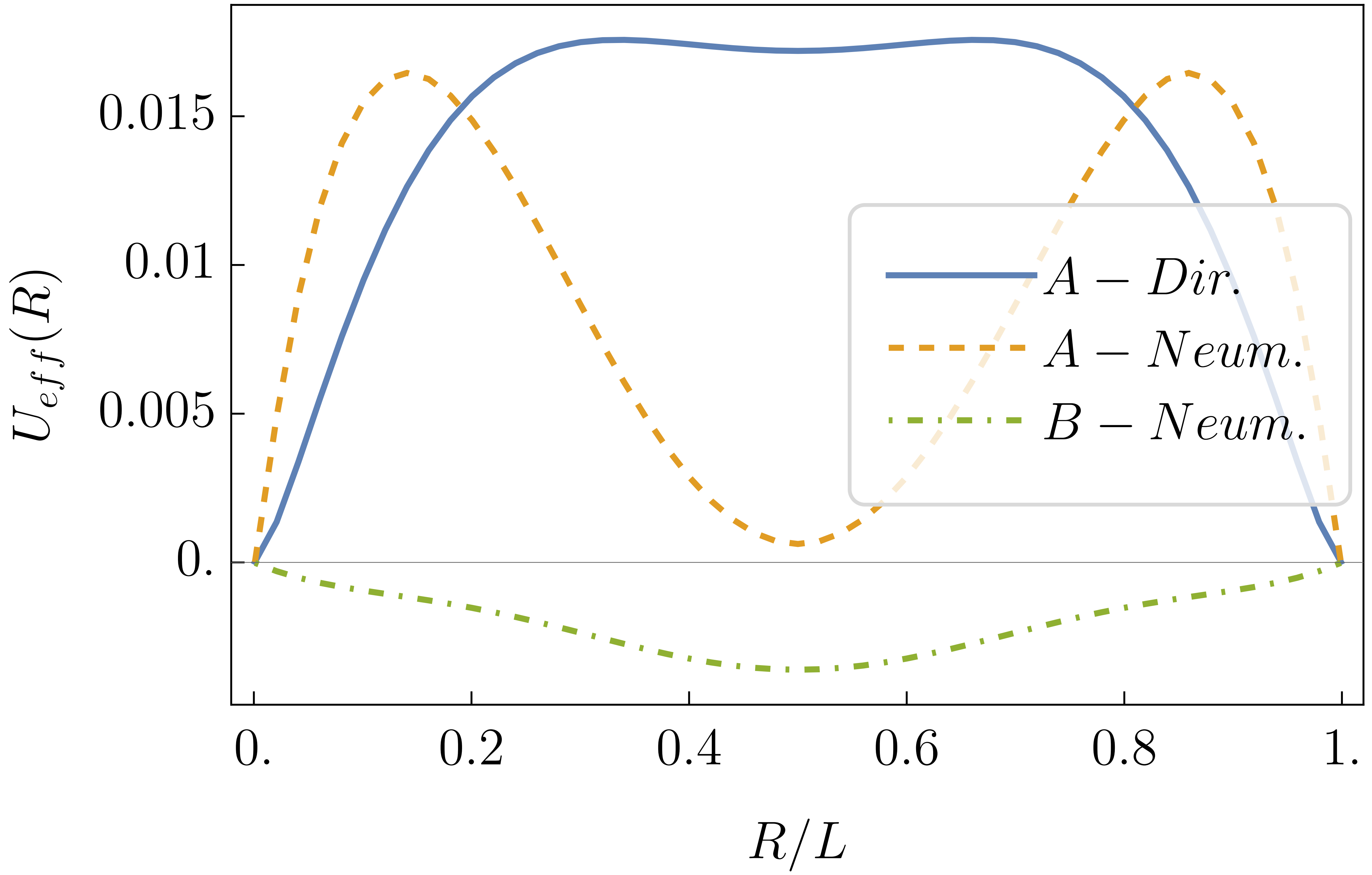
Dynamics & steady-state



Effective F-P description in the adiabatic limit:

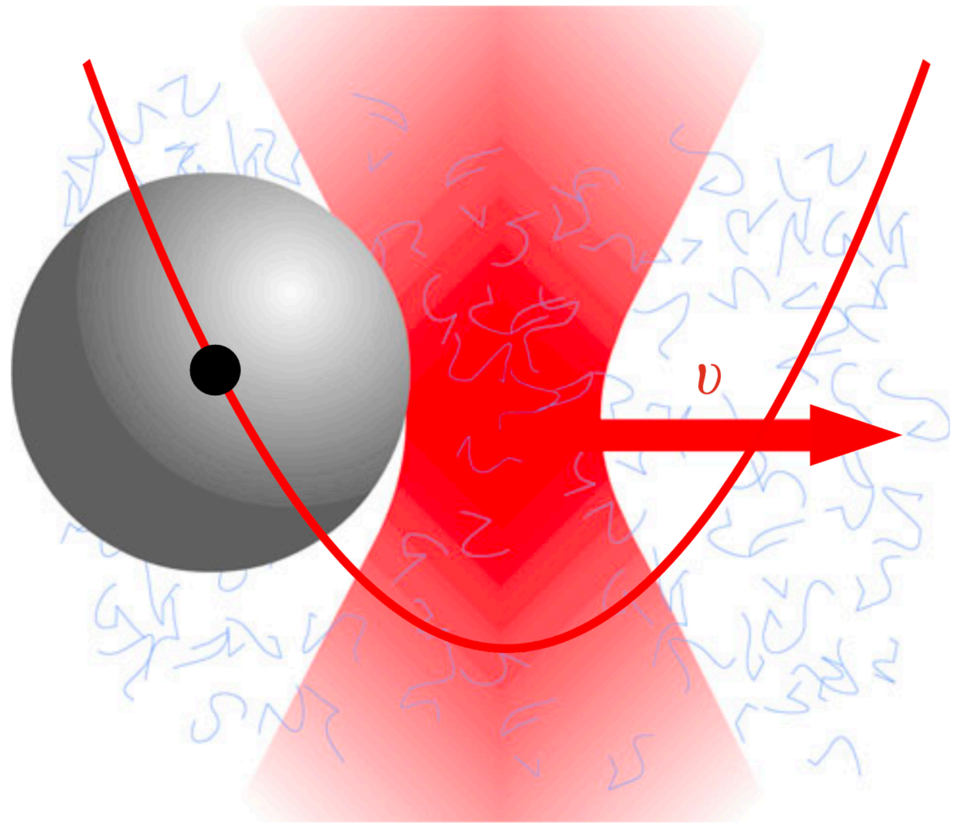
$$\partial_t P(R,t) \simeq -\partial_R \mu(R) P(R) + \partial_R^2 D(R) P(R)$$

BCs-dependent stationary distributions

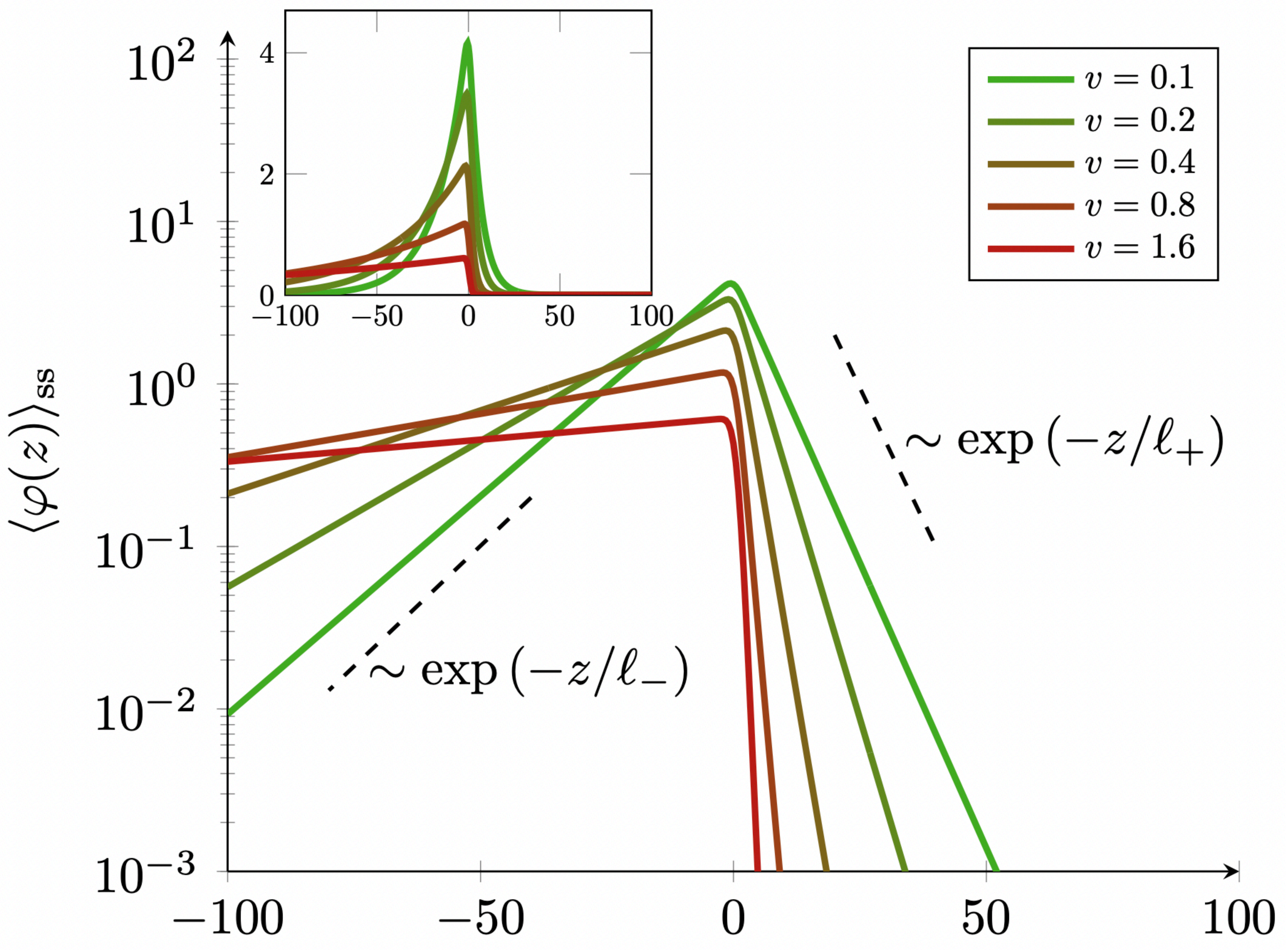


[DV, M. Gross, 2209.10834 (2022)]

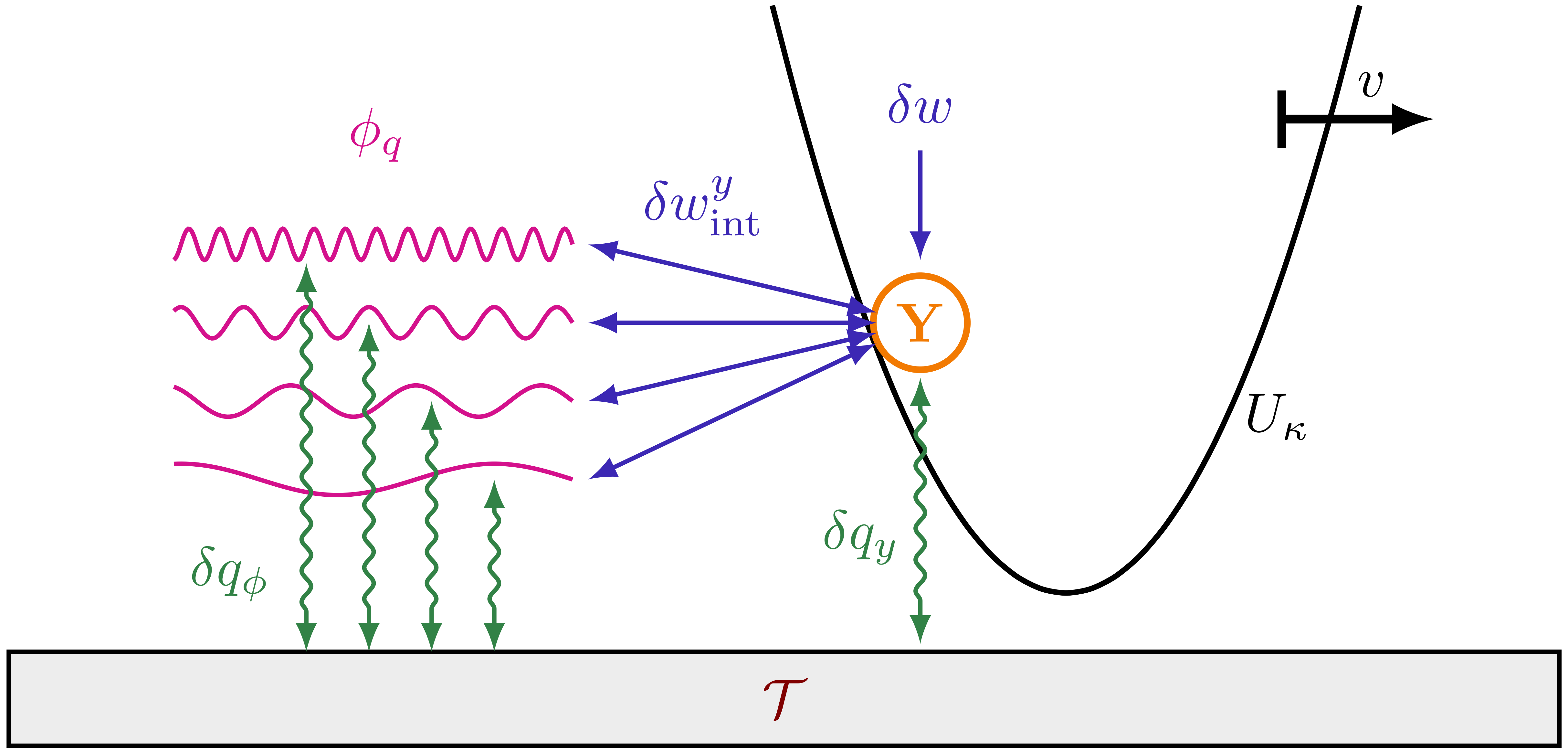
Stochastic thermodynamics



- Framework for field+particle **work/energy** flows
- Full CGF of the dissipated **power**

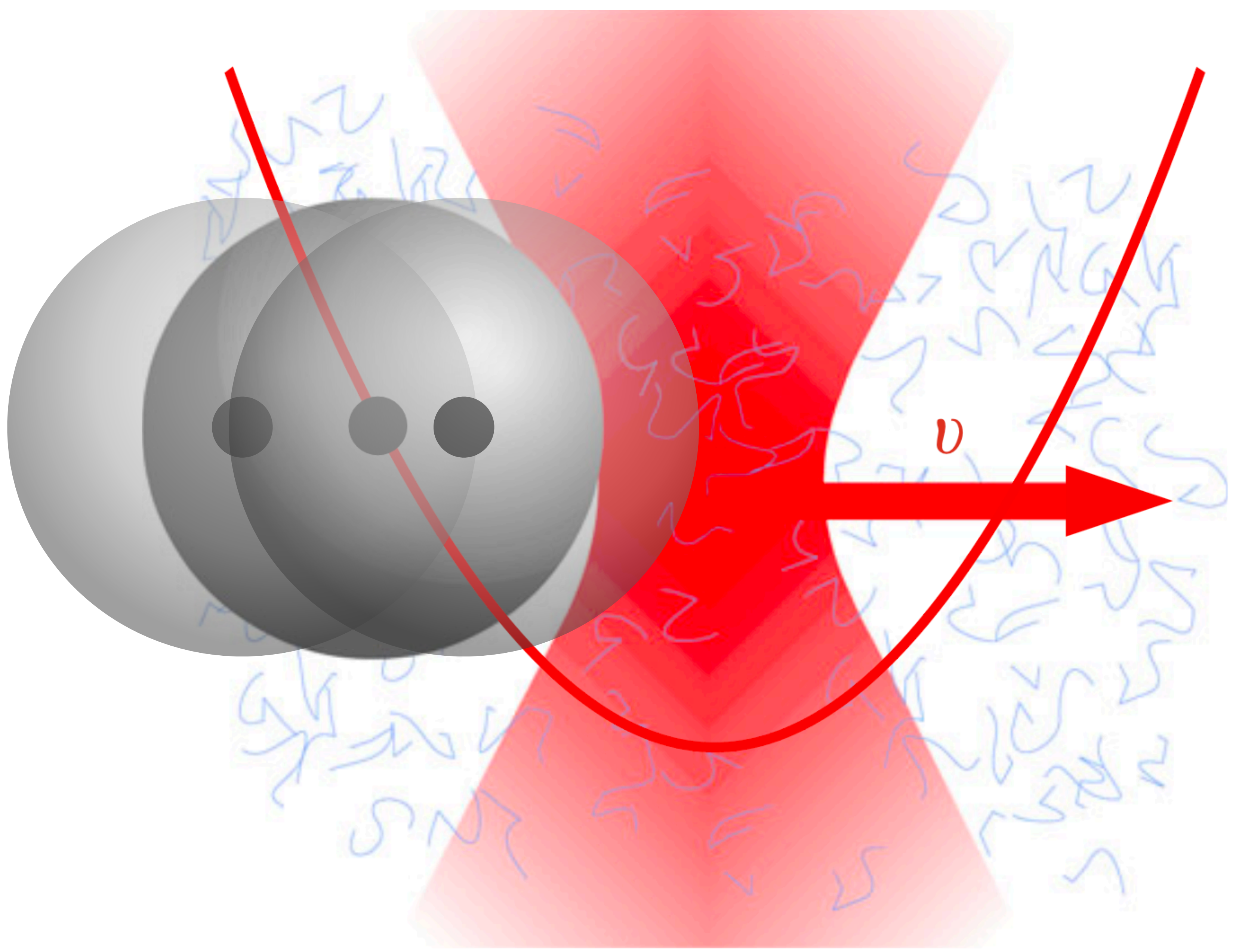


[DV, S.M.Loos, B.Walter, E.Roldan, A.Gambassi (in preparation)]

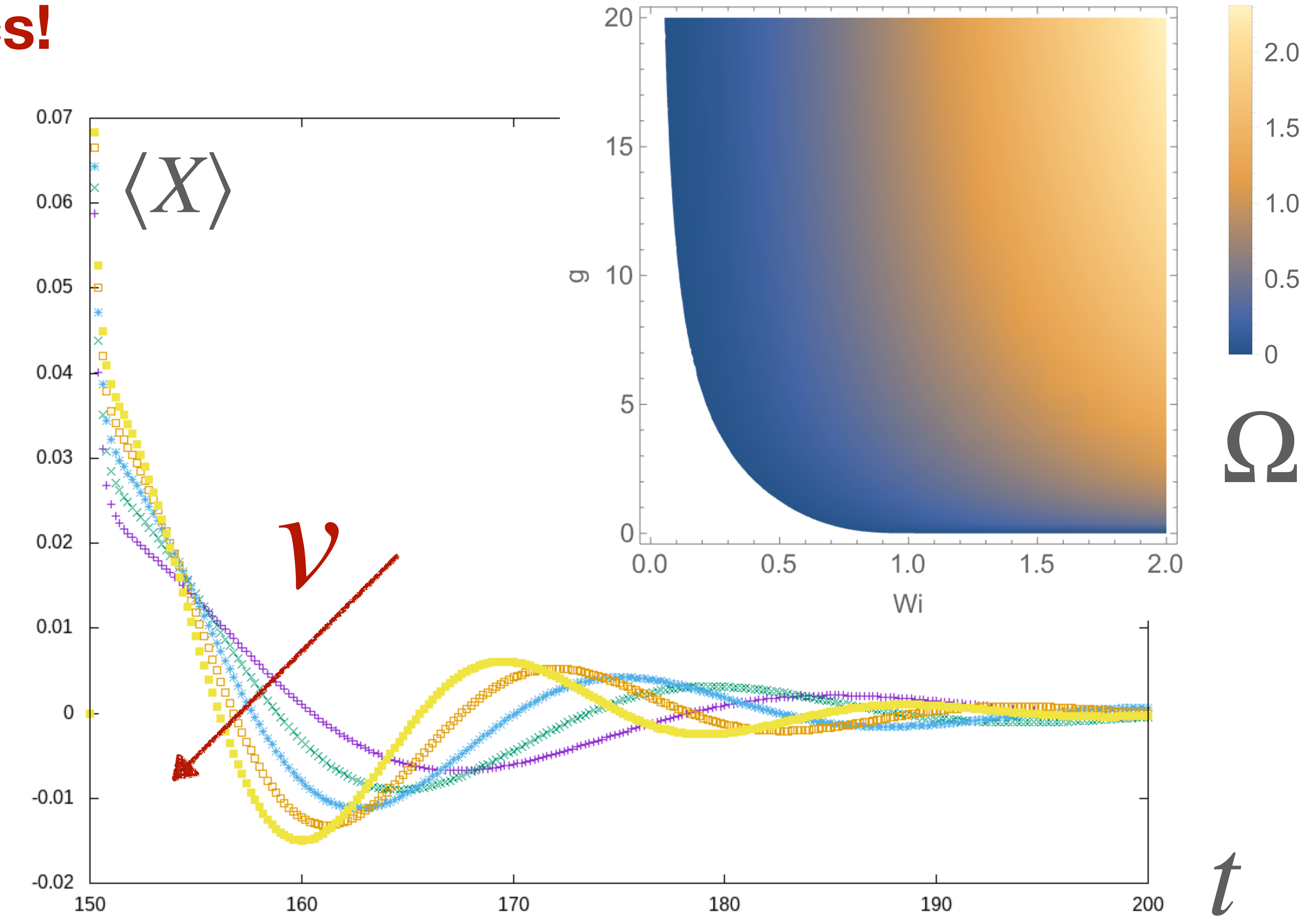


Memory-induced oscillations

in overdamped dynamics!



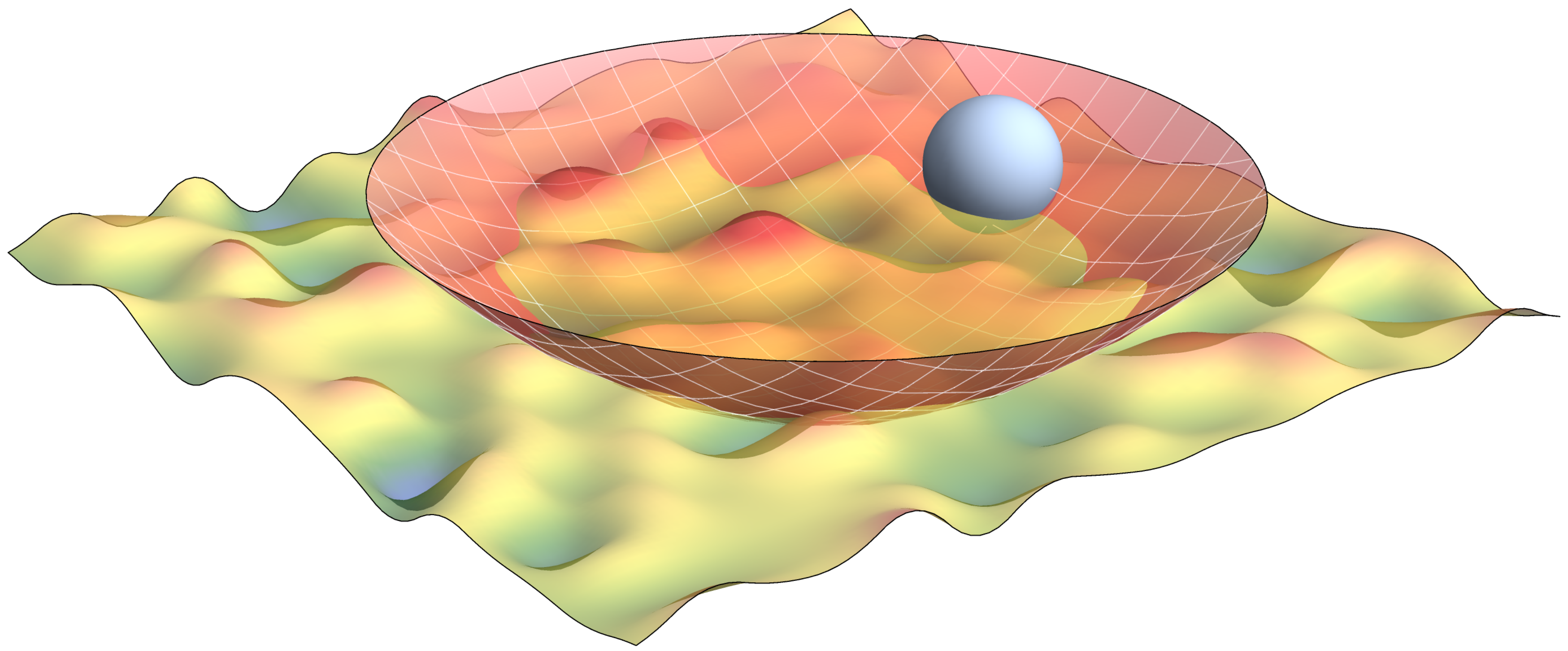
$\mathbf{Wi} = \tau_\phi/\tau_v$



[DV, A.Gambassi (in preparation)]

To sum up

- Relaxation to equilibrium (quench)
- 2-particles nonequilibrium periodic states
- Steady-state in confinement
- Stochastic thermodynamics in NESS
- Memory-induced oscillating modes



Perspectives

- Active field theories
- Self-interacting ϕ^4 field
- Hydrodynamics (model H)

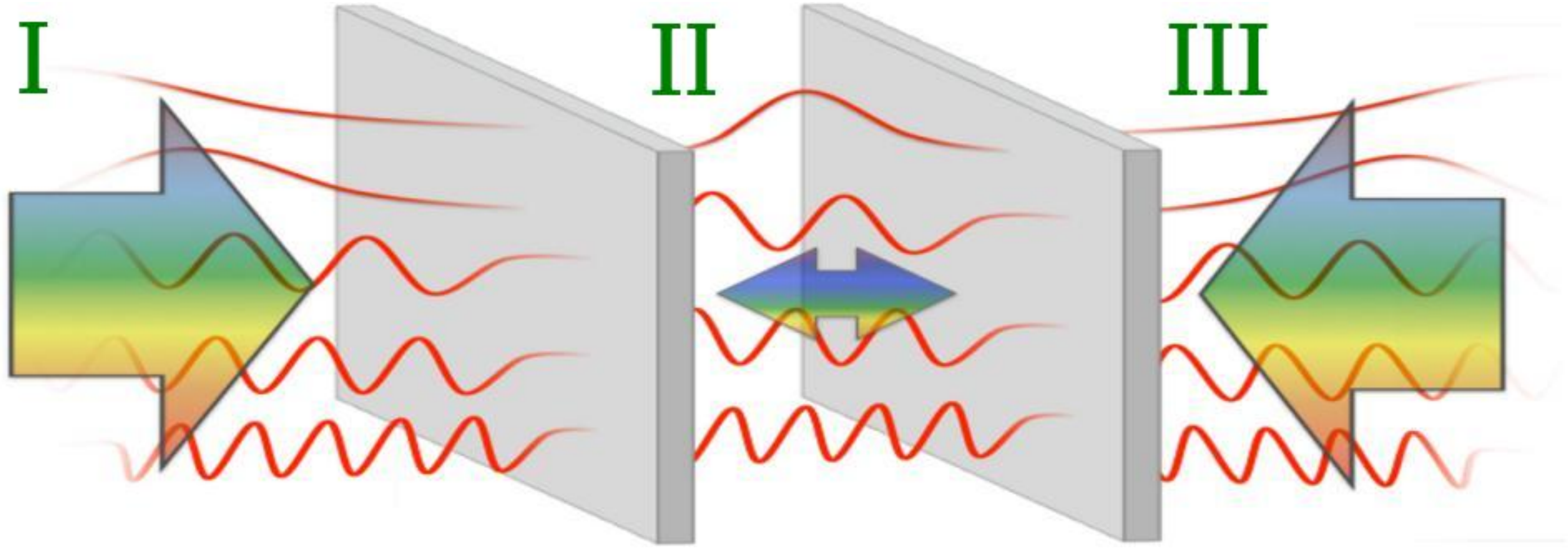
Thank you!

Backup slides

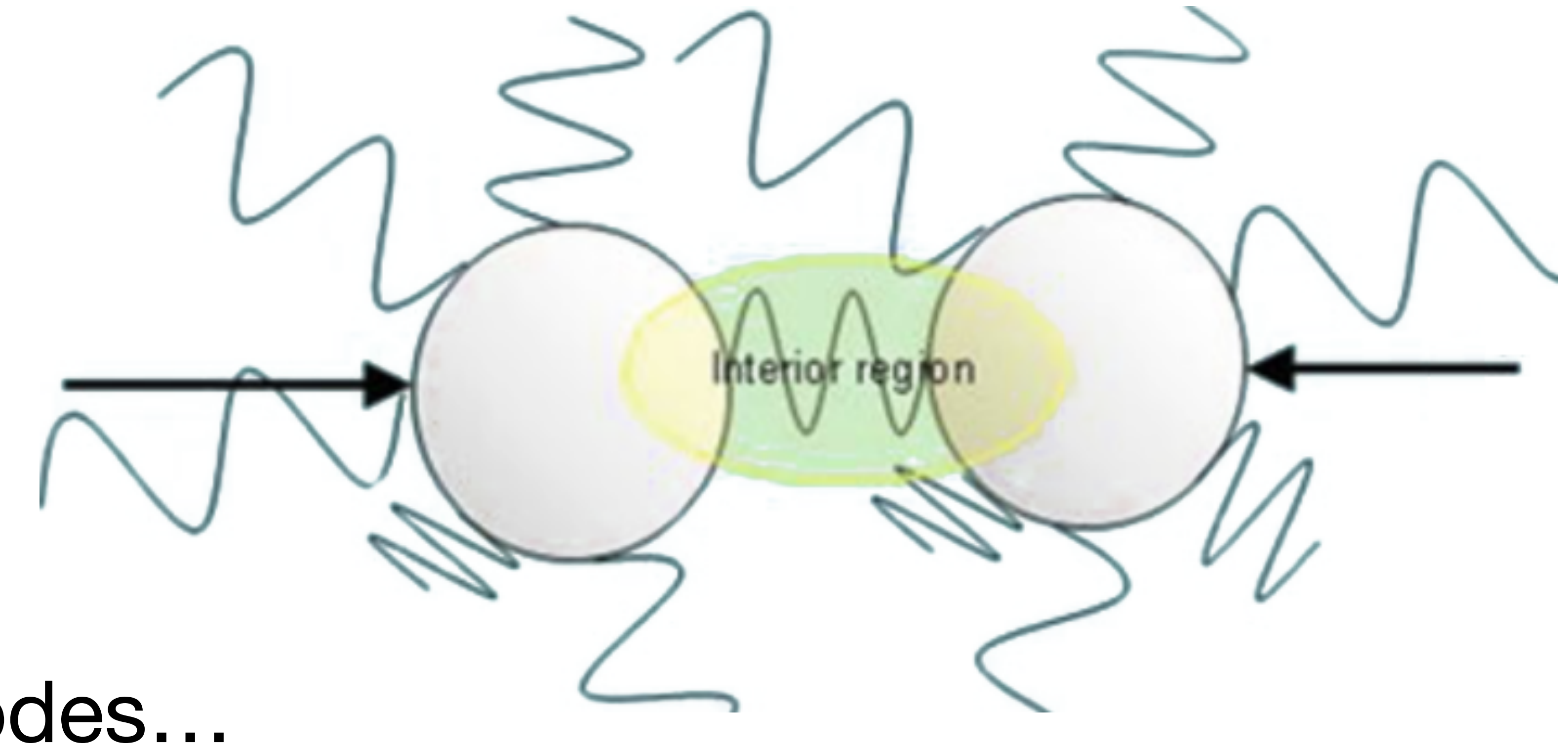
:)

Motivation 1/3

Fluctuation-induced forces

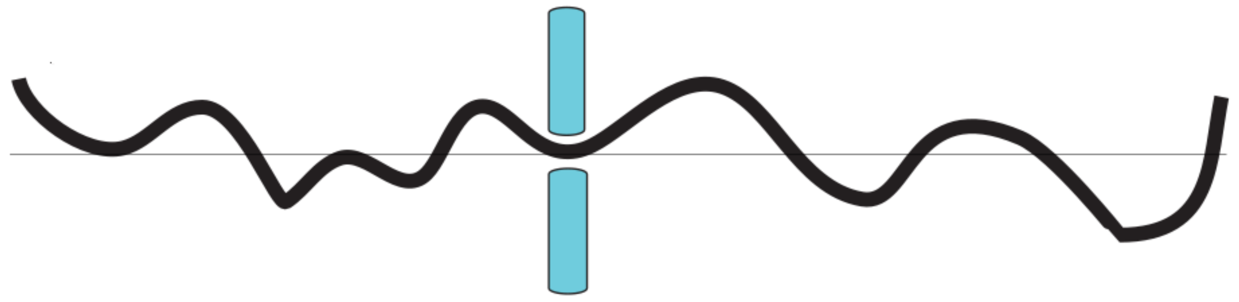


1. A fluctuating **medium**
   - QM: path weight $\sim \exp(i/\hbar \mathcal{S})$
   - StatPhys: weight $\sim \exp(-\beta \mathcal{U}(x))$
2. External **objects** affecting the fluctuations
   - Entropic or energetic origin
   - Examples: QED, CCF, Van Der Waals & dispersion forces, Goldstone modes…



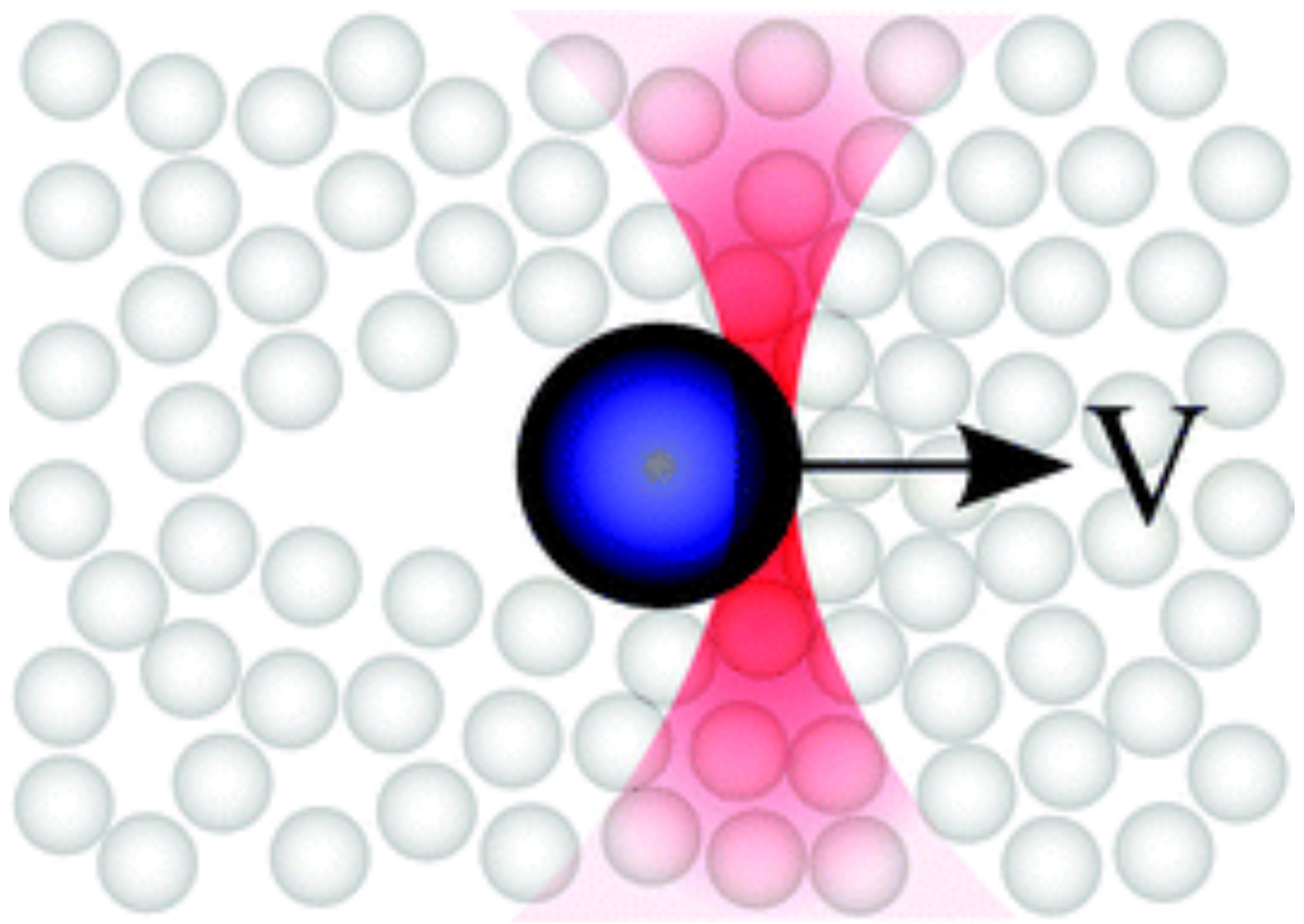
Strength $\propto$ energy of fluctuations $(\hbar, k_B T)$, range $\propto$ range of correlations.

→ What happens in the presence of randomly fluctuating surfaces?

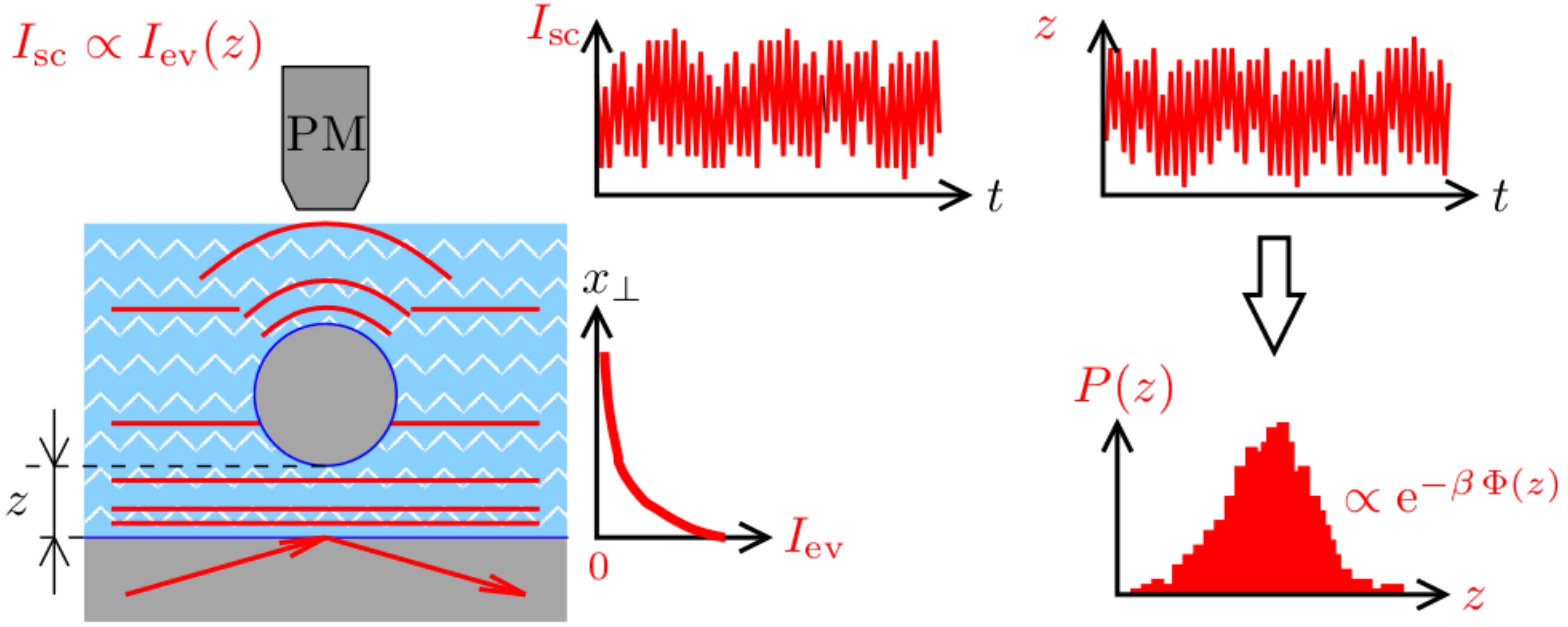


Motivation 2/3

Particle as a probe



- Thermal fluctuations, small forces $(\sim 10^{-7} N)$
  → can affect the motion of colloids!
  → Infer properties of soft-matter materials (microrheology)
- Back-reaction of the particle on the medium
- How does a particle behave in a medium close to a phase transition?



Motivation 3/3

From Brownian motion to non-linear memory

- **Brownian motion**
$$m\,\ddot{x}(t) \;=\; -\gamma\,\dot{x}(t) \;+\; \zeta(t) - V'(x(t))$$
$$\langle \zeta(t)\zeta(t')\rangle = 2\,k_{\mathrm{B}}T\,\gamma\,\delta(t-t')$$

- **GLE**
$$m\,\ddot{x}(t) \;=\; -\int^{t} \mathrm{d}t'\,\Gamma(t-t')\,\dot{x}(t') \;+\; \zeta(t) - V'(x(t))$$
$$\langle \zeta(t)\zeta(t')\rangle = k_{\mathrm{B}}T\,\Gamma(|t-t'|)$$

[e.g. Caldeira&Leggett ’83]

meaningful if $\Gamma(t)$ is independent of the details of $V(x)$
- not always true!

[Daldrop et al., PRX 2017]

[Müller et al., New J. Phys. 2020]

Motivation 3/3

From Brownian motion to non-linear memory

$$m\ddot{X}(t) \;=\; -\gamma_\infty \dot{X}(t) \;+\; \nabla_X \mathcal{H}(t) \;+\; \zeta(t)$$

[Basu, Dèmery, Gambassi ’22]

$$-\kappa X(t) + \int_{-\infty}^{t} \mathrm{d}t'\, F(t-t', X(t)-X(t')) + \Xi(X(t),t) \;+\; \mathcal{O}(\lambda^3)$$

$\propto \dot{X}$ → *non-linear friction!*

$$F_l(t,x) = i\lambda^2 D \int \frac{\mathrm{d}^d q}{(2\pi)^d}\, q_l\, q^2 |U_q|^2\, \mathrm{e}^{iq\cdot x - Dq^2(q^2+r)t}$$

$$\langle \Xi(x,t)\Xi(x',t')\rangle = T \times \left[2\gamma_\infty\, \delta(t-t') \;+ G(x-x', t-t') \right]$$

FDT →

$$\partial_x F(t,x) = -\partial_t G(t,x)$$

Motivation 3/3

From Brownian motion to non-linear memory

[Basu, Dèmery, Gambassi '22]

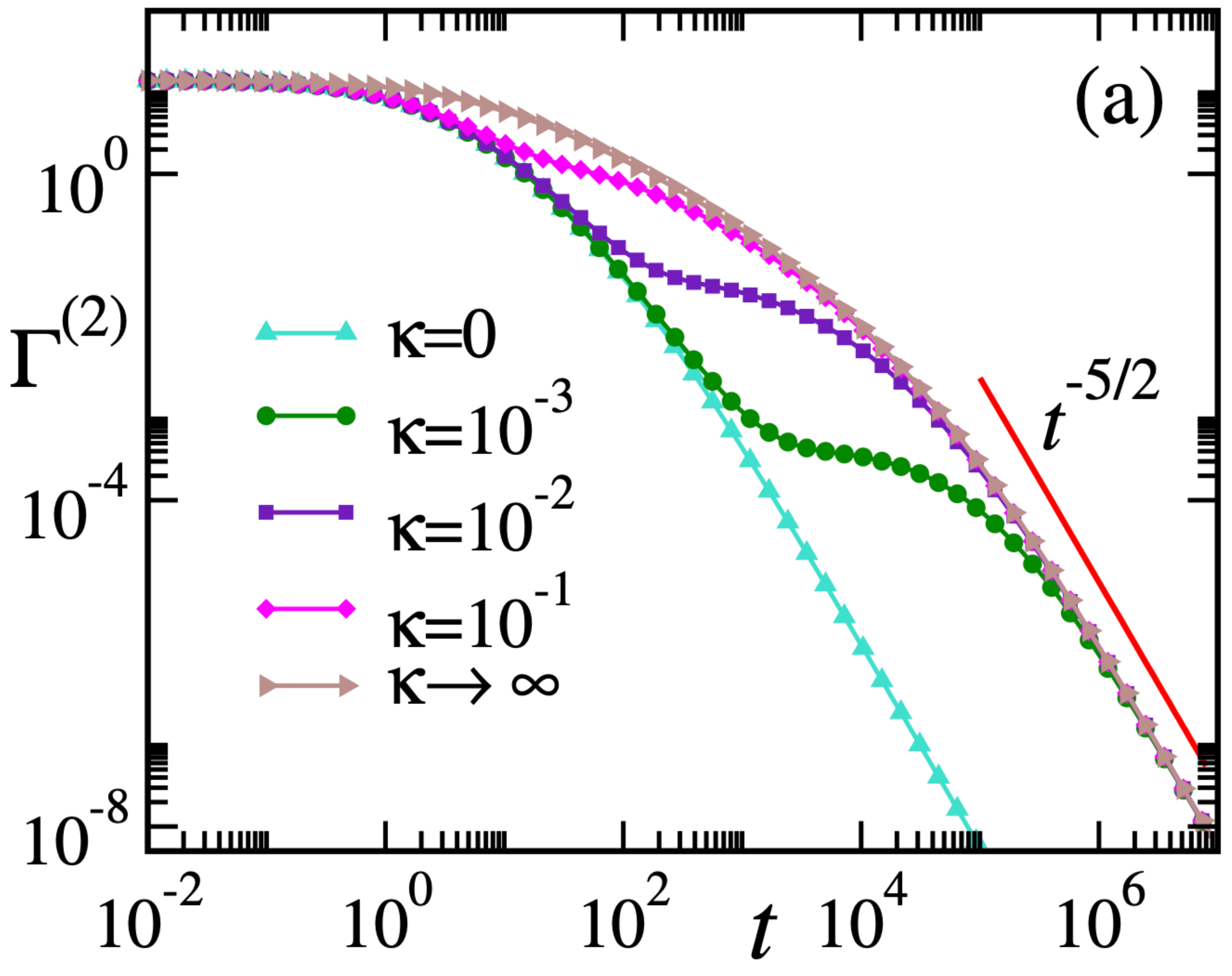
$$\int_{-\infty}^{t} \mathrm{d}t' \, \Gamma(t-t')\dot{X}_j(t') = -\kappa X(t) + \zeta(t)$$

↕

$$\hat{C}(p) = \frac{dT\hat{\Gamma}(p)}{\kappa[\kappa + p\hat{\Gamma}(p)]}$$

↕

$$\hat{\Gamma}(p) = \frac{\kappa\hat{C}(p)}{dT/\kappa - p\hat{C}(p)}$$



Effective particle dynamics

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Two possible approximations:

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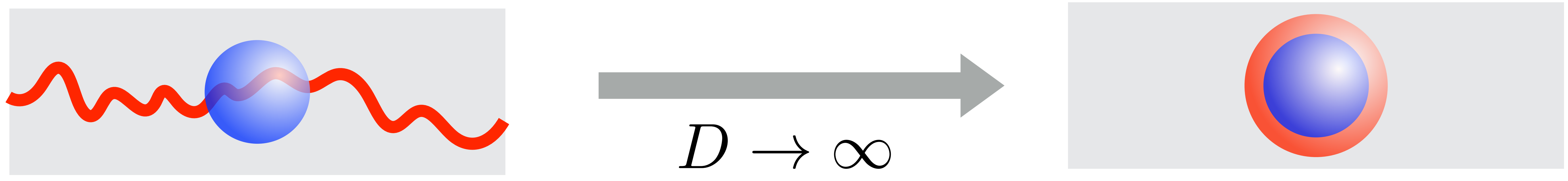
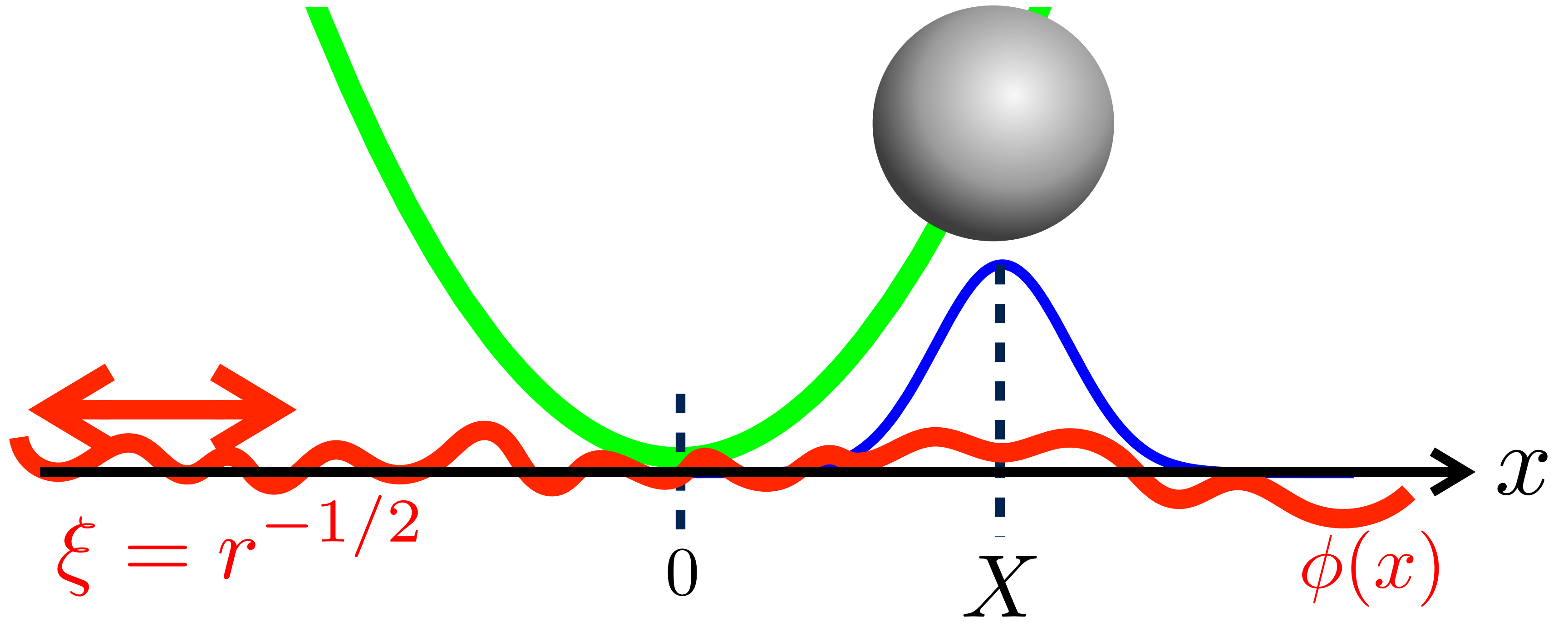
2. Adiabatic approximation



[Kaneko, ’61; Theiss, Titulauer ’85; …. Gross ’21]

Relaxation towards equilibrium

Adiabatic approximation



From Langevin equations

$$\dot{\mathbf{X}}(t) = -\nu k \mathbf{X}(t) + \nu\lambda \int_{\mathbf{R}} \frac{\mathrm{d}^d q}{(2\pi)^d} i\mathbf{q} V_{-q}\phi_q(t) e^{i\mathbf{q}\cdot\mathbf{X}(t)} + \boldsymbol{\xi}(t)$$

$$\dot{\phi}_q^{R,I}(t) = -Dq^\alpha(q^2+r)\phi_q^{R,I}(t) + \lambda D q^\alpha V_q \underbrace{\left[e^{-i\mathbf{q}\cdot\mathbf{X}(t)}\right]^{R,I}}_{g_q} + \zeta_q^{R,I}(t)$$

to the Fokker-Planck equation

$$\partial_t \mathcal{P} = \left\{ \mathcal{L}_X + \sum_{\sigma=R,I} \int_{R^d} \mathrm{d}^d q \, \mathcal{L}_q^\sigma \right\} \mathcal{P}$$

$$\mathcal{L}_X = \nabla \cdot \left\{ \nu k X - \nu\lambda \int_R \mathrm{d}^d q \, q \left(\phi_q^R g_q^I - \phi_q^I g_q^R \right) \right\} + \Gamma_x \nabla^2 ,$$

$$\mathcal{L}_q^\sigma = \frac{\delta}{\delta \phi_q^\sigma} \left\{ \alpha_q \phi_q^\sigma - D\lambda q^\alpha g_q^\sigma(X) \right\} + \frac{\Gamma_\phi}{2} \frac{\delta^2}{\delta(\phi_q^\sigma)^2} .$$

We want to marginalize over the eigenfunctions

$$\mathcal{P}[\phi, X, t] = \sum_n P_n(X,t) \Phi_n[\phi; X] \quad \rightarrow \quad P_0(X,t) = \int \mathcal{D}\phi \mathcal{P}[\phi, X, t]$$

Relaxation towards equilibrium

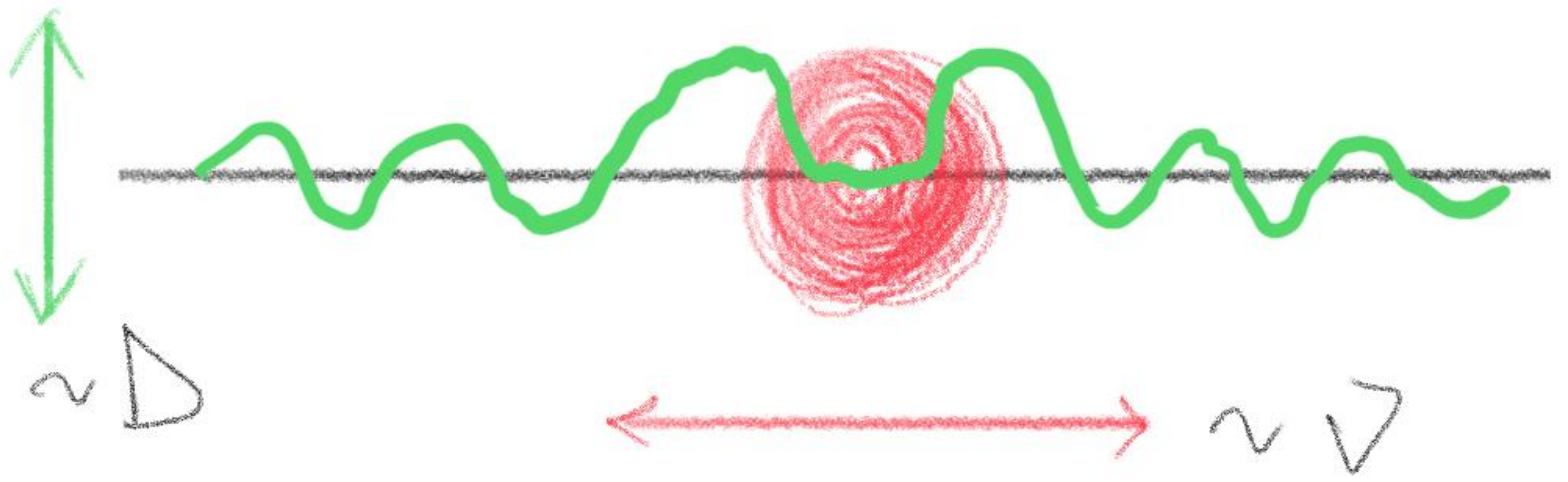
Adiabatic approximation

Effective F-P equation:

$$\partial_t P(\mathbf{X},t) = \mathcal{L}_X^{\text{eff}} P(\mathbf{X},t)$$

$$\mathcal{L}_X^{\text{eff}} = \chi \left[\boldsymbol{\nabla} \cdot (\nu k \mathbf{X}) + \nu T \nabla^2 \right] + \mathcal{O}\left(\frac{1}{D^2}\right)$$

$$\chi \equiv 1 - \frac{\lambda^2 \nu}{Dd} \int_{\mathbf{R}} \frac{\mathrm{d}^d q}{(2\pi)^d} \frac{q^{2-\alpha}}{(q^2+r)^2} |V_q|^2$$



Solutions decay to X=0 exponentially

[DV, Ferraro, Gambassi '22]

Relaxation towards equilibrium

Weak-coupling approximation

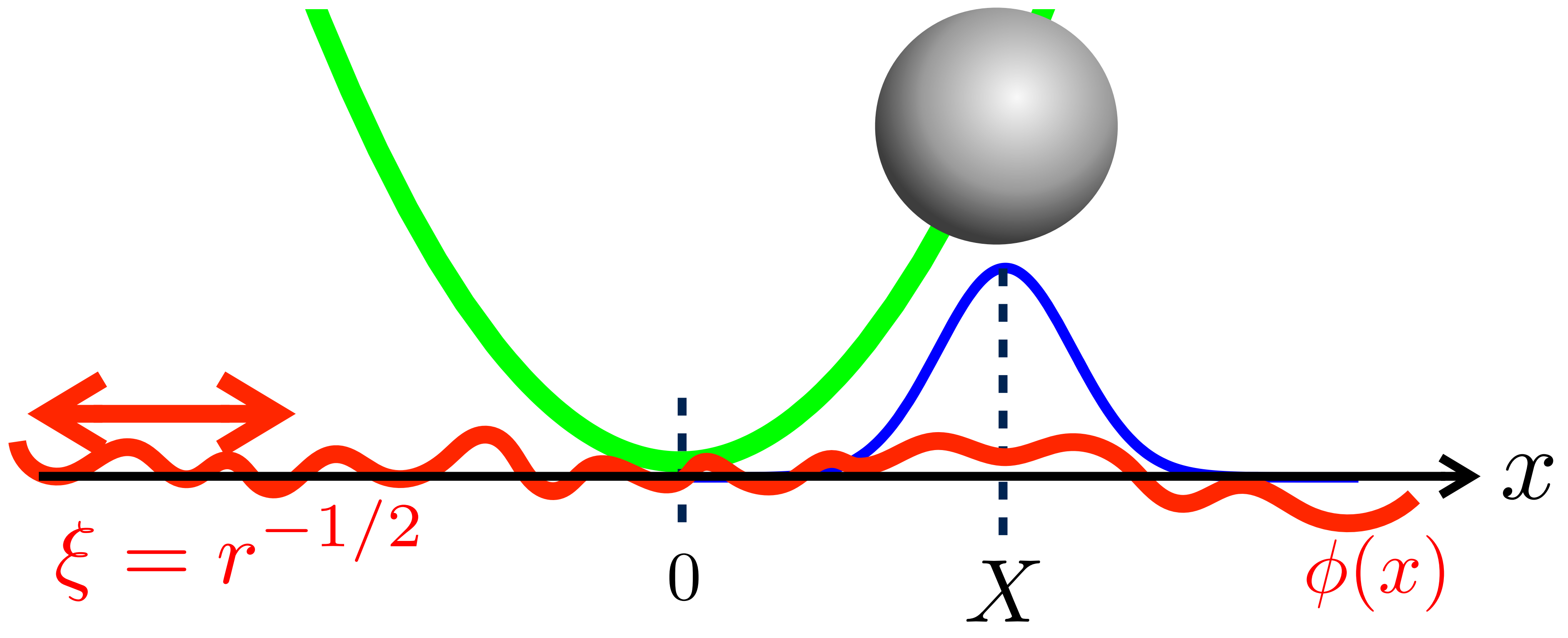
$$X(t) = X^{(0)}(t) + \lambda^2 X^{(2)}(t) + \mathcal{O}(\lambda^4)$$

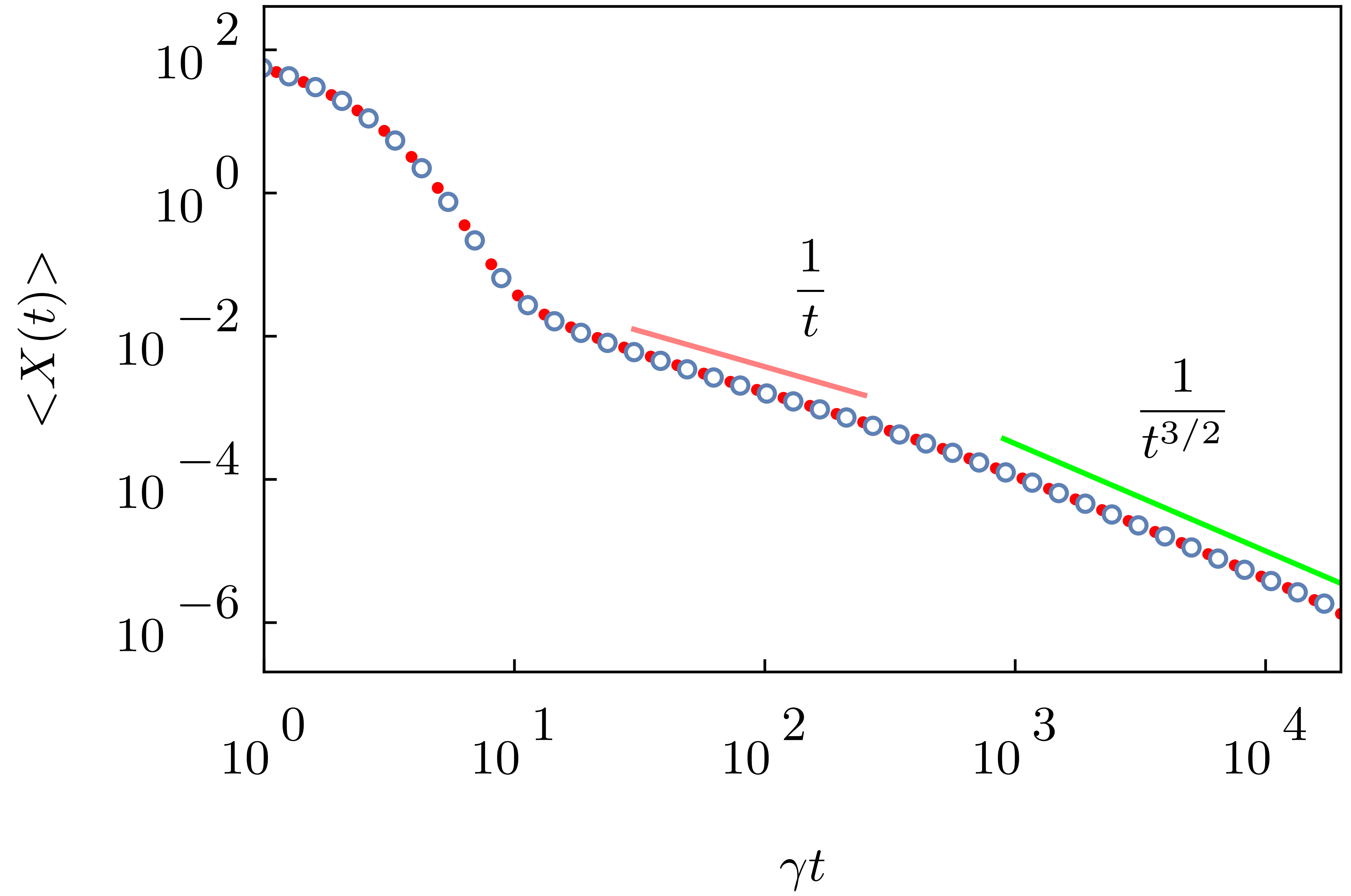
@ long times,

$$\langle X^{(2)}(t)\rangle \sim \begin{cases} t^{-\left(1+\frac{d}{2}\right)}, & \text{Model A, } r=0 \\ t^{-\left(1+\frac{d}{4}\right)}, & \text{Model B, } r=0 \\ t^{-\left(2+\frac{d}{2}\right)}, & \text{Model B, } r>0 \end{cases}$$

A matter of timescales:

$$\tau_X^{-1} = \nu k$$
$$\tau_\phi^{-1} = D q^\alpha (q^2 + r)$$





Relaxation towards equilibrium

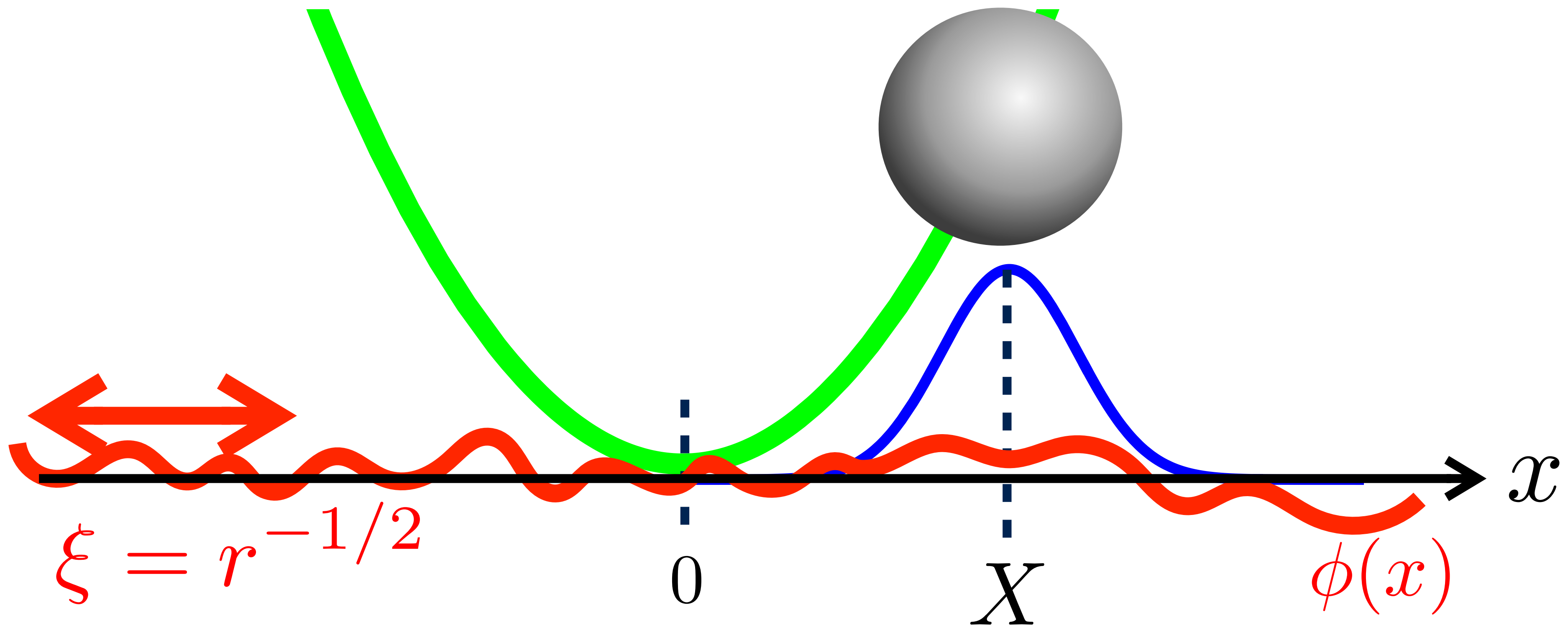
Weak-coupling approximation

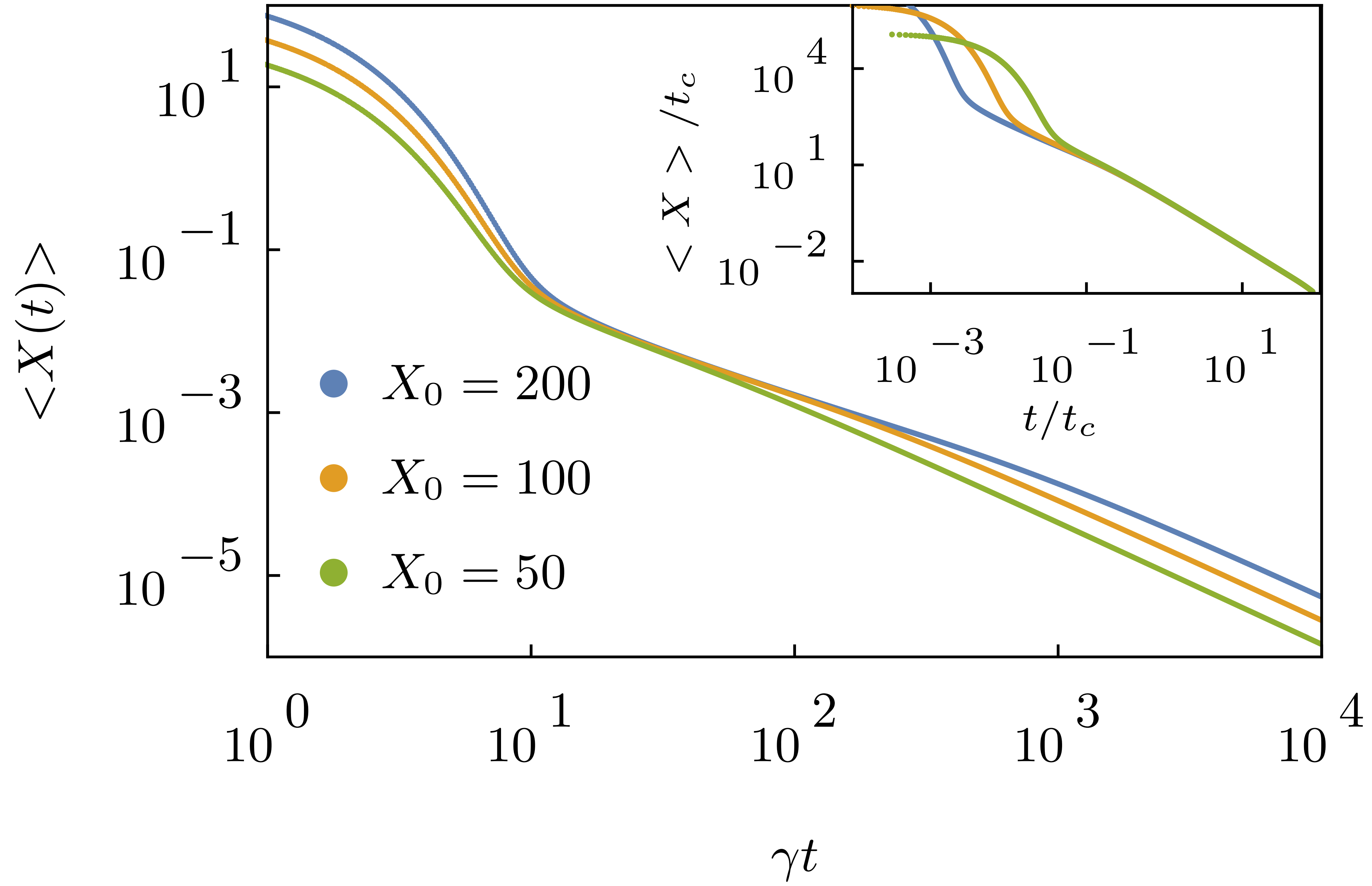
For $t > \tau_X$,

$$\langle X(t) \rangle \simeq c_0 t^{-\alpha_0} f\left(t/t_c\right)$$

$$f(\tau) \sim \begin{cases} \tau^{-\beta_0} & \text{for } \tau \gg 1 \\ \text{const.} & \text{for } \tau \lesssim 1 \end{cases}$$

$$t_c = \tau_\phi^{-1}(q \sim 1/X_0)$$





Before the crossover, the amplitude is X_0 - independent!

Autocorrelation function

Weak-coupling approximation

$$C(t) \equiv \langle X(0)X(t)\rangle = C_0(t) + \lambda^2 C_2(t) + \mathcal{O}(\lambda^4)$$

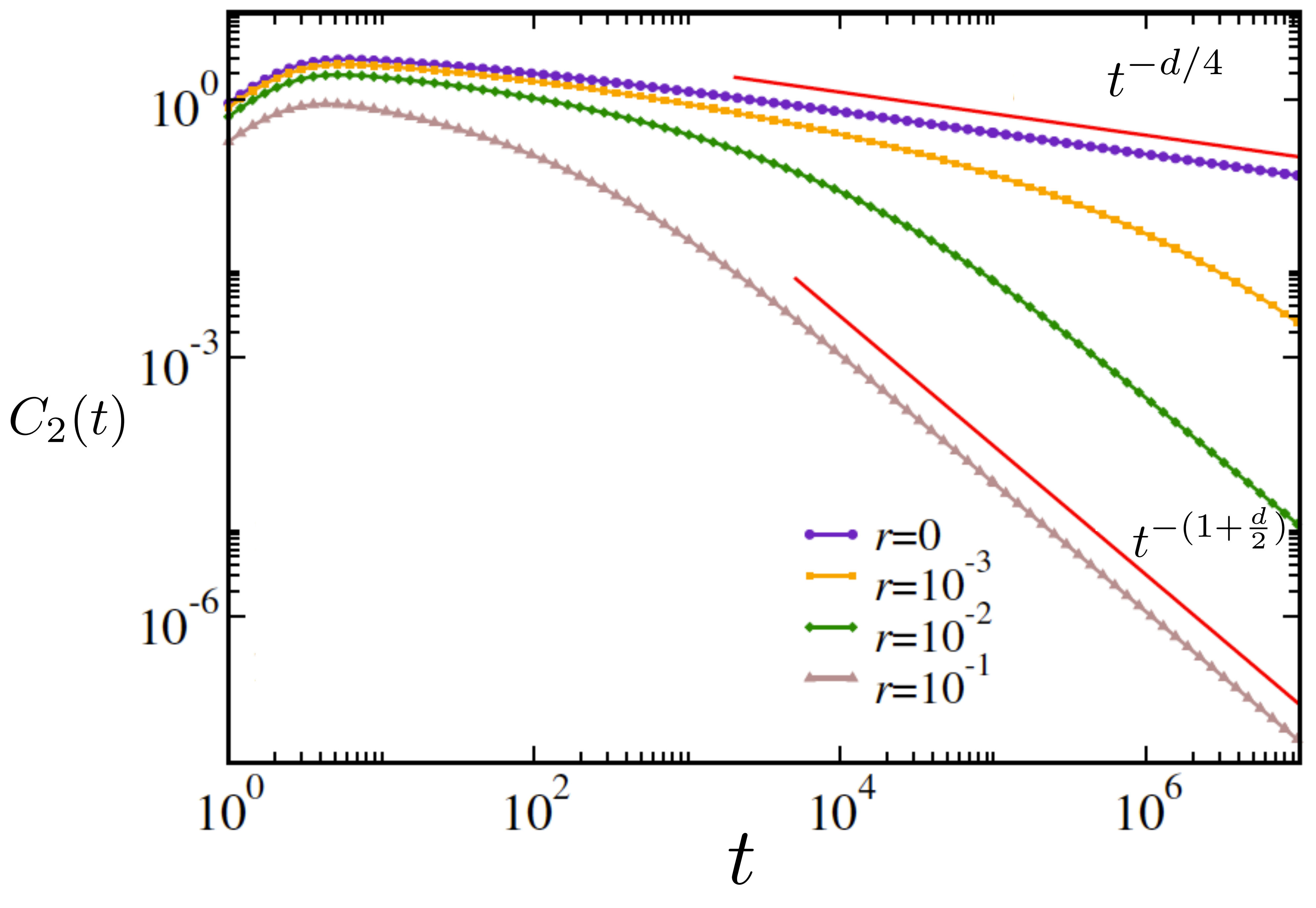
@ long times t,

$$C(t) \sim \begin{cases} t^{-\frac{d}{2}}, & \text{Model A}, \ r = 0 \\ t^{-\frac{d}{4}}, & \text{Model B}, \ r = 0 \\ t^{-\left(1+\frac{d}{2}\right)}, & \text{Model B}, \ r > 0 \end{cases}$$

Connection comes from FDT,

$$\langle X(t)\rangle = X_0 R(t) \longrightarrow R(t>0) = -\frac{1}{k_B T}\frac{\mathrm{d}C(t)}{\mathrm{d}t}$$

$d = 1$

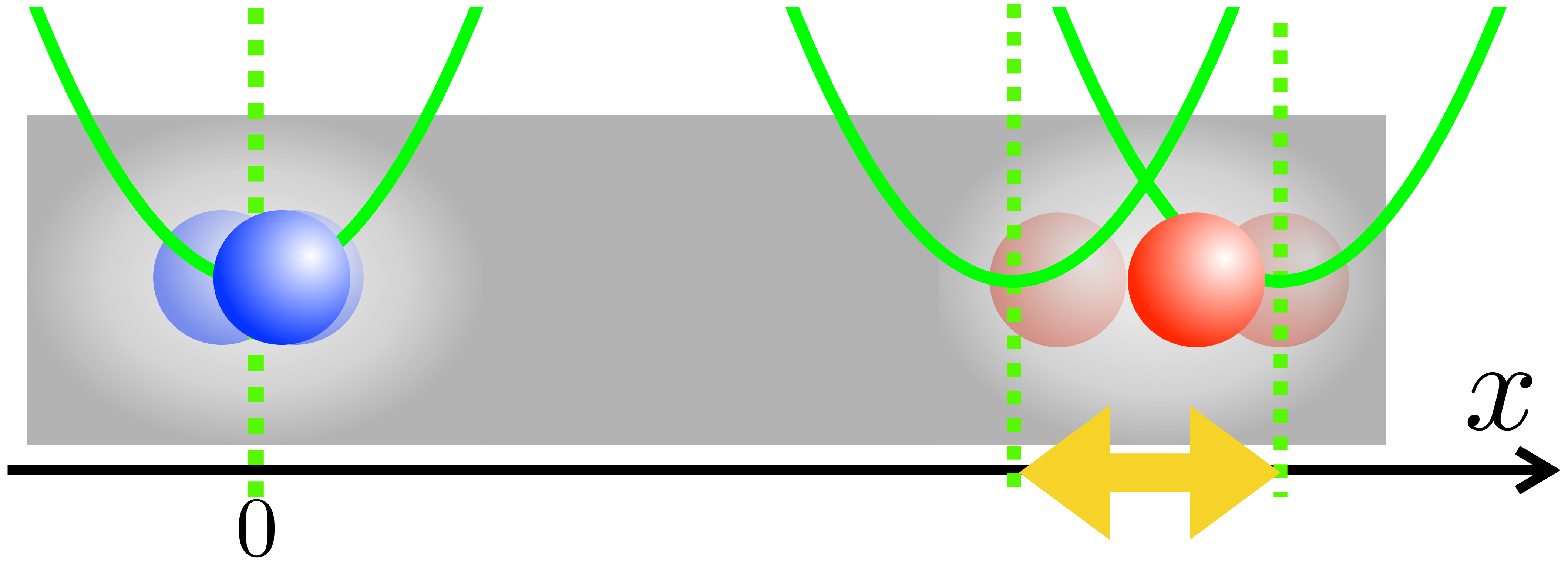


[Basu, Dèmery, Gambassi '22]

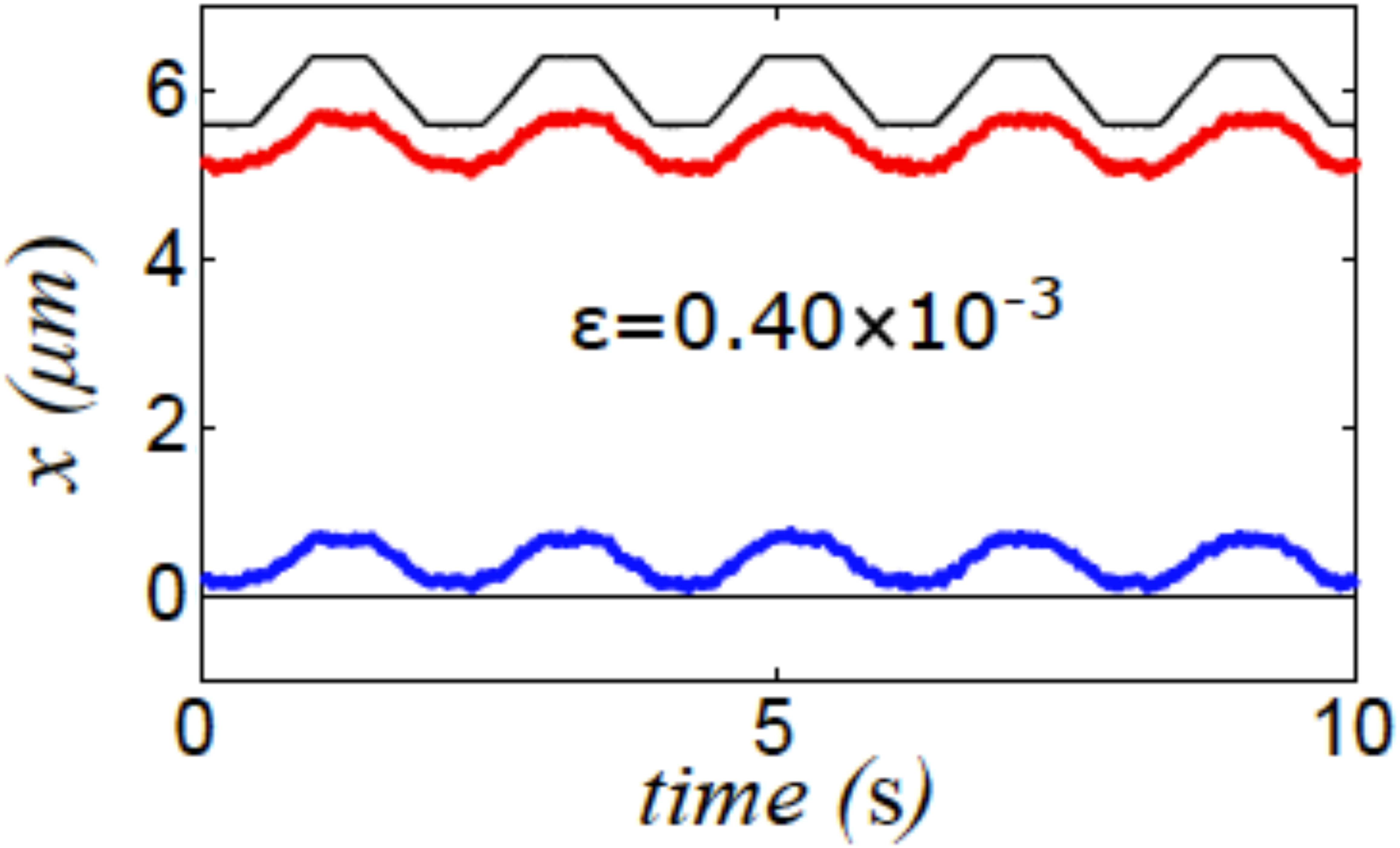
[DV, Ferraro, Gambassi '22]

Energy Transfer between Colloids via Critical Interactions

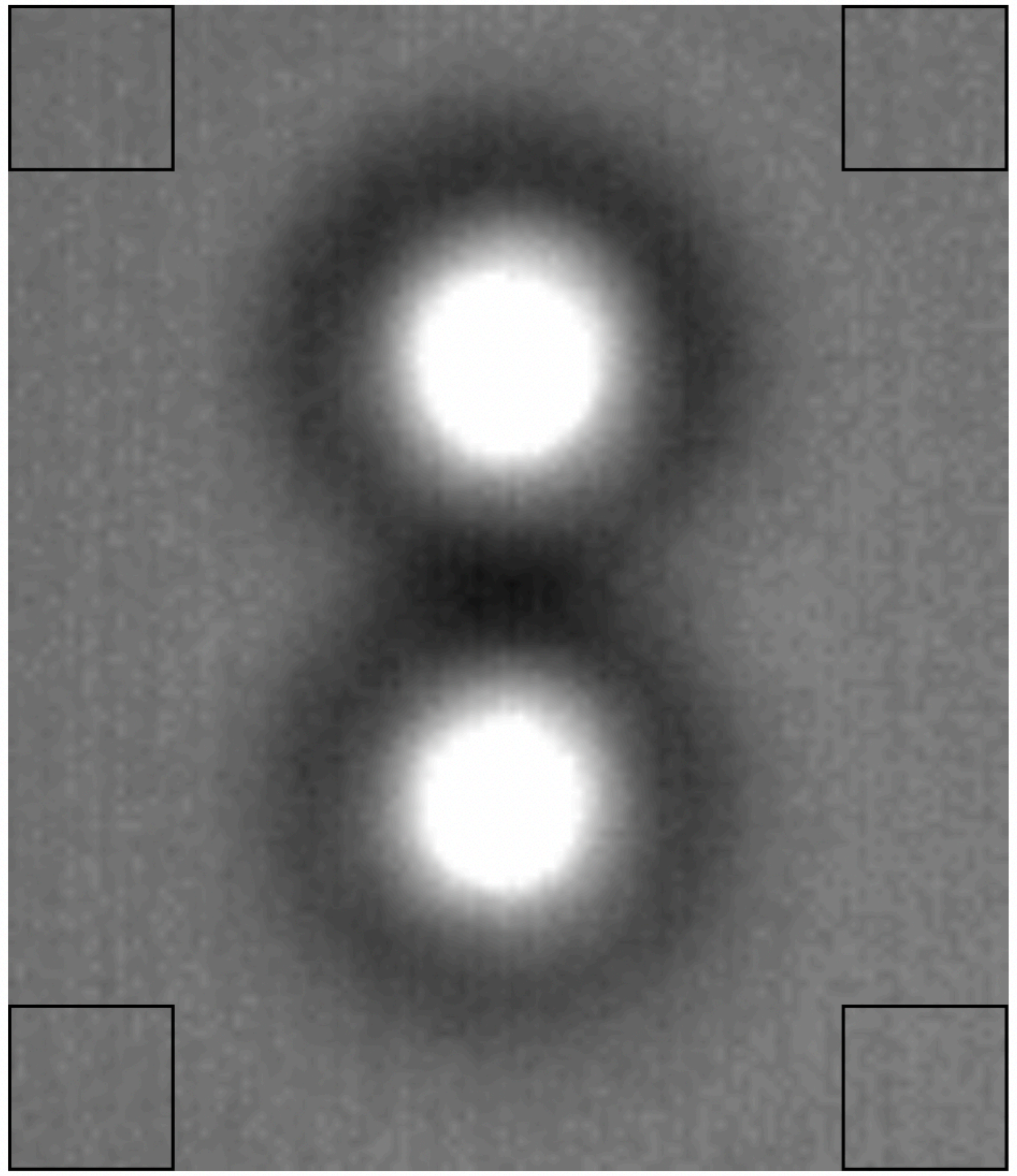
Ignacio A. Martínez [1,2,*], Clemence Devailly [1,3], Artyom Petrosyan [1] and Sergio Ciliberto [1,*]



cartoon not in scale



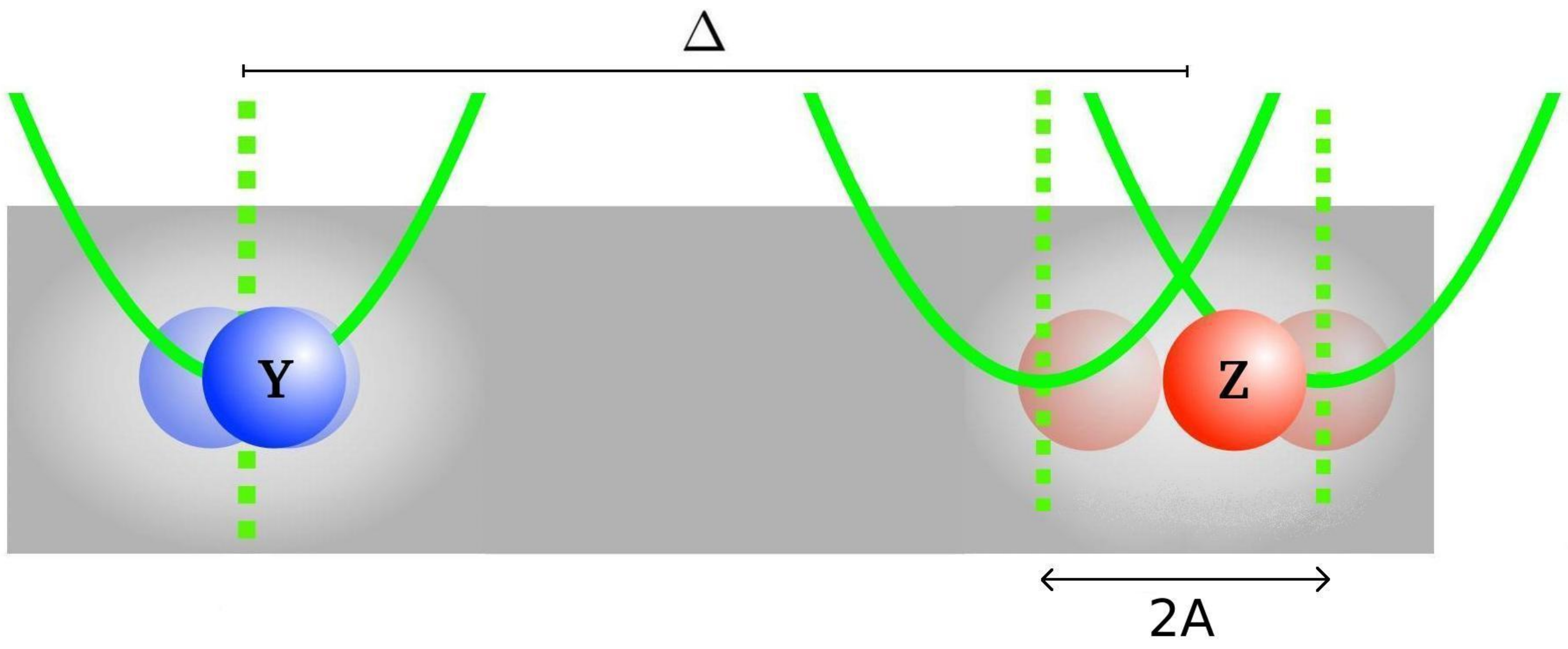
$\varepsilon = (T_c - T)/T_c$



[2017]

Two particles

Model



Two particles independently interact with the field,

$$\mathcal{H} = \mathcal{H}_\phi + \mathcal{U}_Y + \mathcal{U}_Z - \lambda \left(\mathcal{H}^Y_{\text{int}} + \mathcal{H}^Z_{\text{int}} \right)$$

$$\int \mathrm{d}^d \mathbf{x}\, \phi(\mathbf{x}) \left[V^{(z)}(\mathbf{x} - \mathbf{Z}) + V^{(y)}(\mathbf{x} - \mathbf{Y}) \right]$$

One of them is **driven** periodically,

$$\mathcal{U}_Z = \frac{k_z}{2} \left[\mathbf{Z} - \mathbf{Z}_F(t) \right]^2$$

$$\mathbf{Z}_F(t) = \boldsymbol{\Delta} + \mathbf{A} \sin(\Omega t)$$

→ how does $Y(t)$ respond?

Two particles

Adiabatic approximation

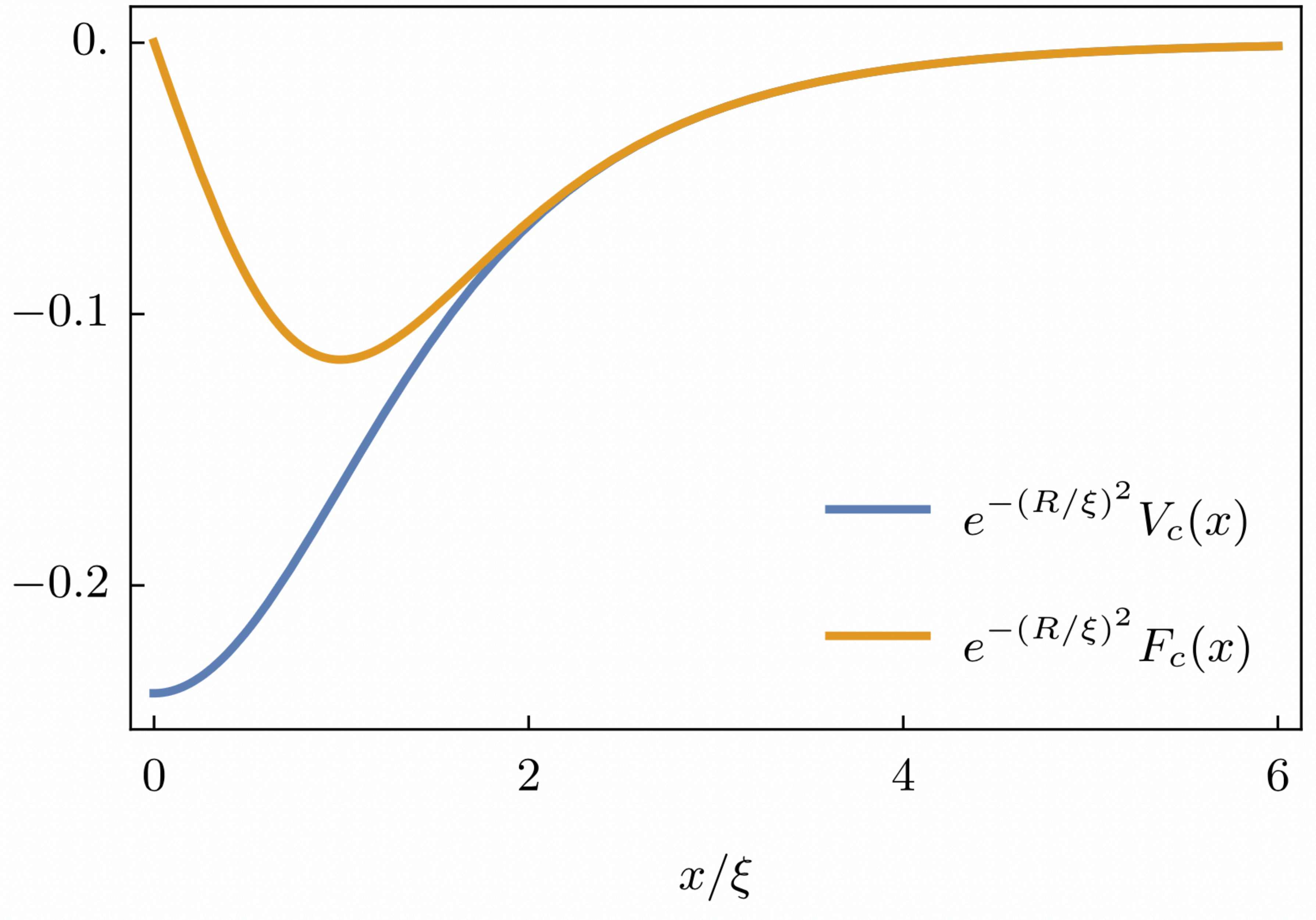
@ equilibrium, the two particles interact via

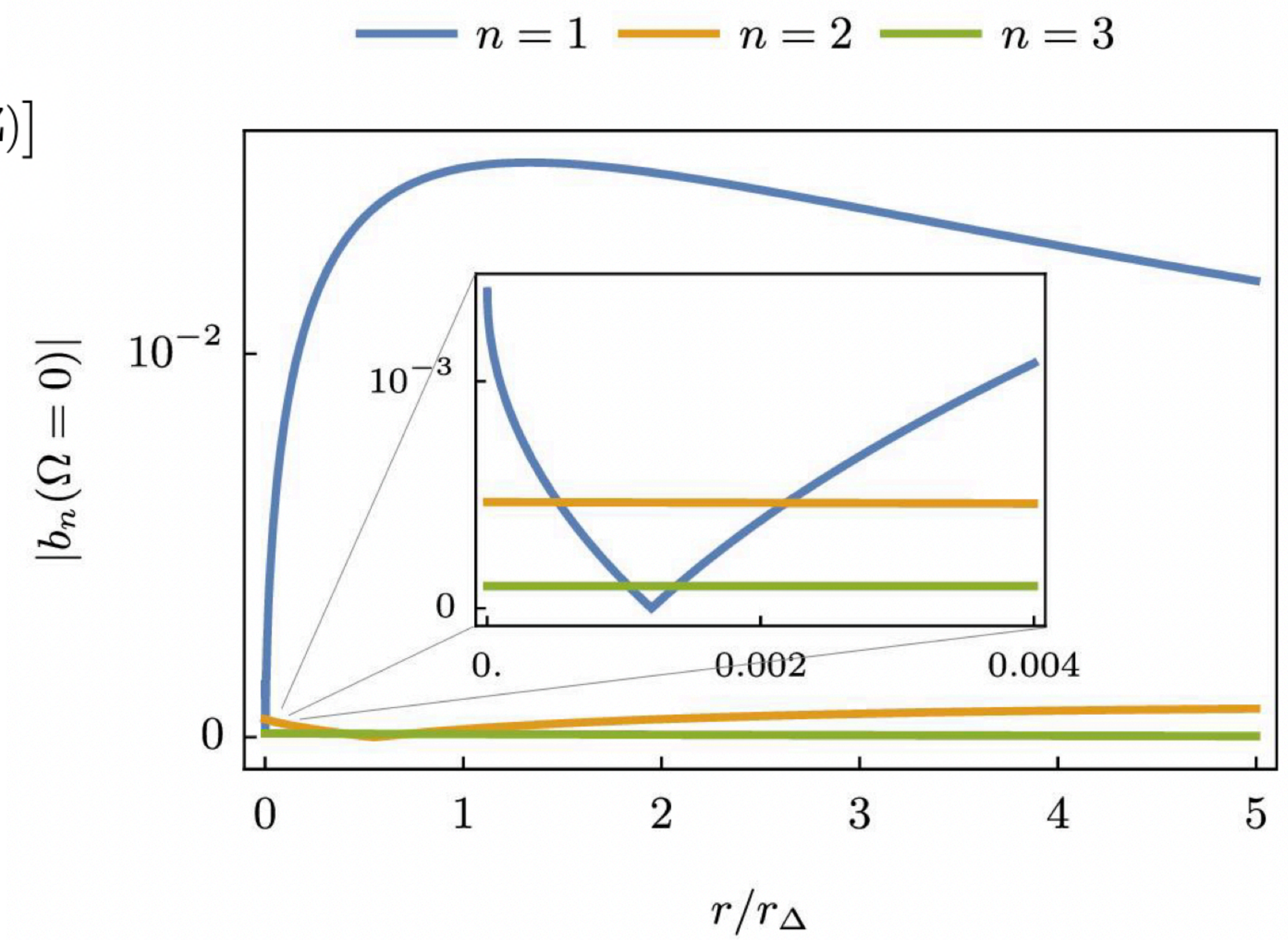
$$V_c(\mathbf{Y},\mathbf{Z}) = -\int \frac{\mathrm{d}^d q}{(2\pi)^d} \frac{|V_q|^2}{q^2+r} e^{i\mathbf{q}\cdot(\mathbf{Z}-\mathbf{Y})}$$

$$\mathcal{P}(\mathbf{Y},\mathbf{Z}) \propto e^{-\beta(\mathcal{U}_y+\mathcal{U}_z)} \int \mathcal{D}\phi\, e^{-\beta(\mathcal{H}_\phi - \lambda \mathcal{H}_{\text{int}})} \propto e^{-\beta[\mathcal{U}_Y+\mathcal{U}_Z+\lambda^2 V_c(\mathbf{Y},\mathbf{Z})]}$$

This gives an effective, overdamped Langevin dynamics (non-linear, Markovian)

$$\dot{\mathbf{Y}}_{\text{ad}}(t) = -\nu_y \nabla_y \left[\mathcal{U}_y(\mathbf{Y}) + \lambda^2 V_c(\mathbf{Y},\mathbf{Z})\right] + \boldsymbol{\xi}(t)$$





Two particles

Weak-coupling approximation

Look for an effective **master equation** ruling $P_1(\mathbf{y},t) = \langle \delta(\mathbf{y}-\mathbf{Y}(t)) \rangle$

Using definition + Novikov thm, → $\partial_t P_1(\mathbf{y},t) = -\boldsymbol{\nabla}_{\mathbf{y}} \cdot \left\langle \delta(\mathbf{y}-\mathbf{Y}(t))\dot{\mathbf{Y}}(t) \right\rangle$

$$\mathcal{L}_0 = \boldsymbol{\nabla}_{\mathbf{y}} \cdot (\nu k \mathbf{y} + \nu T \boldsymbol{\nabla}_{\mathbf{y}})$$

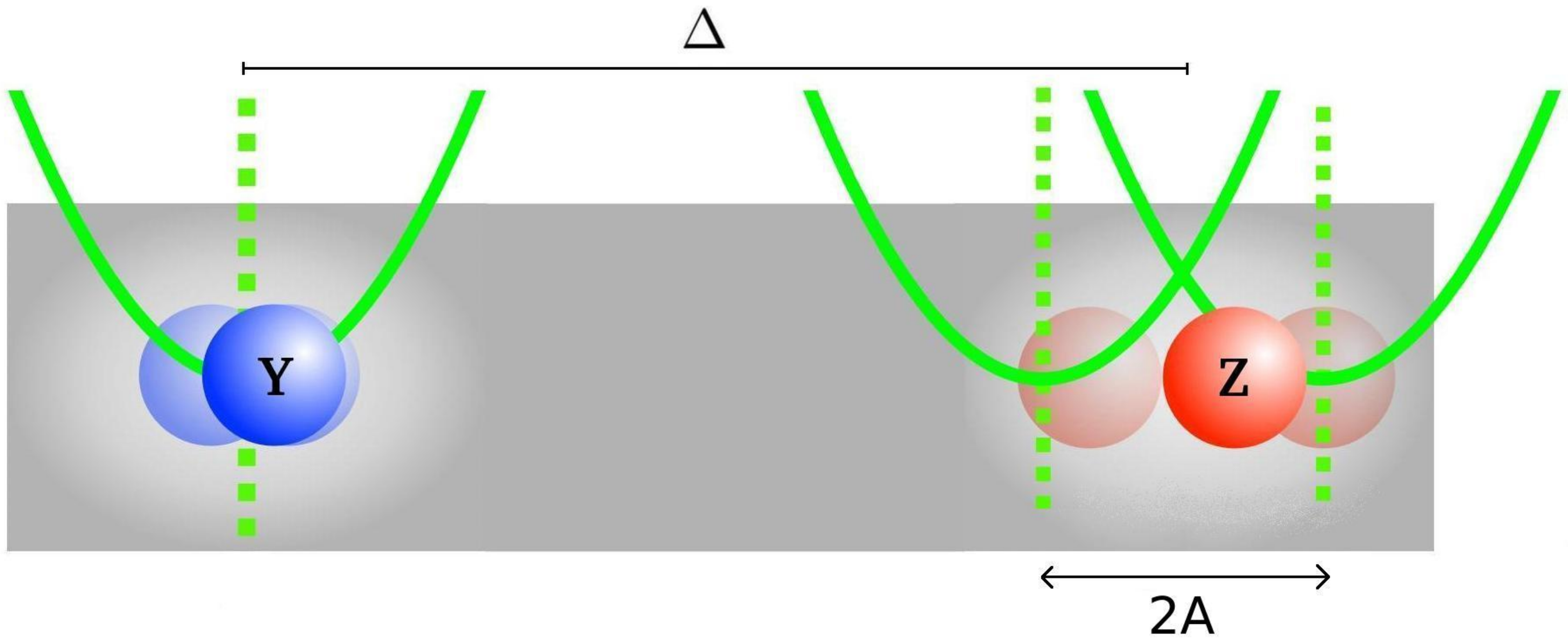
$$\mathcal{L}_z(t) = \boldsymbol{\nabla}_{\mathbf{y}} \cdot \nu \int \frac{\mathrm{d}^d q}{(2\pi)^d} i\mathbf{q} |V_q|^2 e^{-i\mathbf{q}\cdot\mathbf{y}} \int_{t_0}^{t} \mathrm{d}s\, \chi_q(t-s) e^{i\mathbf{q}\cdot\mathbf{Z}(s)}$$

$$\mathcal{L}(\mathbf{r};t,s) \equiv \boldsymbol{\nabla}^k \nu_y \int \frac{\mathrm{d}^d q}{(2\pi)^d} i q_k |V_q^{(y)}|^2 e^{-i\mathbf{q}\cdot\mathbf{r}} \times \left[\chi_q(t-s) - i\nu_y C_q(s,t;t_0) e^{-\gamma_y(t-s)} q_j \boldsymbol{\nabla}^j\right]$$

$$\partial_t P_1(\mathbf{y},t) = \mathcal{L}_0 P_1(\mathbf{y},t) + \lambda^2 \mathcal{L}_z(t) P_1(\mathbf{y},t) + \lambda^2 \int_{t_0}^{t} \mathrm{d}s \int \mathrm{d}\mathbf{x}\, \mathcal{L}(\mathbf{y}-\mathbf{x};t,s) P_2(\mathbf{y},t;\mathbf{x},s) + \mathcal{O}(\lambda^4)$$

Two particles

Weak-coupling approximation



- Memory vanishes in the periodic state → seemingly **Markovian**!
- Full **cumulant gen. func.**
- $Y(t)$ is practically immersed into the **effective field** generated by the driven particle which acts as a source term,

$$\partial_t P_1(\mathbf{y},t) = \mathcal{L}_0 P_1(\mathbf{y},t) + \lambda^2 \mathcal{L}_z(t) P_1(\mathbf{y},t) + \mathcal{O}\left(\lambda^4\right)$$

$$\mathcal{L}_0 = \boldsymbol{\nabla}_{\mathbf{y}} \cdot (\nu k \mathbf{y} + \nu T \boldsymbol{\nabla}_{\mathbf{y}})$$

$$\mathcal{L}_z(t) = \boldsymbol{\nabla}_{\mathbf{y}} \cdot \nu \int \frac{\mathrm{d}^d q}{(2\pi)^d} i\mathbf{q} |V_q|^2 e^{-i\mathbf{q}\cdot\mathbf{y}} \int_{t_0}^{t} \mathrm{d}s\, \chi_q(t-s) e^{i\mathbf{q}\cdot\mathbf{Z}(s)}$$

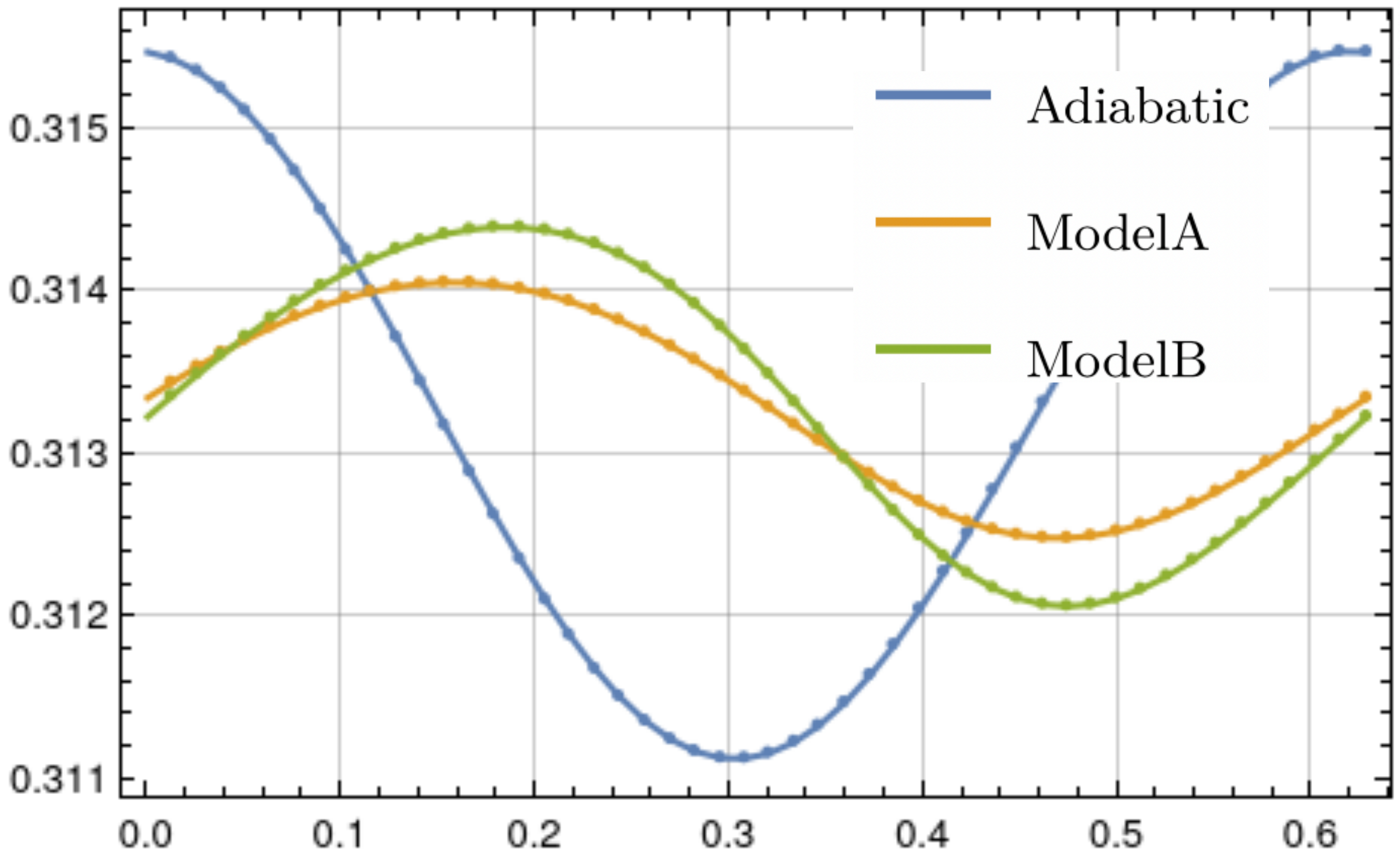
$$\langle \phi_q^{\mathrm{eff}}(t) \rangle = \lambda \int_{-\infty}^{t} \mathrm{d}s\, \chi_q(t-s) V_q^{(z)} \left\langle e^{-i\mathbf{q}\cdot\mathbf{Z}(s)} \right\rangle$$

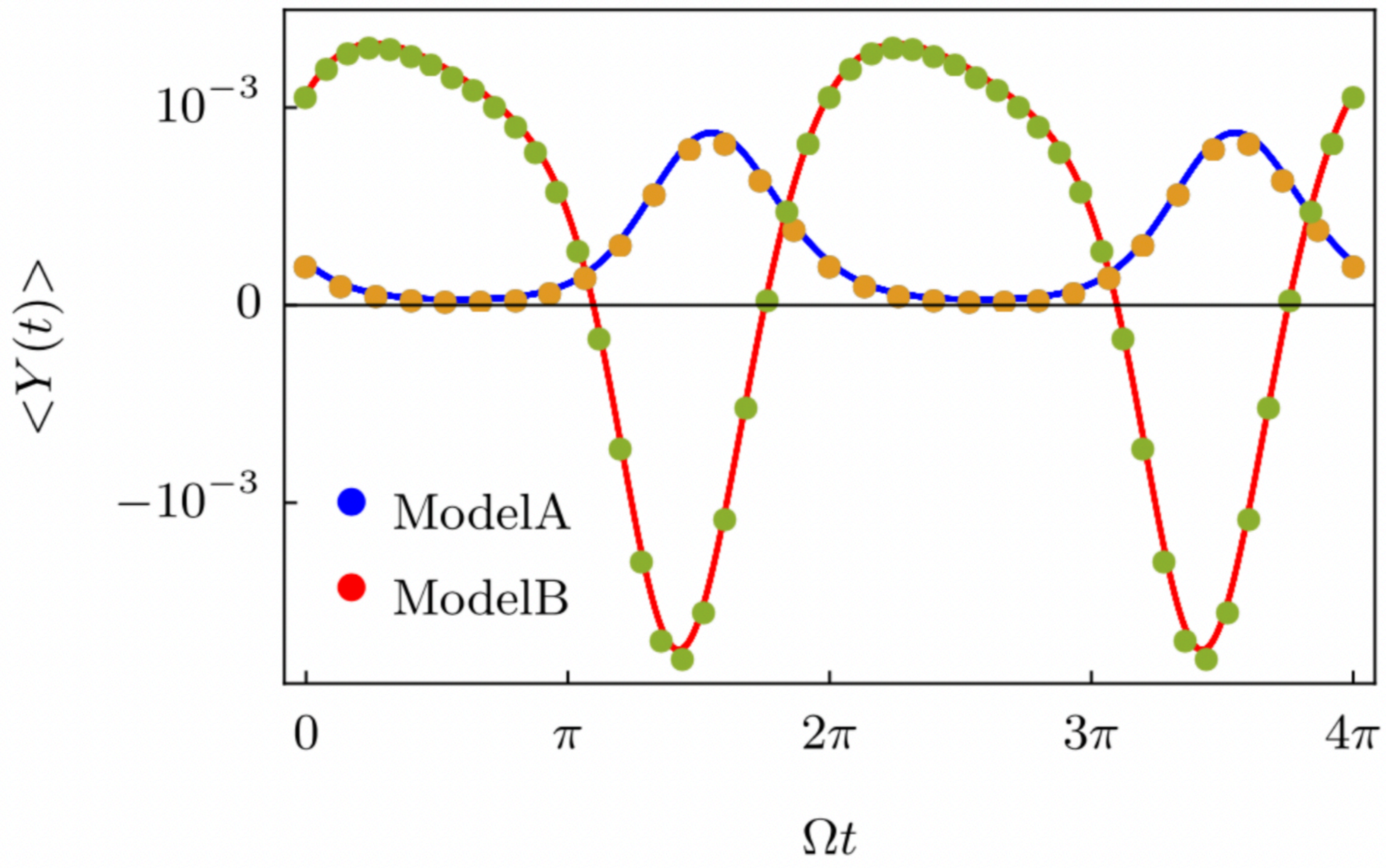
$$\dot{\mathbf{Y}}(t) = -\gamma_y \mathbf{Y}(t) + \boldsymbol{\xi}^{(y)}(t) + \nu_y \lambda \int \frac{\mathrm{d}^d q}{(2\pi)^d} i\mathbf{q} V_{-q}^{(y)} \langle \phi_q^{\mathrm{eff}}(t) \rangle e^{i\mathbf{q}\cdot\mathbf{Y}(t)}$$

Two particles

A comparison

Predictions
(adiabatic **vs** weak-coupling)



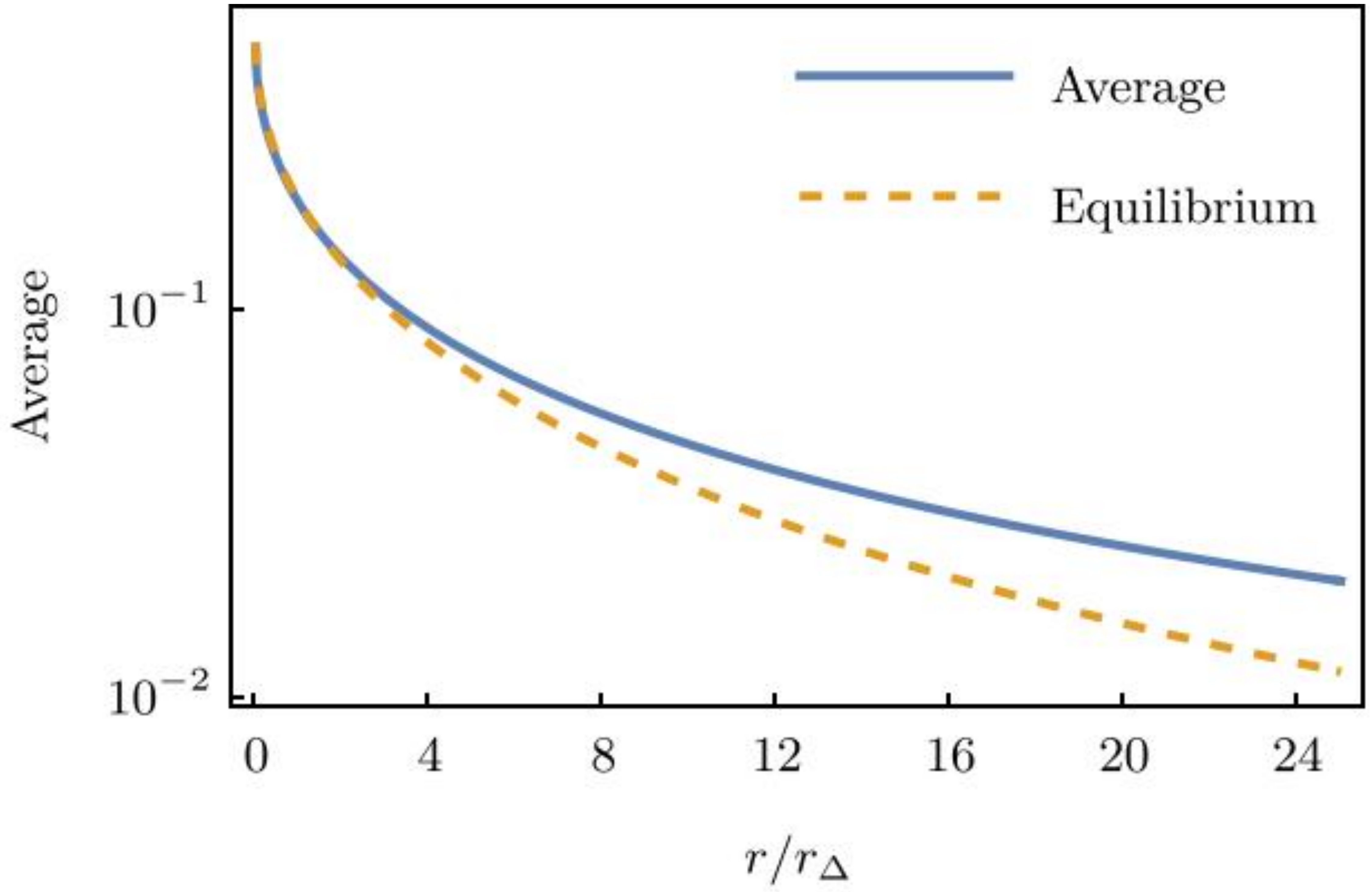


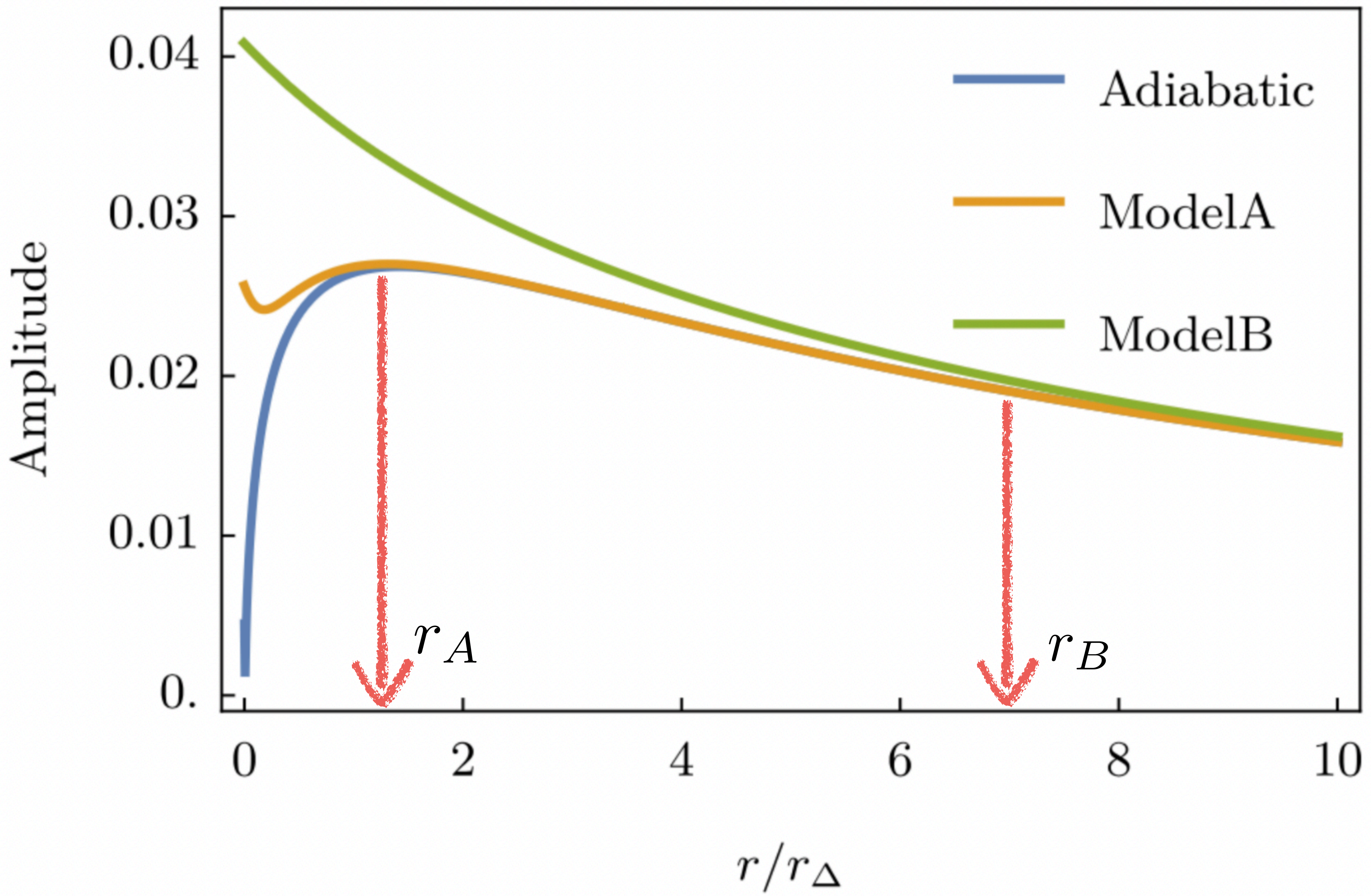
Numerical simulation
(d=2)

Two particles

A comparison

Temporal average and amplitude of the oscillations





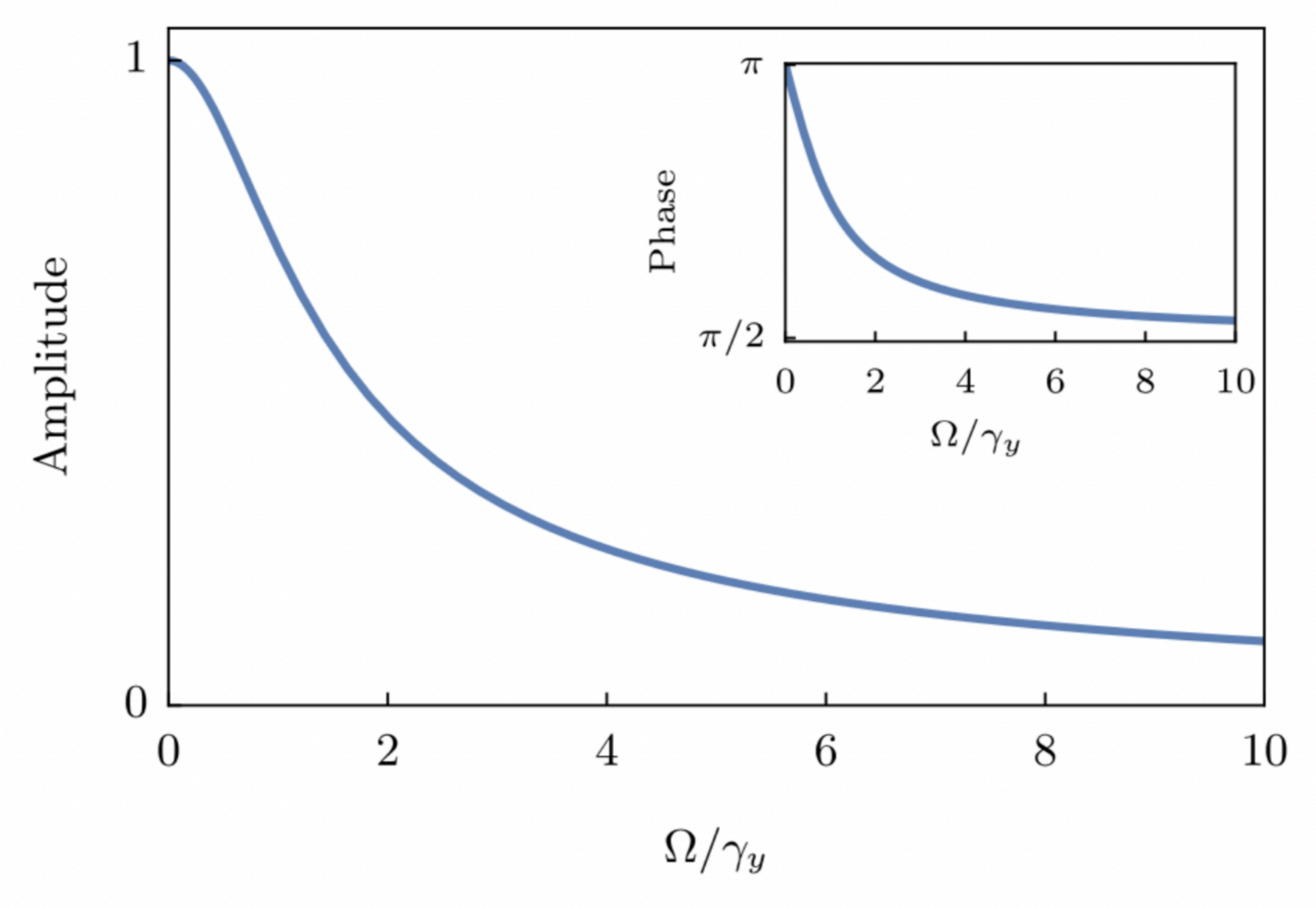
Competing timescales
$$\begin{cases} \tau_\phi^{-1} \sim Dq^\alpha(q^2+r) \\ \tau_\Omega^{-1} \sim \Omega \end{cases}$$

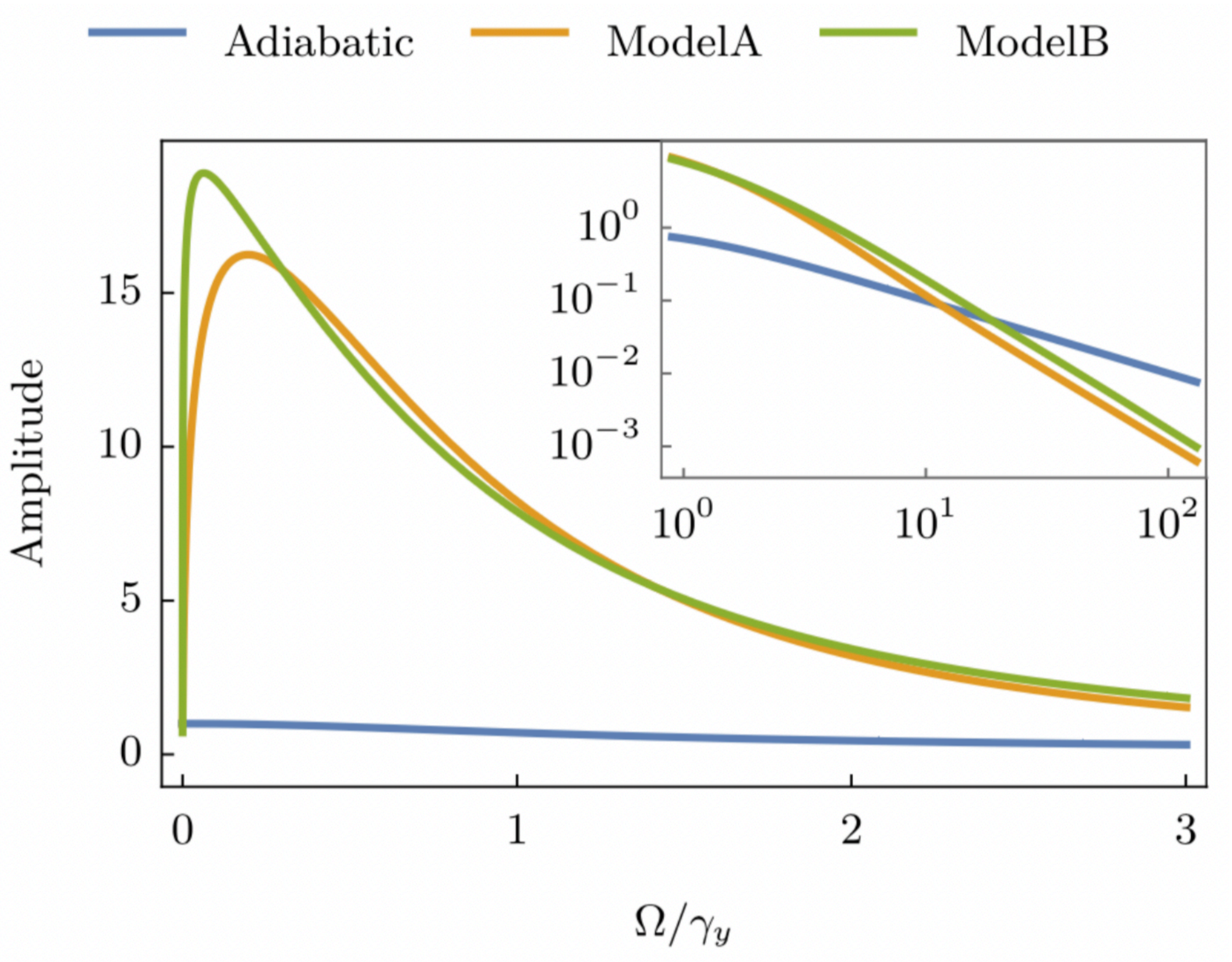
Choosing $q \sim r^{1/2} = 1/\xi \quad \Rightarrow \quad r_A \sim \Omega\,,\ r_B \sim \Omega^{1/2}$

Two particles

A comparison

Behavior as a function of Ω (frequency response)





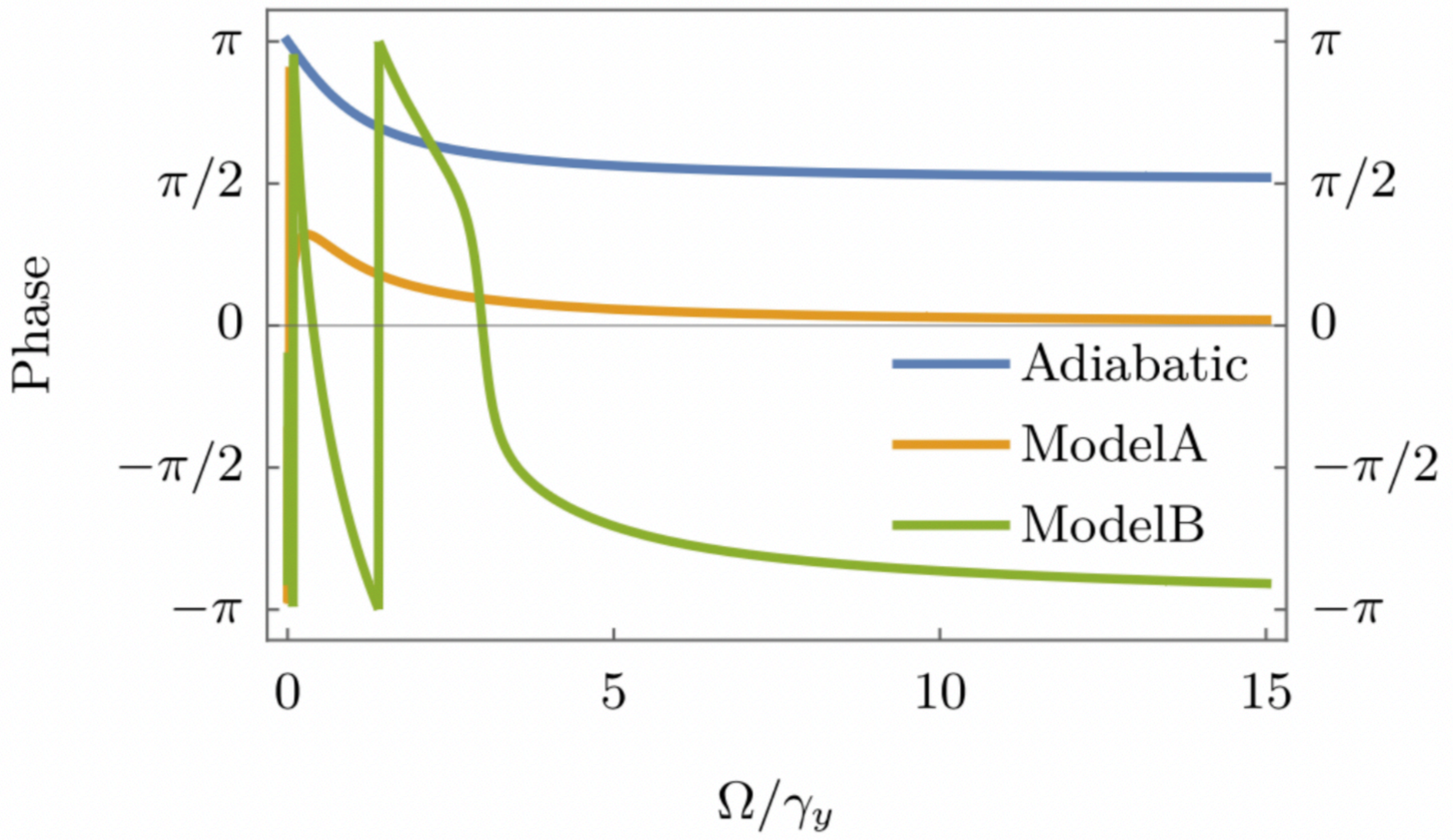
When $\xi \gg \Delta$, the non-equilibrium response is peaked around

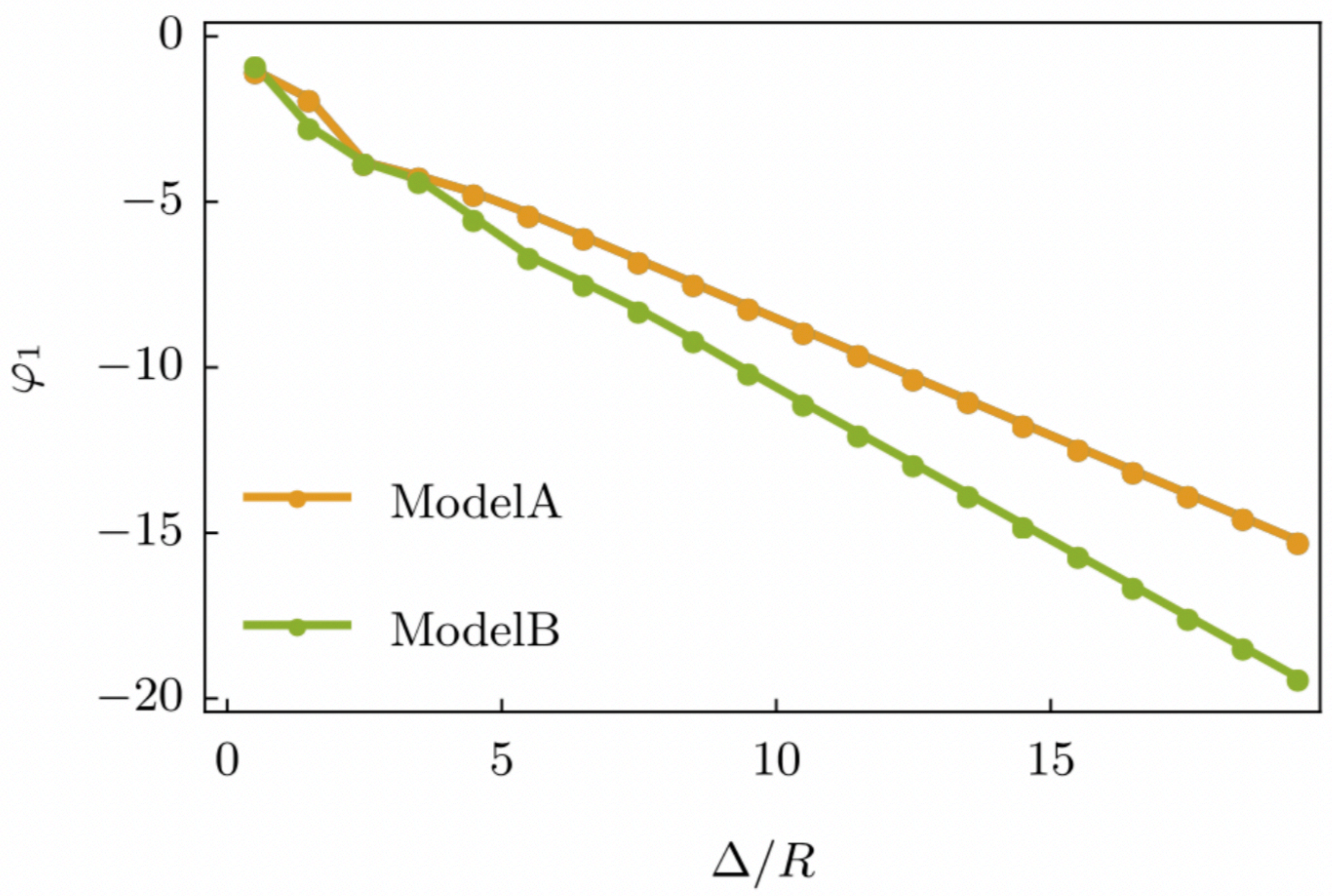
$$\Omega_{\text{peak}} \sim \tau_\phi^{-1}(q \simeq 1/\Delta) \simeq D/\Delta^z$$

Two particles

A comparison

Phase shift





Within the "effective field picture",

$$\langle \mathbf{Y}(t) \rangle \simeq \mathbf{R}(\Omega) \cos(\Omega t + \kappa \Delta + \varphi_0)$$

[DV, Gambassi '22]

Take-home messages

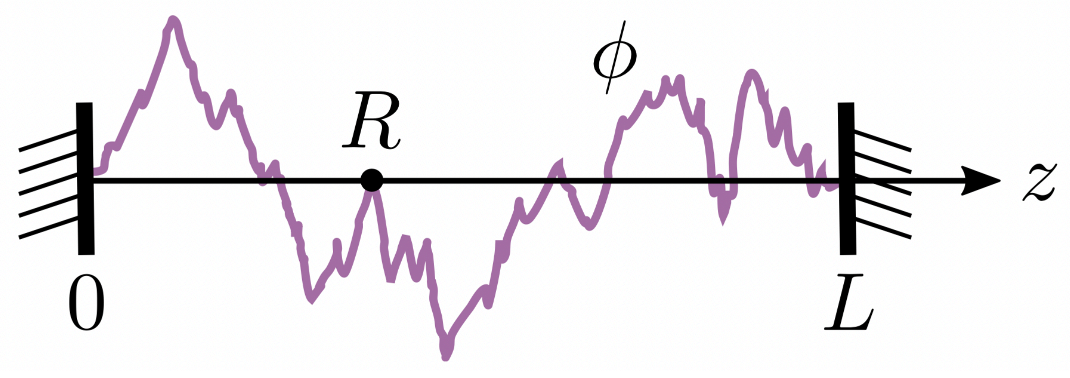
(but this is not my last slide)

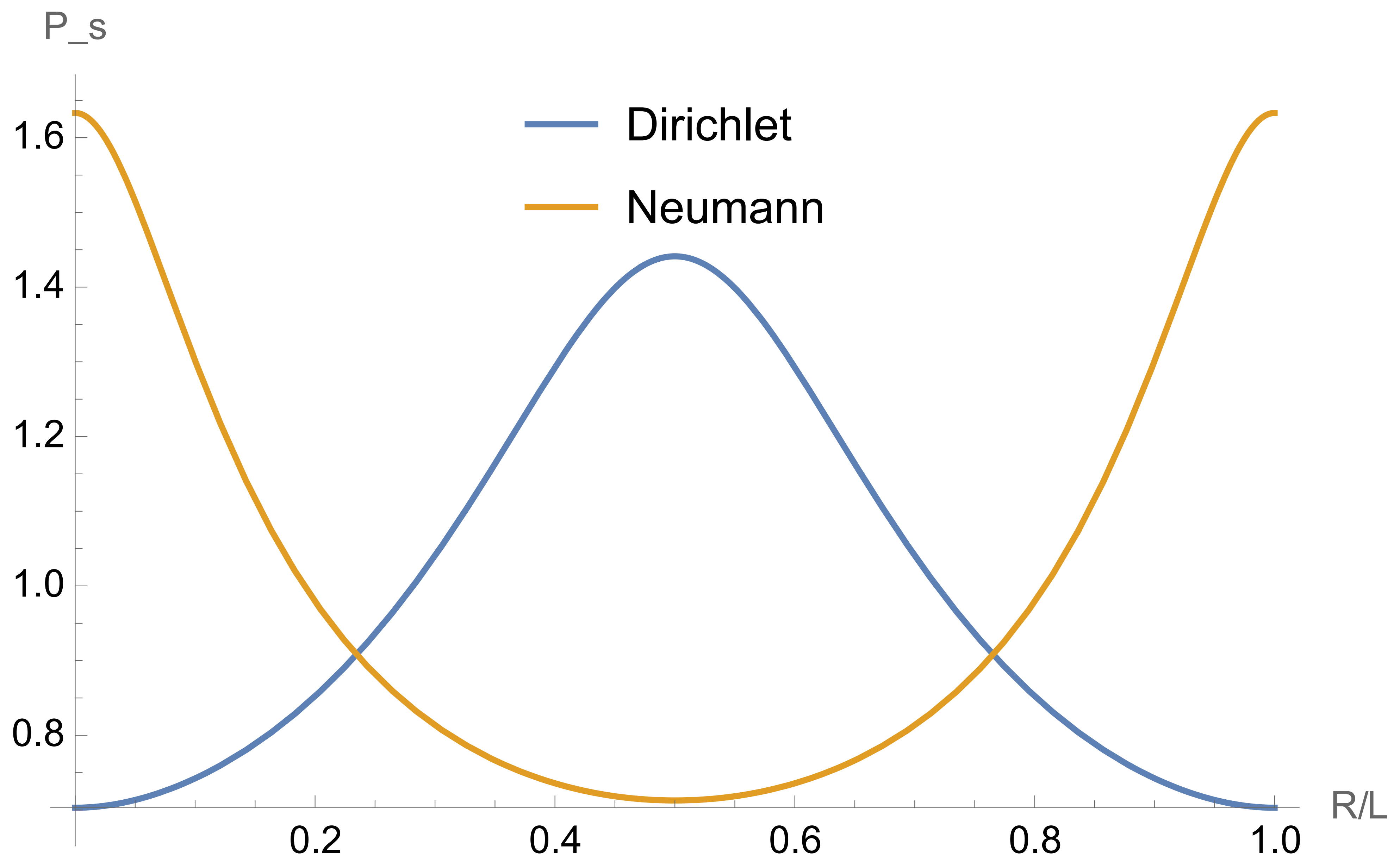
- Particles interacting with a field experience **field-induced** reciprocal **interactions**,
- but even a single particle feels a **self-interaction**, not visible in equilibrium.
- A medium close to a phase transition relaxes slowly, and **non-equilibrium** effects become prominent,
- and this affects the motion of **tracer particles** in contact with the medium.
- **Adiabatic** approximations and **linear response** analysis sometimes fail to capture these funny effects.

Work in progress

1/2

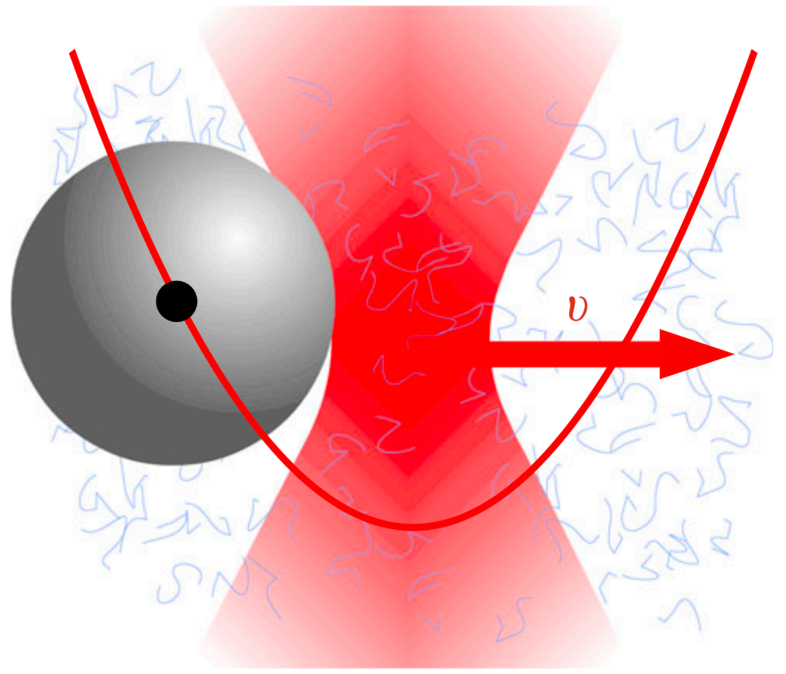
Effective (adiabatic) particle dynamics in a **confined** geometry

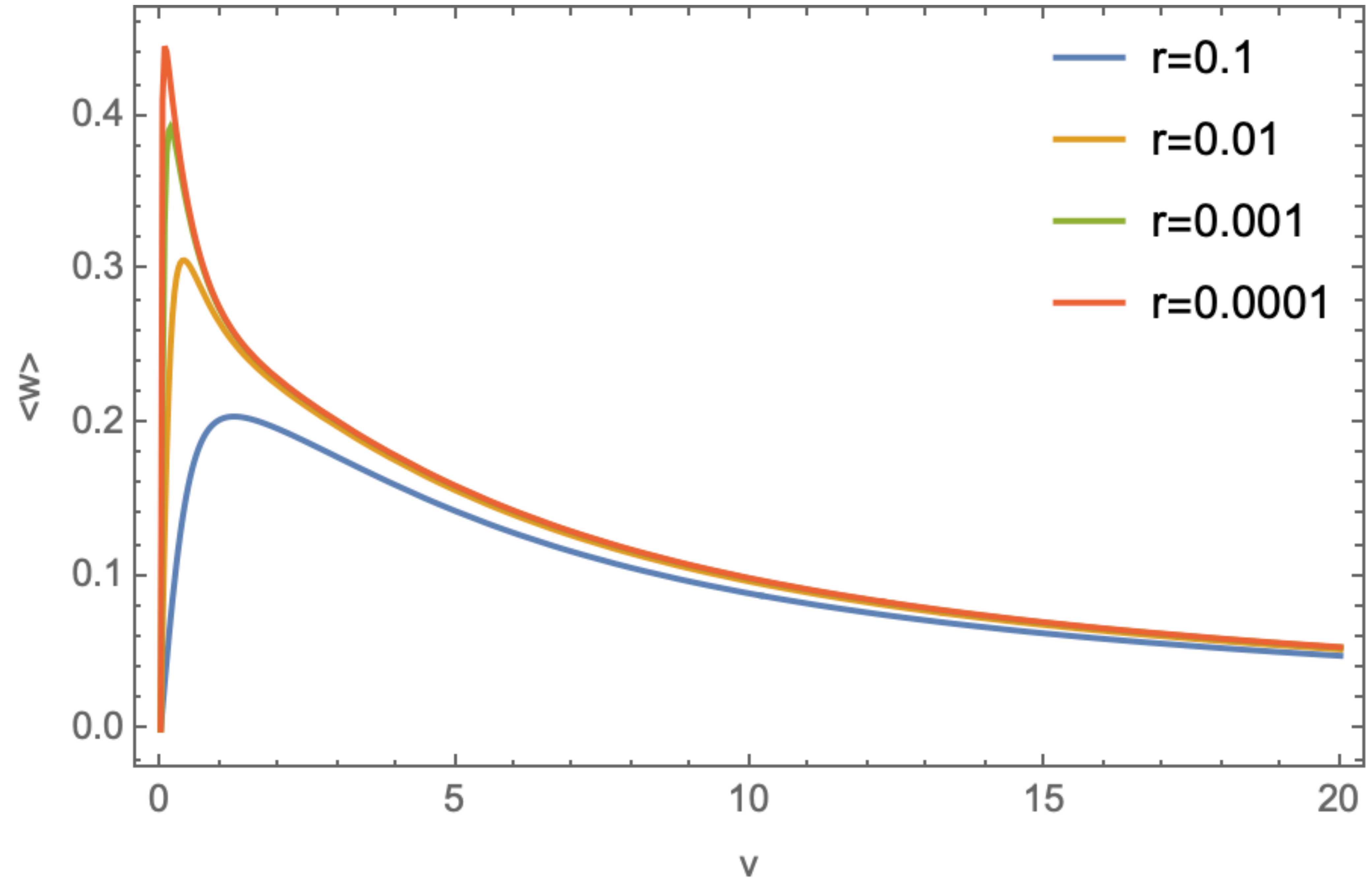




ft. M. Gross (Max Planck - Stuttgart)

Stochastic thermodynamics of field+particle





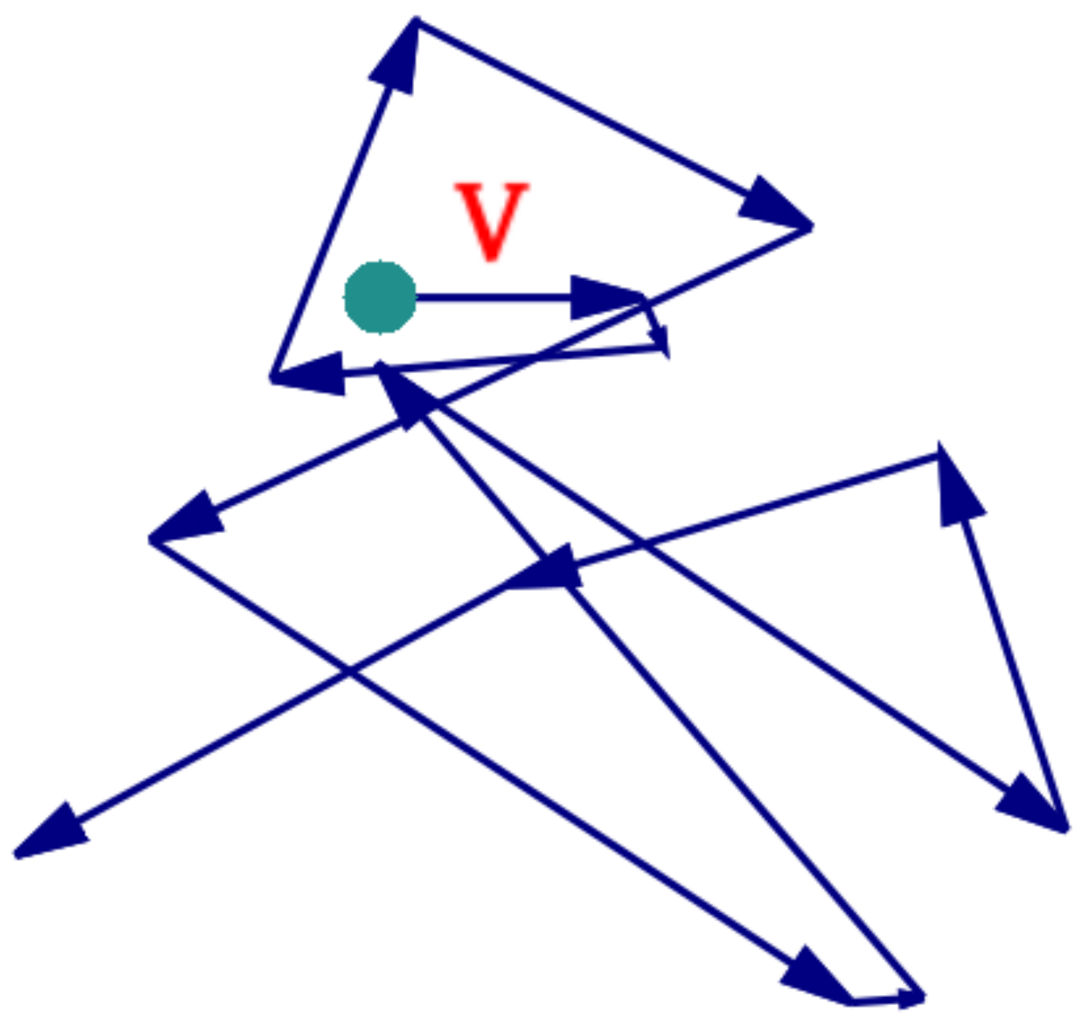
ft. B. Walter, S. Loos & E. Roldan (SISSA & ICTP)

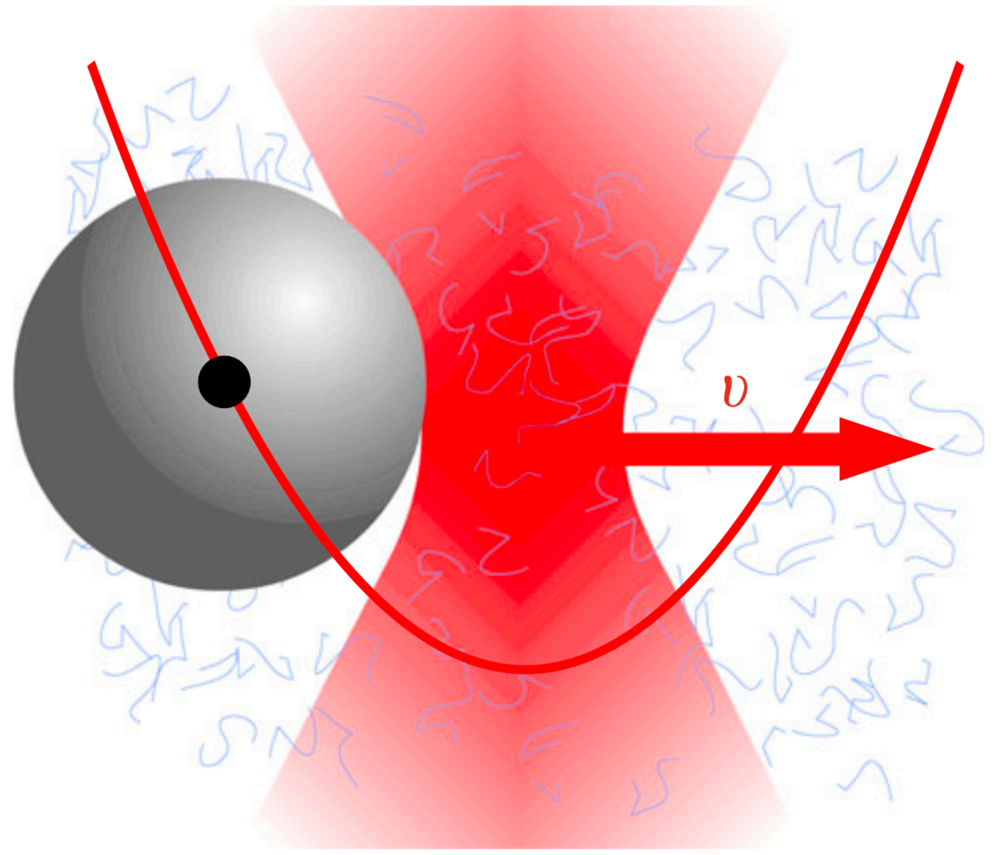
Work in progress

2/2

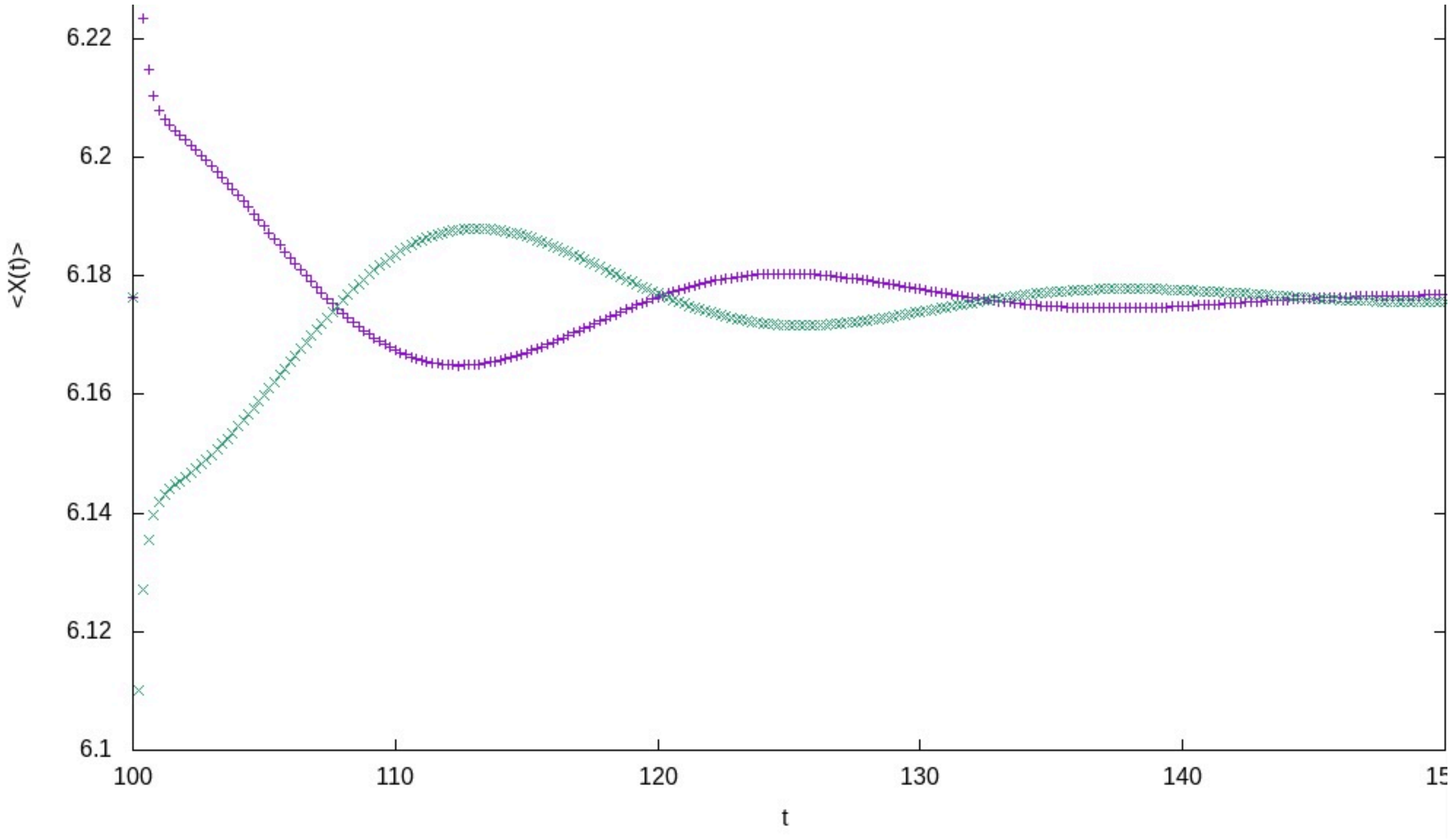
Active particles in a Gaussian field

ft. U. Basu (Bose Center - Kolkata)





Oscillating modes from overdamped dynamics



Future perspectives

(*now* this is my last slide)

- **Active** field theories
- Self-interacting ϕ^4 field
- Hydrodynamics (**model H**)
- Quadratic type interaction $\sim \phi^2(X)$

Thank you!