# Dynamics of tracer particles in a fluctuating correlated medium

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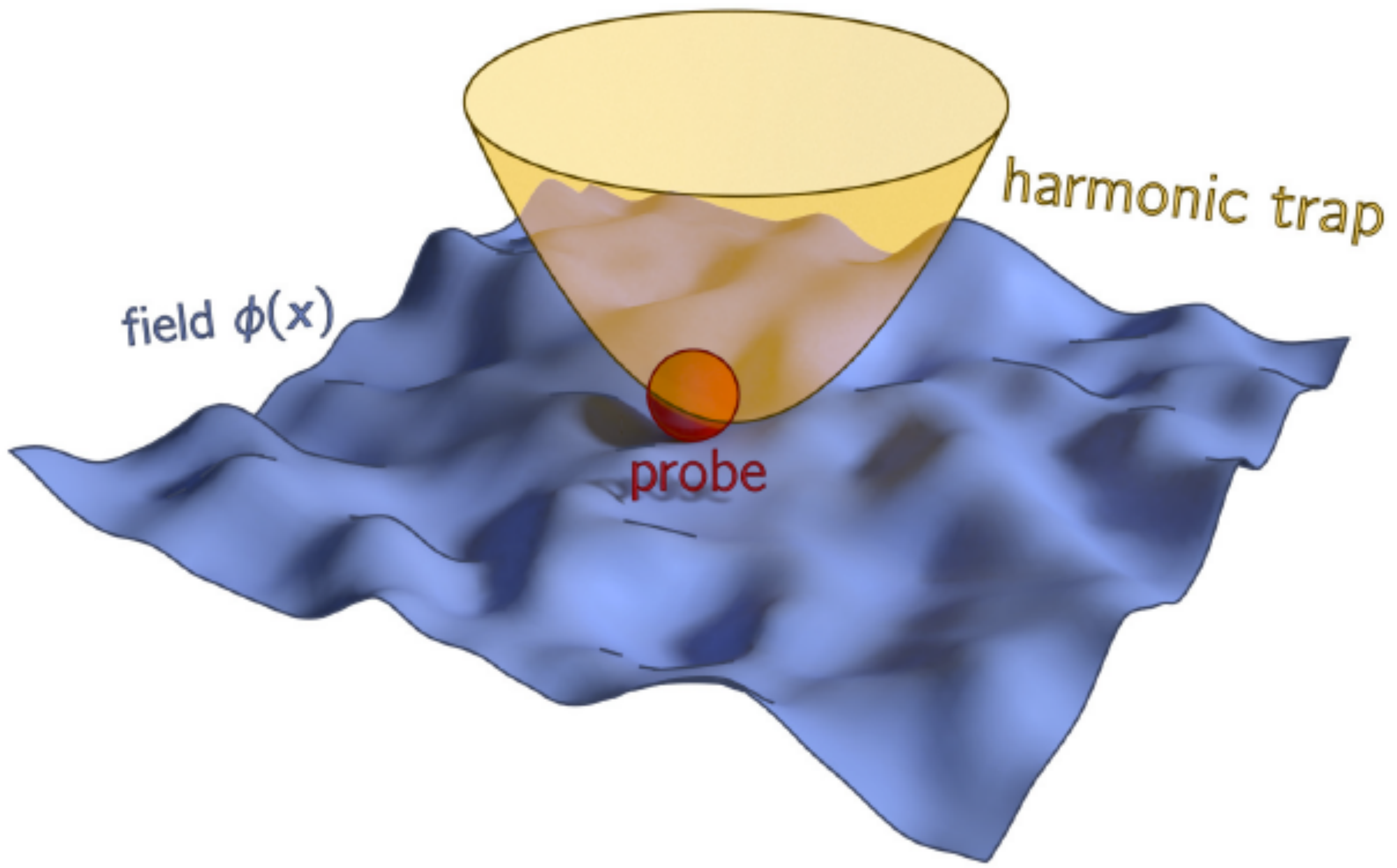
StatPhys28, Tokyo, August 9th 2023

SISSA



# Particle in a complex medium

## From Brownian motion to correlated media

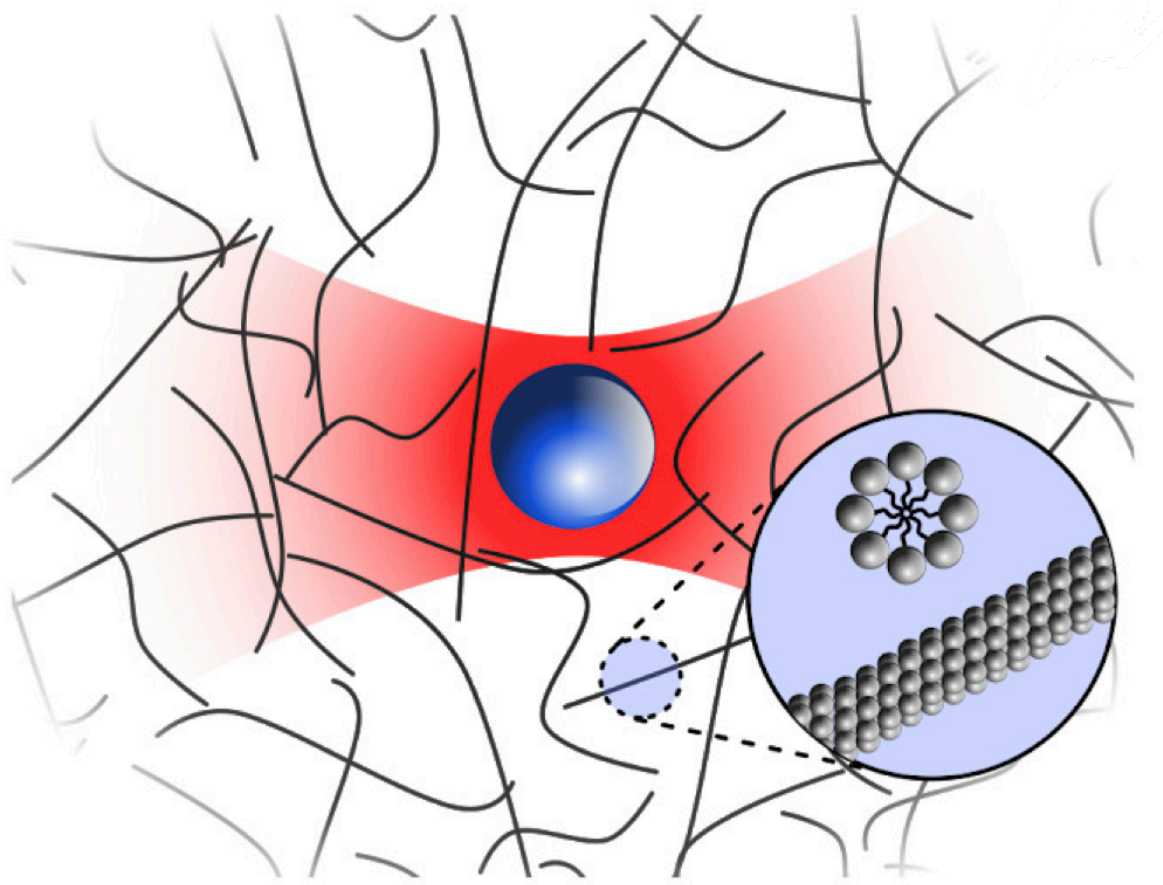
- **Brownian motion**

$$m\,\ddot{x}(t) \;=\; -\gamma\,\dot{x}(t) \;+\; \zeta(t) - V'(x(t))$$

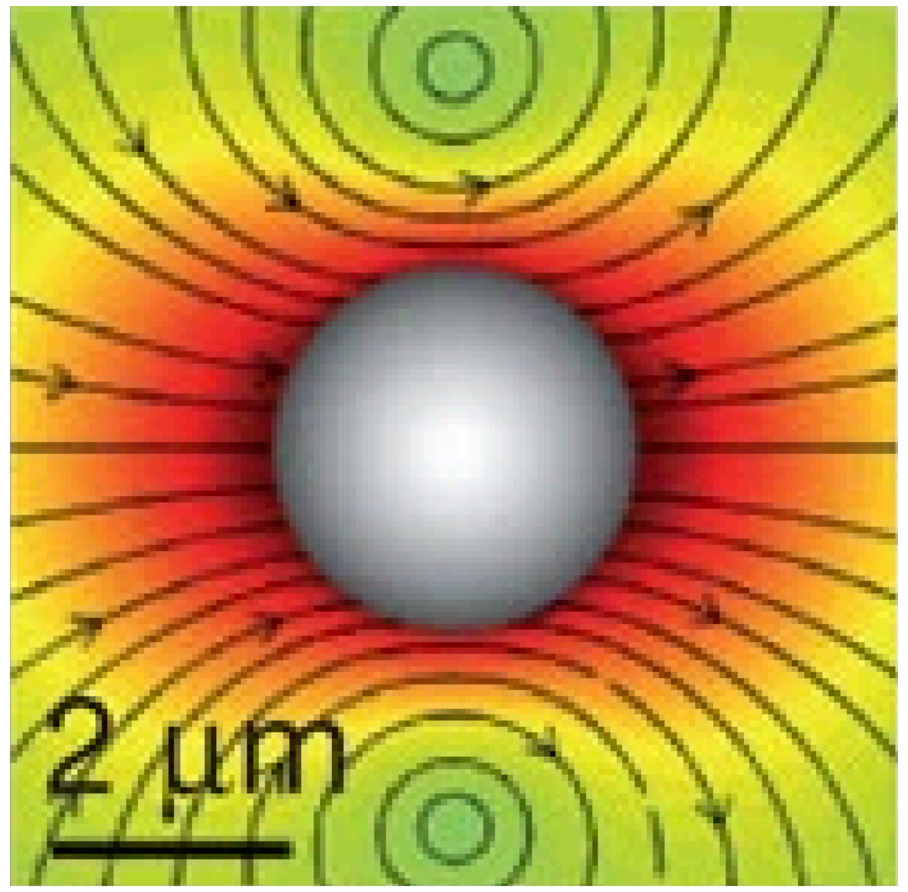
$$\langle \zeta(t)\zeta(t')\rangle = 2\,k_{\mathrm{B}}T\,\gamma\,\delta(t-t')$$

**Separation of timescales challenged by, e.g.,**

- Viscoelastic fluids
- Near-critical media
- Hydrodynamic memory



[R. Jain et al., J. Chem. Phys. (2021)]




[T. Franosh et al., Nature (2011)]

# Particle in a complex medium

## From Brownian motion to correlated media

- **Brownian motion**

$$m\,\ddot{x}(t) = -\gamma\,\dot{x}(t) + \zeta(t) - V'(x(t))$$

$$\langle \zeta(t)\zeta(t') \rangle = 2\,k_{\mathrm{B}}T\,\gamma\,\delta(t-t')$$

- **GLE**

$$m\,\ddot{x}(t) = -\int^{t} \mathrm{d}t'\,\Gamma(t-t')\,\dot{x}(t') + \zeta(t) - V'(x(t))$$

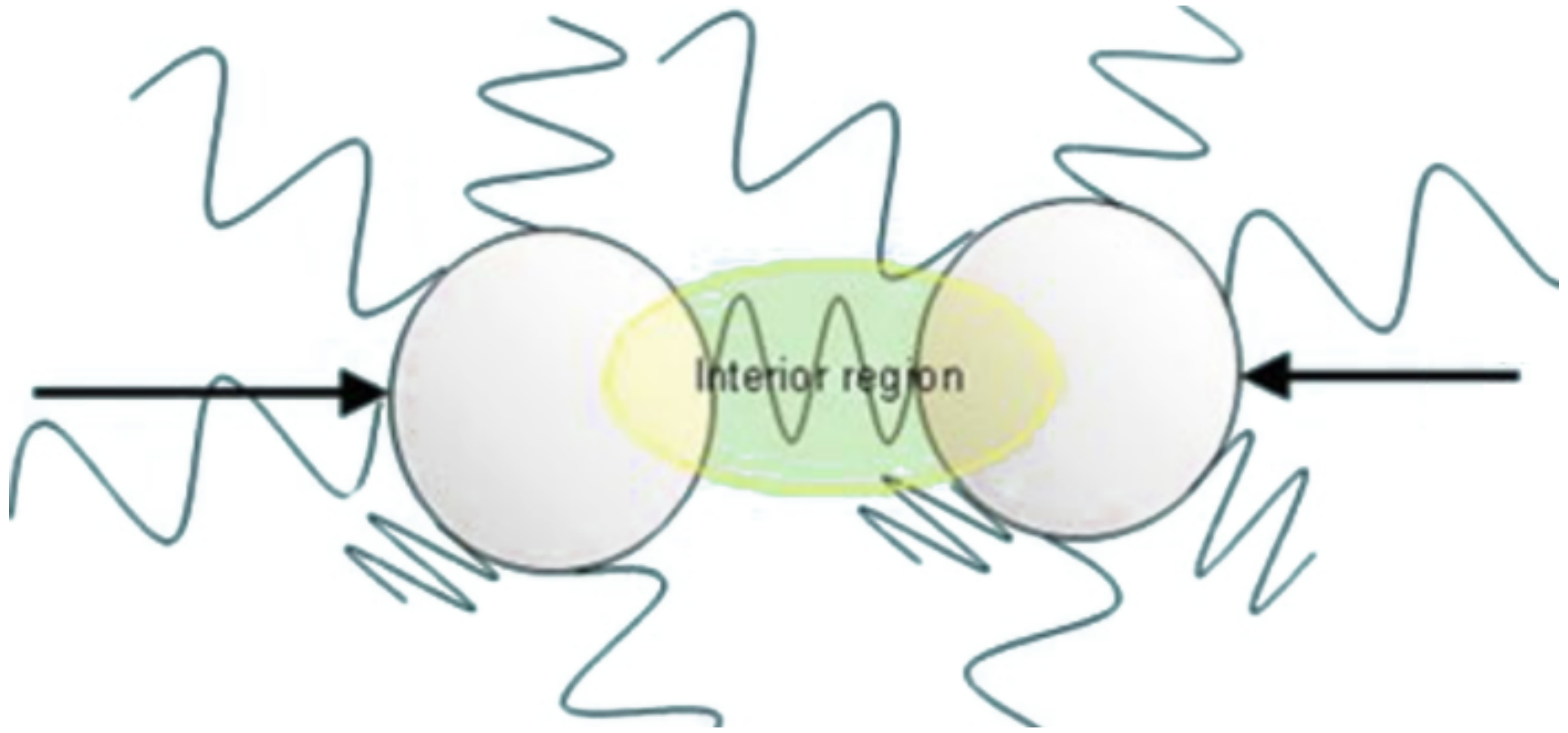
$$\langle \zeta(t)\zeta(t') \rangle = k_{\mathrm{B}}T\,\Gamma(|t-t'|)$$

[e.g. Caldeira&Leggett ’83]

→ **Spatial structure? Energy flows? Fluctuation-induced forces?**

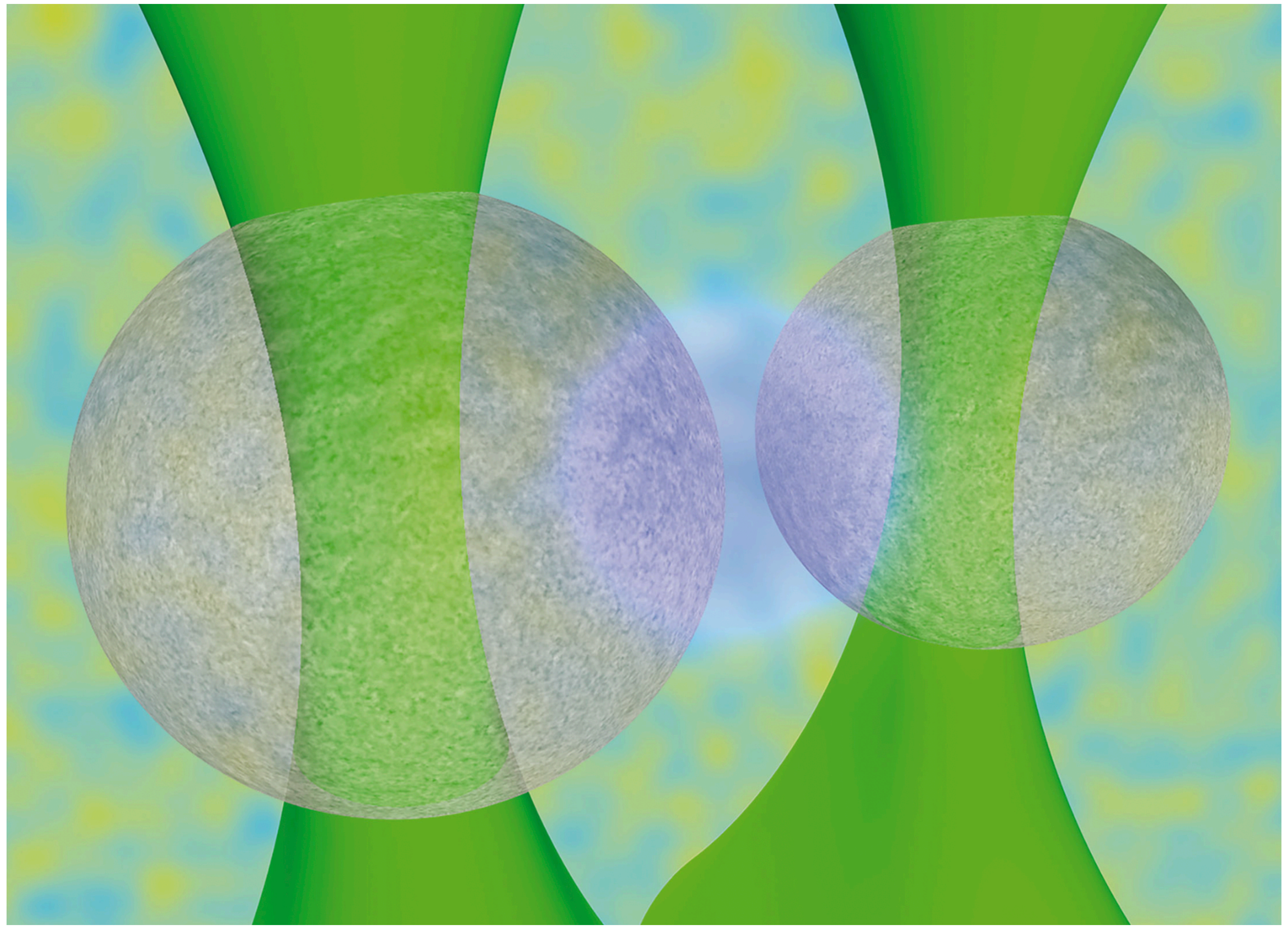
# Fluctuation-induced forces

## in correlated systems




[C. Hertlein et al., Nature (2008)]

[A. Magazzù et al., Soft Matter (2018)]



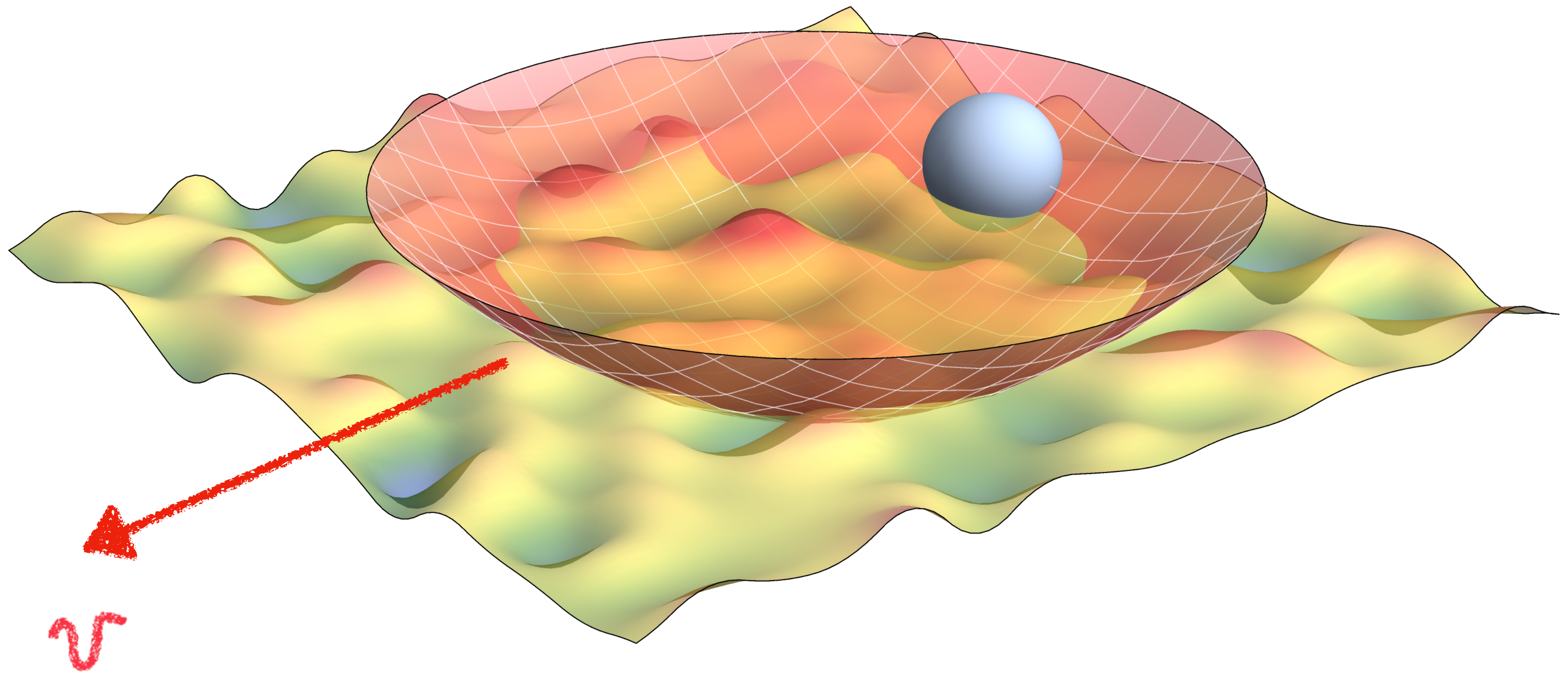
- Critical Casimir forces
- Near-critical media (e.g., binary liquid mixtures)
- Retardation effects (not instantaneous)

# A toy model

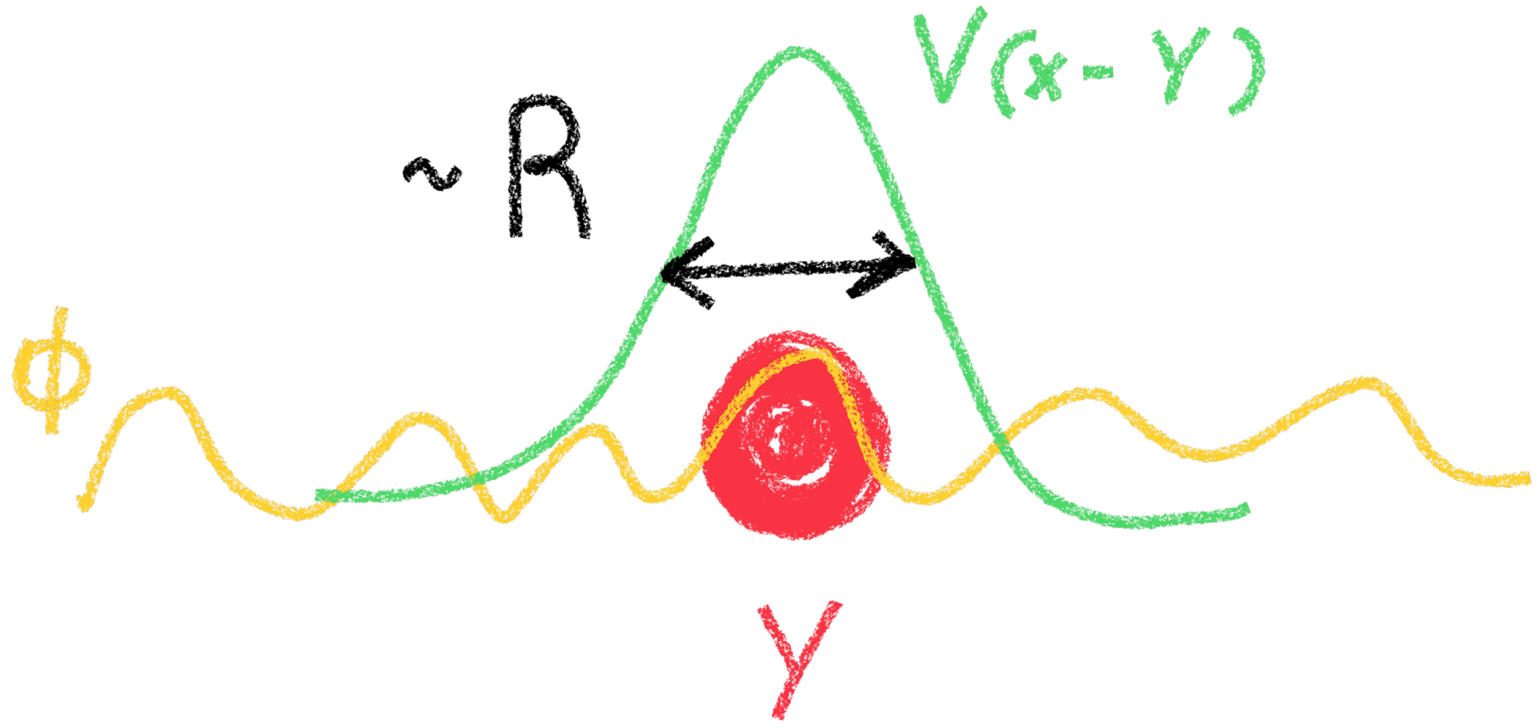
$\xi = r^{-1/2}$ **sets the range of spatial correlations of** $\phi(x,t)$

$$\mathcal{H}[\phi,\mathbf{Y}] = \mathcal{H}_\phi[\phi] + \mathcal{U}(\mathbf{Y},t) + \mathcal{H}_{\rm int}[\phi,\mathbf{Y}]$$

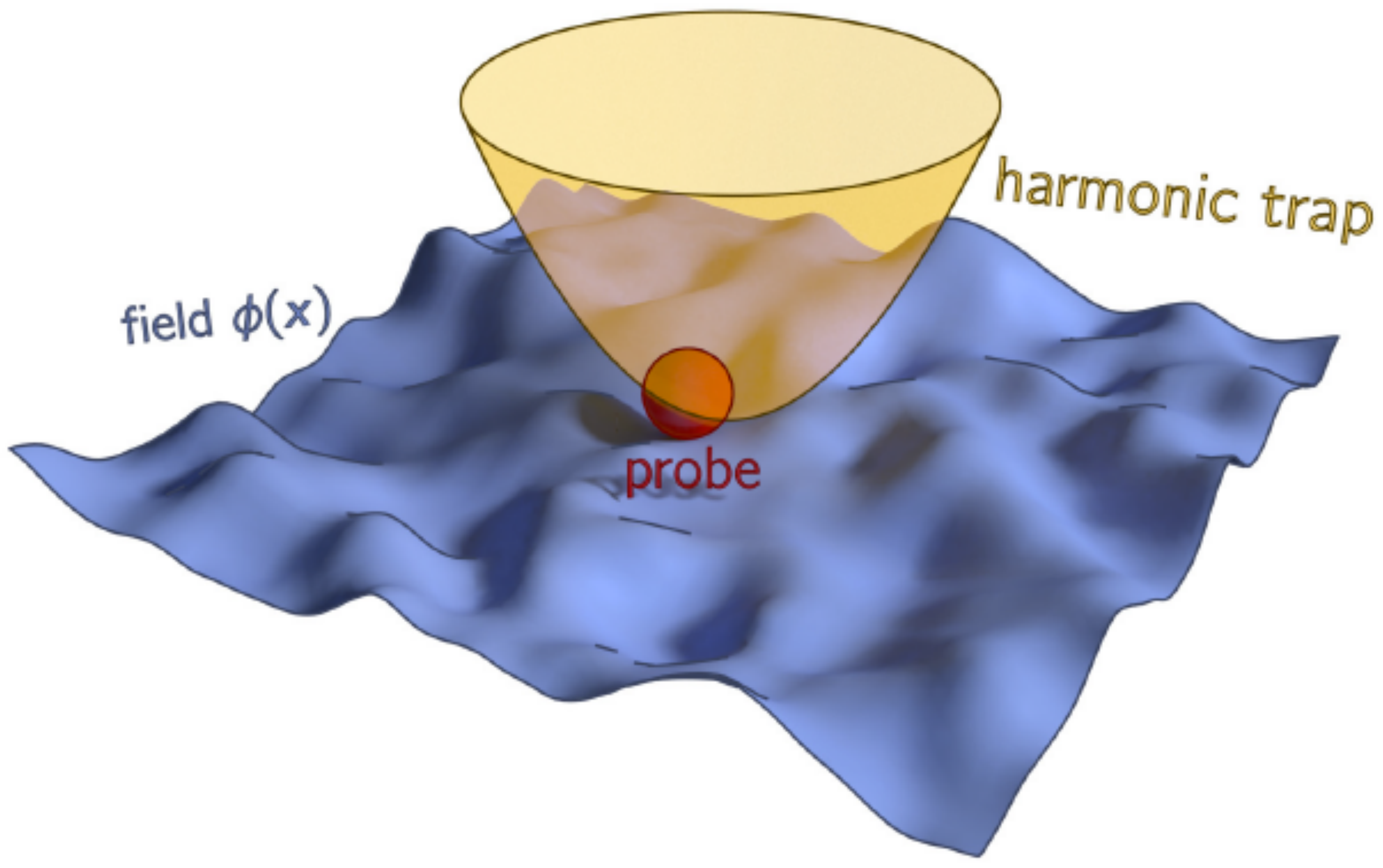
$$\begin{cases} \mathcal{H}_\phi = \int \mathrm{d}^d\mathbf{x} \left[\frac{1}{2}(\nabla\phi)^2 + \frac{1}{2}r\phi^2\right] \\ U(\mathbf{Y},t) = \frac{\kappa}{2}(\mathbf{Y}-\mathbf{v}t) \\ \mathcal{H}_{\rm int} = -\lambda \int \mathrm{d}^d x\, V(\mathbf{x}-\mathbf{Y})\phi(\mathbf{x}) \end{cases}$$




$V(\mathbf{x}-\mathbf{Y})$ **extends within the size R of the particle**



# Dynamics




$\phi, \mathbf{Y}$ influence each other:

$$\gamma_y \dot{\mathbf{Y}}(t) = -\boldsymbol{\nabla}_{\mathbf{Y}} \mathcal{H} + \boldsymbol{\xi}(t)$$

$$\gamma_\phi \dot{\phi}(\mathbf{x}, t) = -(-\nabla^2)^{\alpha/2} \frac{\delta \mathcal{H}}{\delta \phi(\mathbf{x}, t)} + \eta(\mathbf{x}, t)$$

+ thermal bath @T, $\begin{cases} \langle \xi_i(t) \xi_j(t') \rangle = 2\gamma_y T \delta_{ij} \delta(t - t') \\ \langle \eta(\mathbf{x}, t) \eta(\mathbf{x}', t') \rangle = 2\gamma_\phi T (-\boldsymbol{\nabla}^2)^{\alpha/2} \delta^d(\mathbf{x} - \mathbf{x}') \delta(t - t') \end{cases}$

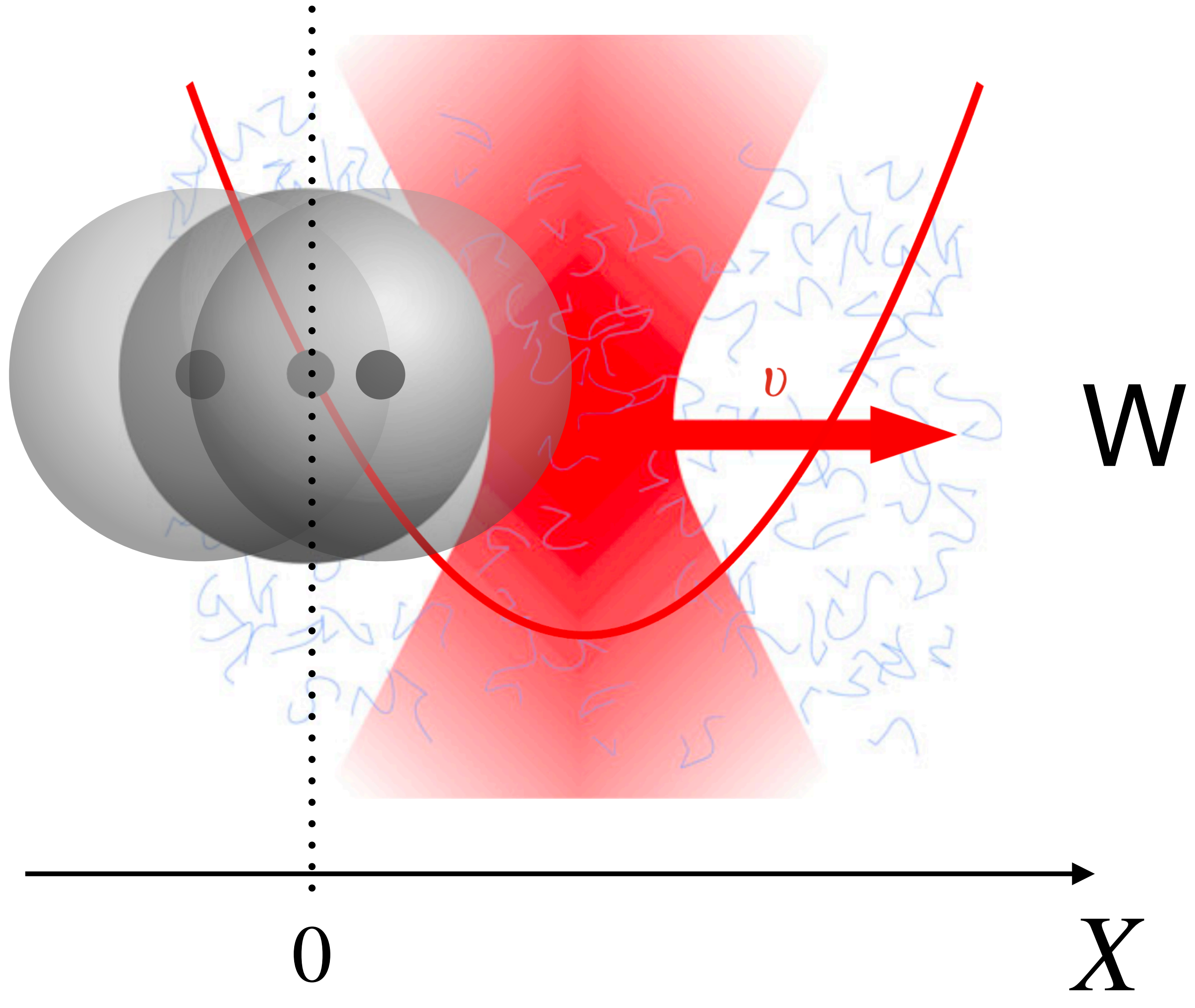
Go in Fourier:

$$\gamma_y \dot{\mathbf{Y}} = -\kappa(\mathbf{Y} - \mathbf{v}t) + \lambda \int_{\mathbb{R}} \frac{\mathrm{d}^d q}{(2\pi)^d} i\mathbf{q} V_{-q} \phi_q e^{i\mathbf{q}\cdot\mathbf{Y}} + \boldsymbol{\xi}$$

$$\gamma_\phi \dot{\phi}_q = -q^\alpha (q^2 + r) \phi_q + \cancel{\lambda} q^\alpha V_q e^{-i\mathbf{q}\cdot\mathbf{Y}} + \eta_q \longrightarrow \tau_\phi^{-1}(q) \equiv \gamma_\phi^{-1} q^\alpha (q^2 + r)$$

# Memory-induced oscillations

## in viscoelastic media



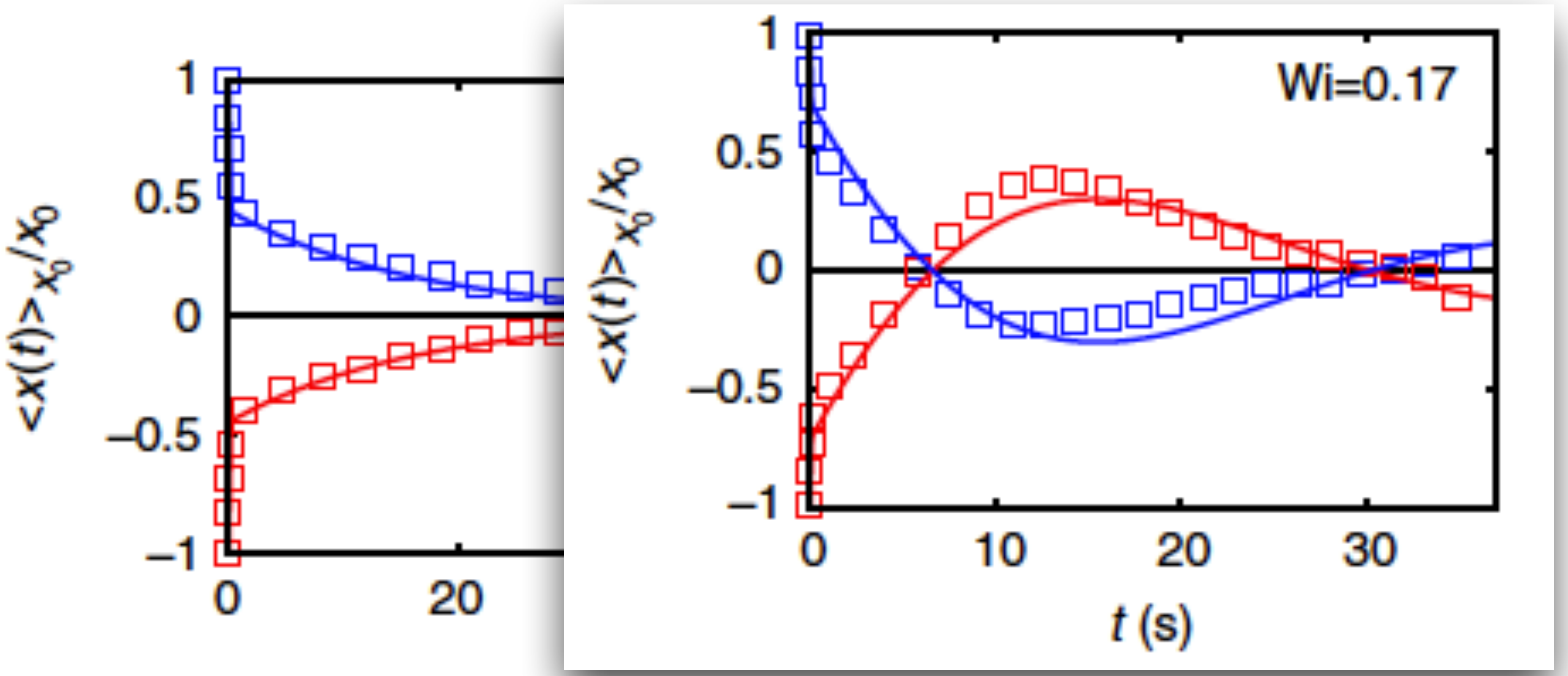

$$\mathrm{Wi}_R = \tau_s \underbrace{v/R}_{\text{shear rate}}$$

Micellar solution (overdamped!)

nature COMMUNICATIONS [2018]

Oscillating modes of driven colloids in overdamped systems

Johannes Berner[1,2], Boris Müller[3,4], Juan Ruben Gomez-Solano[1,2], Matthias Krüger[3,4,5] & Clemens Bechinger[1,2,4]



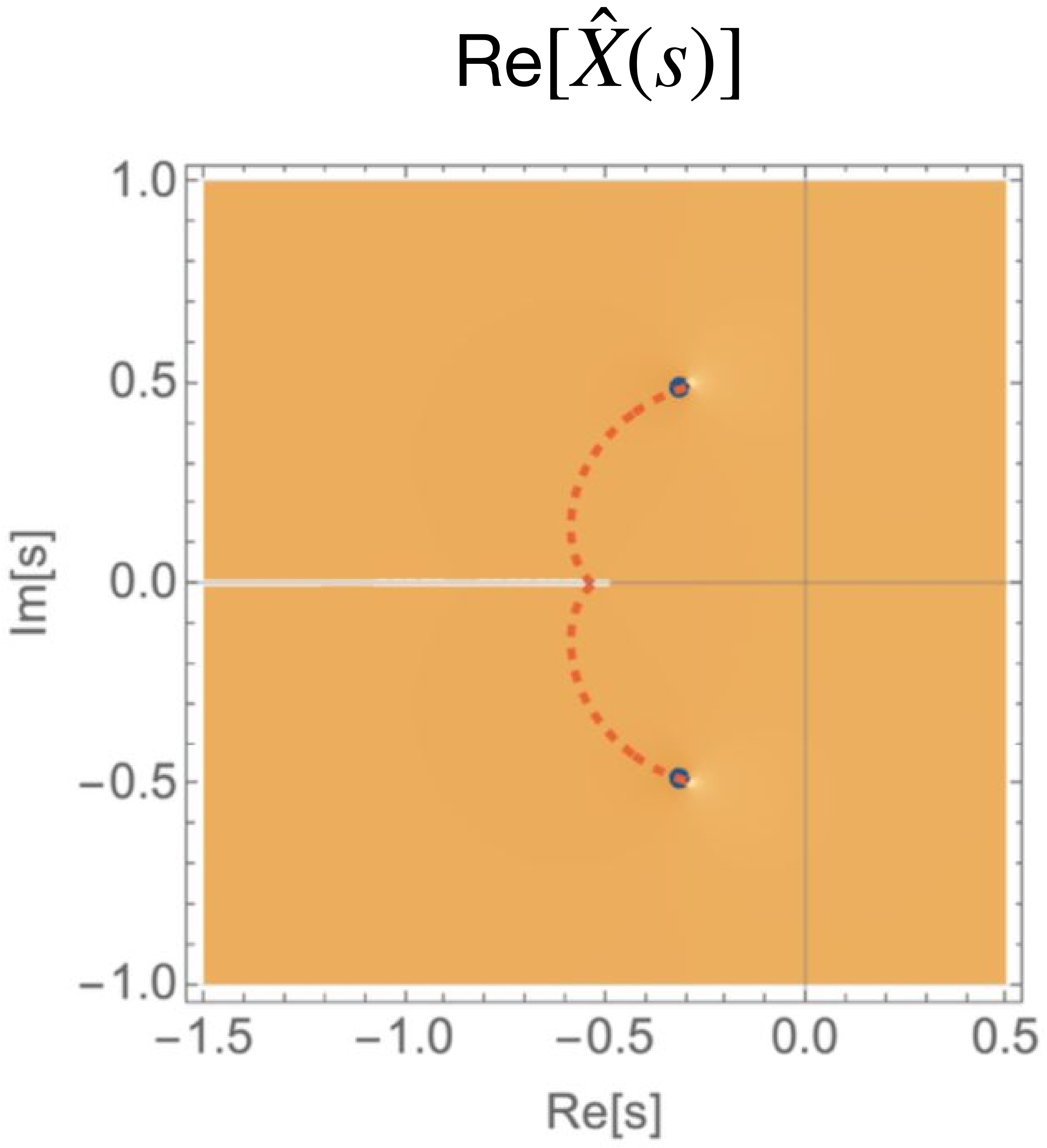
# Memory-induced oscillations
## in correlated media

Linearized effective dynamics:

$\phi$

$$\partial_t \langle X_j(t)\rangle = -\langle X_j(t)\rangle \left[\gamma + \hat{\Gamma}_j(s=0)\right] + \int_{-\infty}^{t} du\, \Gamma_j(t-u)\, \langle X_j(u)\rangle$$

$$\left\langle \hat{X}_j(s) \right\rangle = \frac{X_0}{s + \gamma - \left[\hat{\Gamma}_j(s) - \hat{\Gamma}_j(0)\right]}$$




Complex poles for sufficiently large:

- Field-particle coupling
- $\mathrm{Wi}_R = \tau_R v / R$, with $\tau_R \equiv \tau_\phi(q \sim \pi/R)$

[DV, A.Gambassi, arXiv:2304.09684]

# Memory-induced oscillations

## in correlated media

Linearized effective dynamics:

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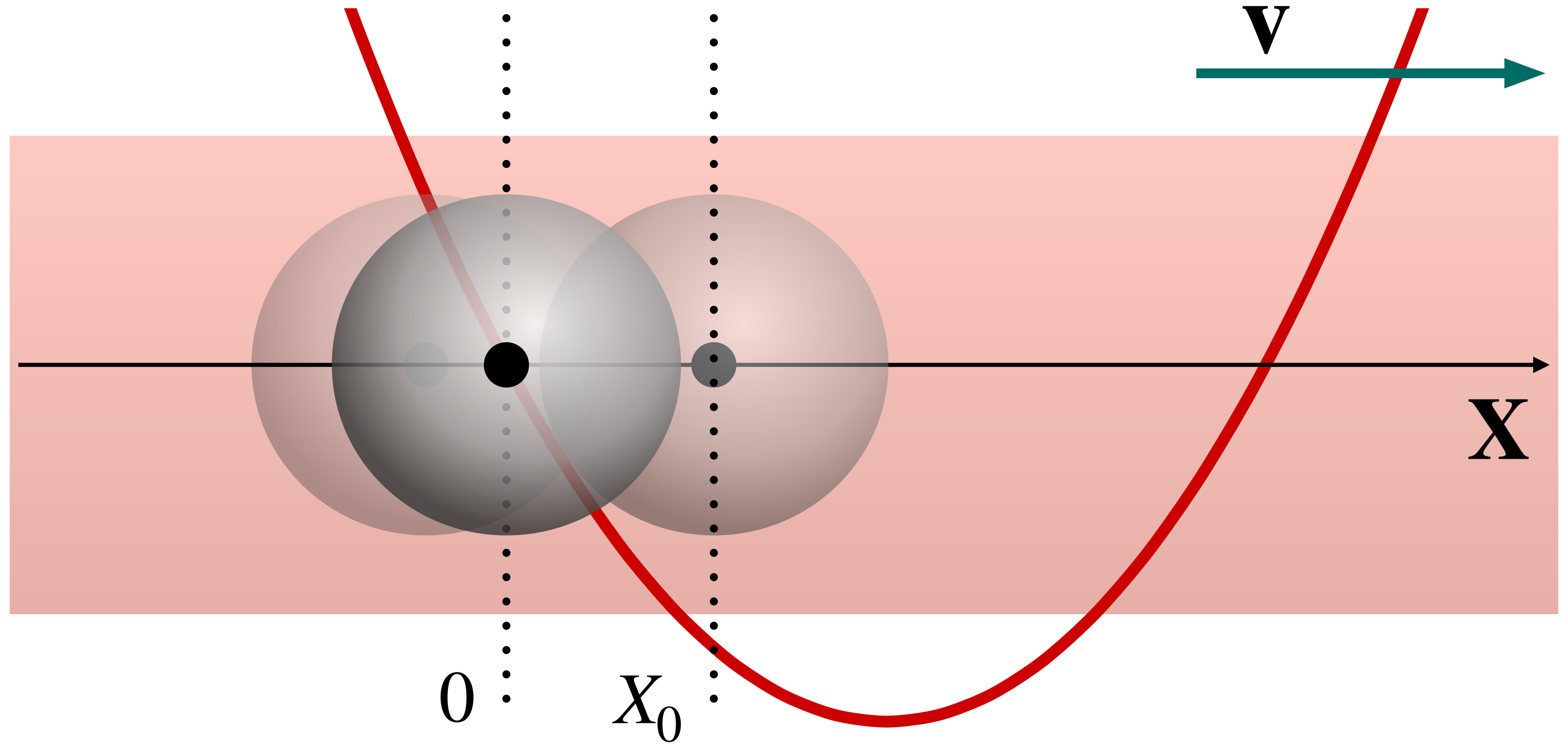
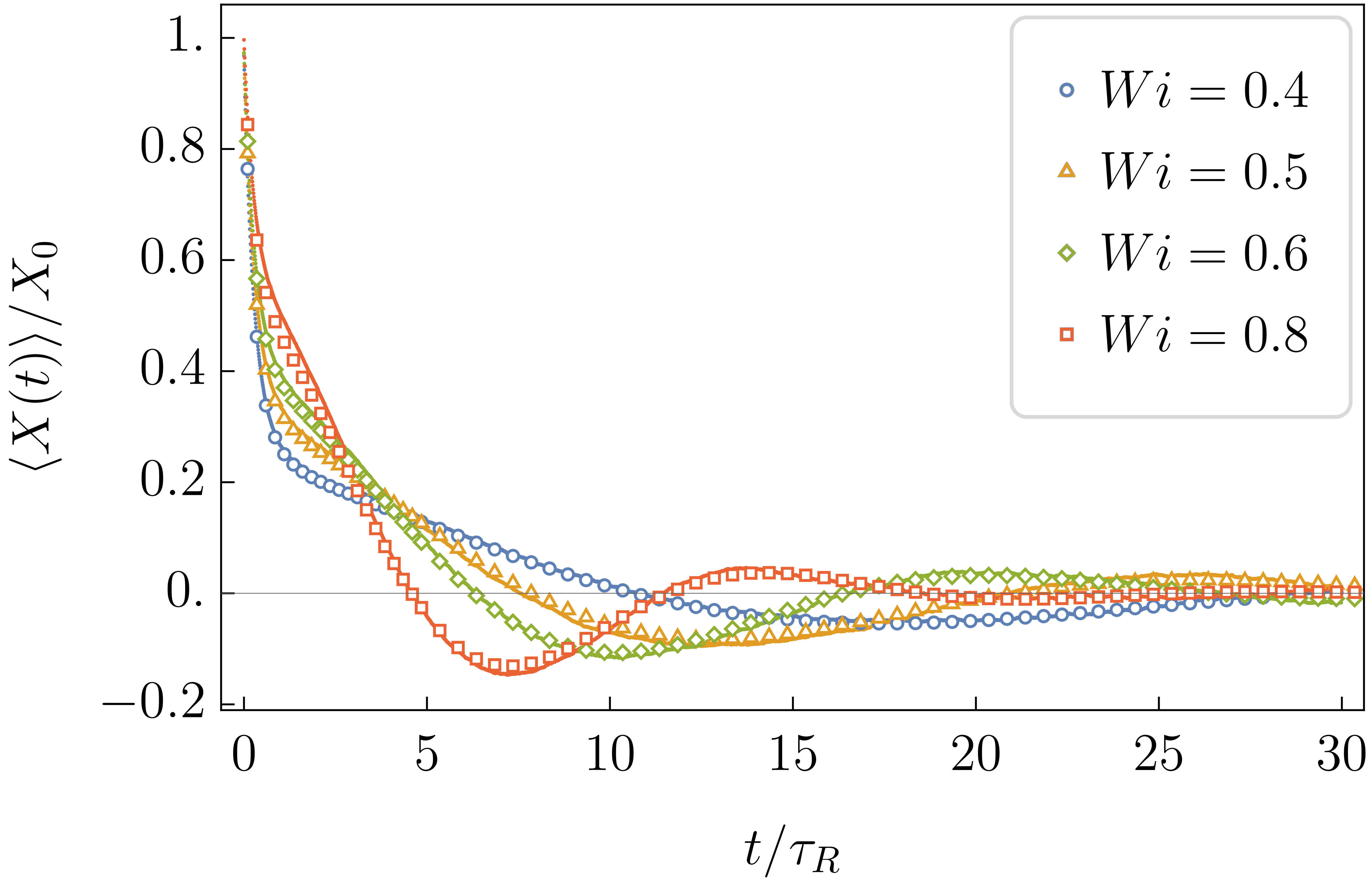
$$\left\langle \hat X_j(s)\right\rangle = \frac{X_0}{s+\gamma-\left[\hat\Gamma_j(s)-\hat\Gamma_j(0)\right]}$$

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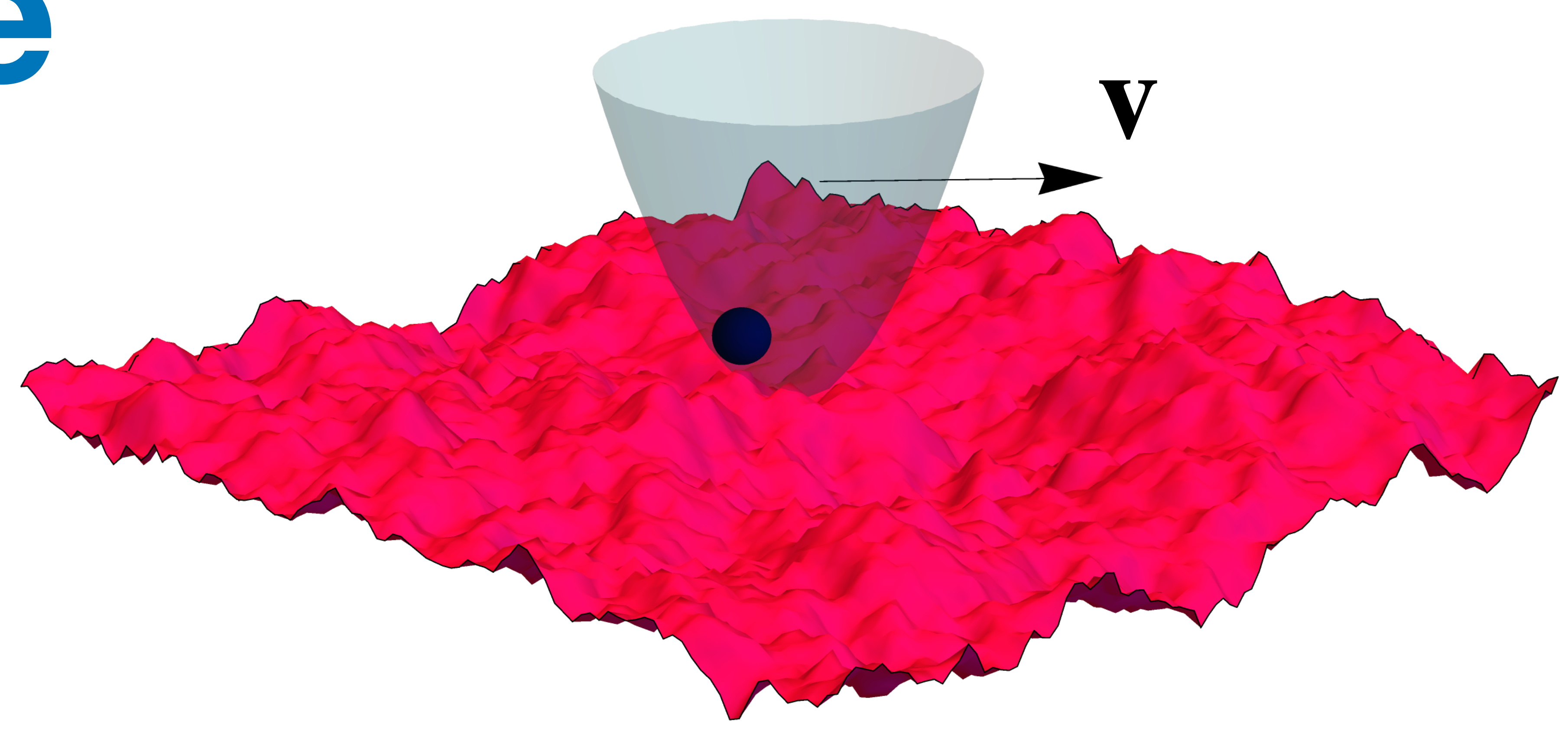
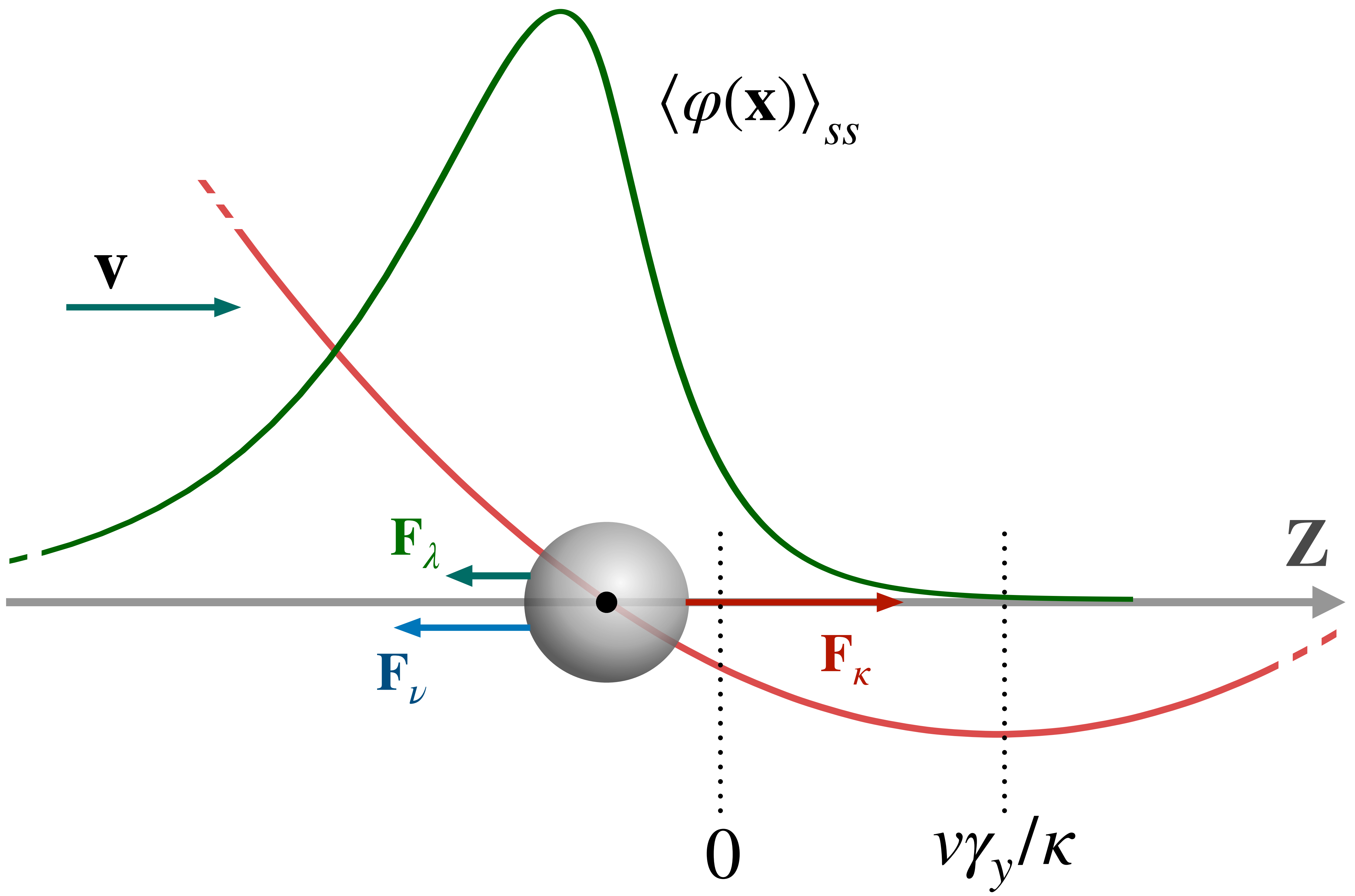
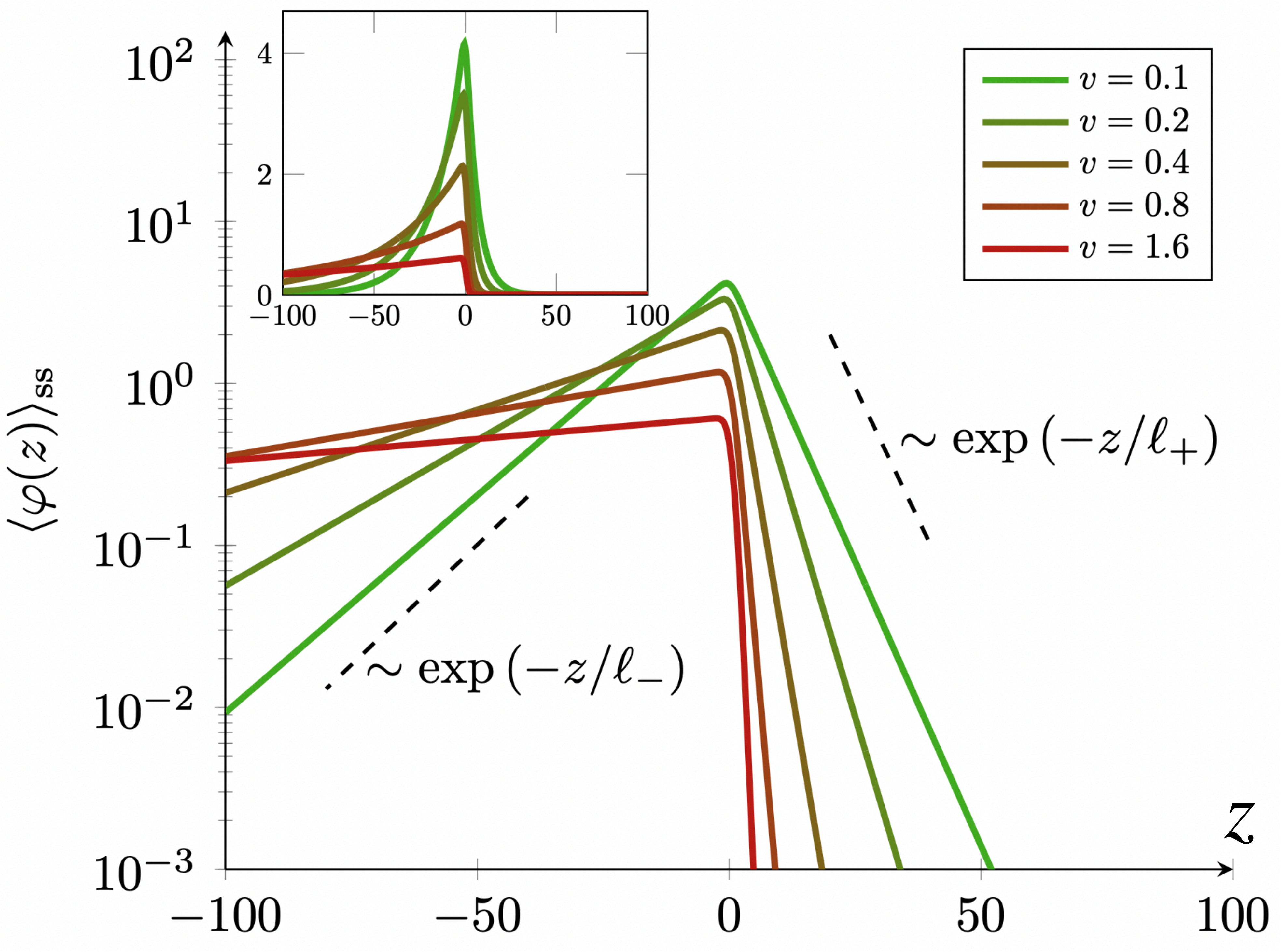
# Nonequilibrium steady state

## Particle driven at constant velocity

- $\mathbf{Z}$ = particle position in comoving frame
- $\langle \varphi \rangle_{ss}$ = avg field in comoving frame (*shadow*)



# Stochastic thermodynamics

## First law

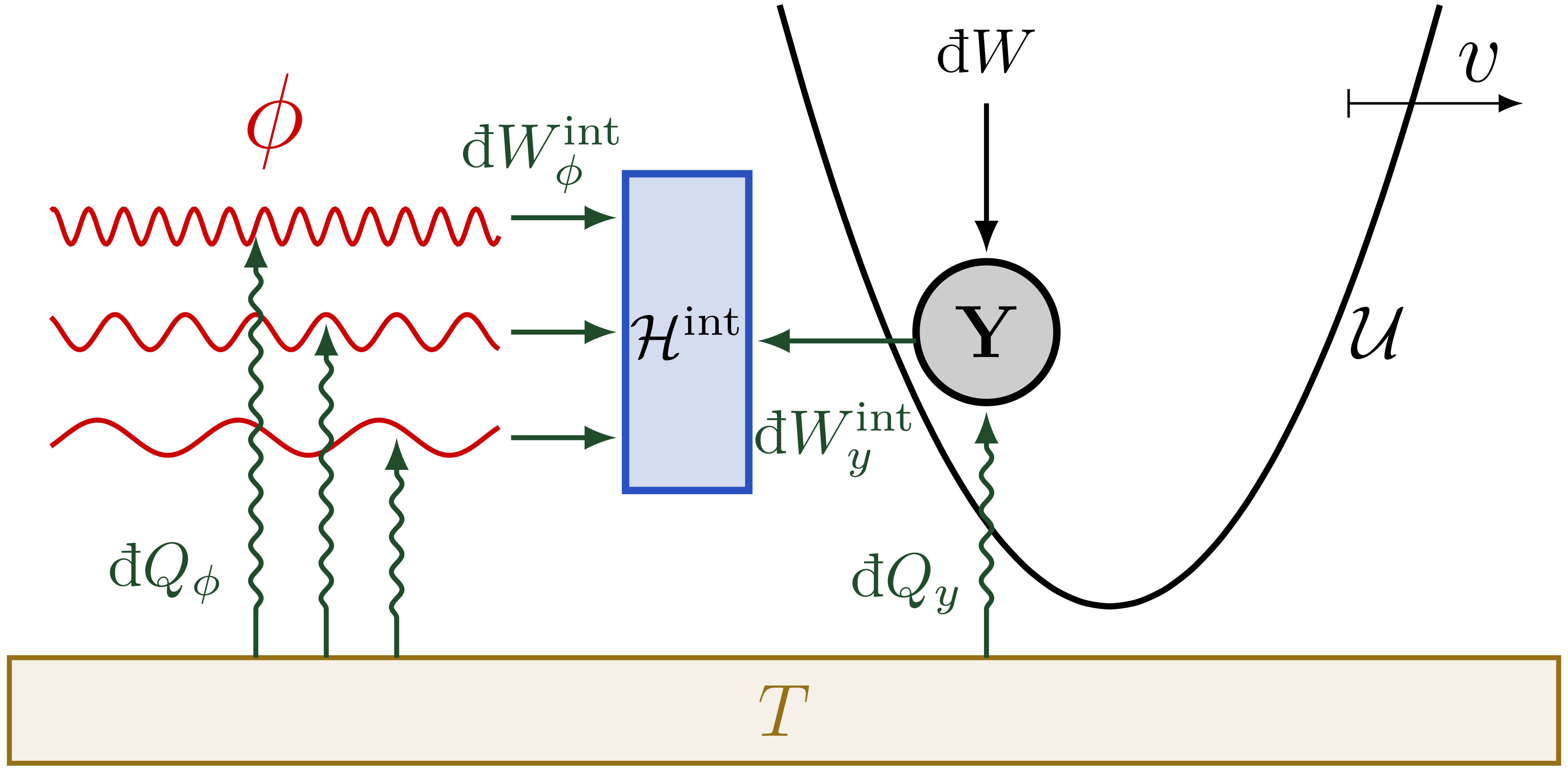
$$\mathrm{d}\mathcal{U} = đQ_y + đW_y^{\mathrm{int}} + đW$$

$$\mathrm{d}\mathcal{H}_\phi = \int \mathrm{d}^d\mathbf{x}\left[đQ_\phi(\mathbf{x}) + đW_\phi^{\mathrm{int}}(\mathbf{x})\right]$$

$$đW_y^{\mathrm{int}} := -\nabla_{\mathbf{Y}}\mathcal{H}^{\mathrm{int}} \circ \mathrm{d}\mathbf{Y}\,,$$

$$đW_\phi^{\mathrm{int}}(\mathbf{x}) := -\frac{\delta\mathcal{H}^{\mathrm{int}}}{\delta\phi(\mathbf{x})} \circ \mathrm{d}\phi(\mathbf{x}).$$

$$\mathcal{H}[\phi,\mathbf{Y},t] = \mathcal{H}_\phi[\phi] + \mathcal{H}^{\mathrm{int}}[\phi,\mathbf{Y}] + \mathcal{U}(\mathbf{Y},t)$$




[DV, S. M. Loos, B. Walter, É. Roldán, A. Gambassi, arxiv:2305.16235]

# Stochastic thermodynamics

## Average dissipated power

- Full CGF of the dissipated **power**

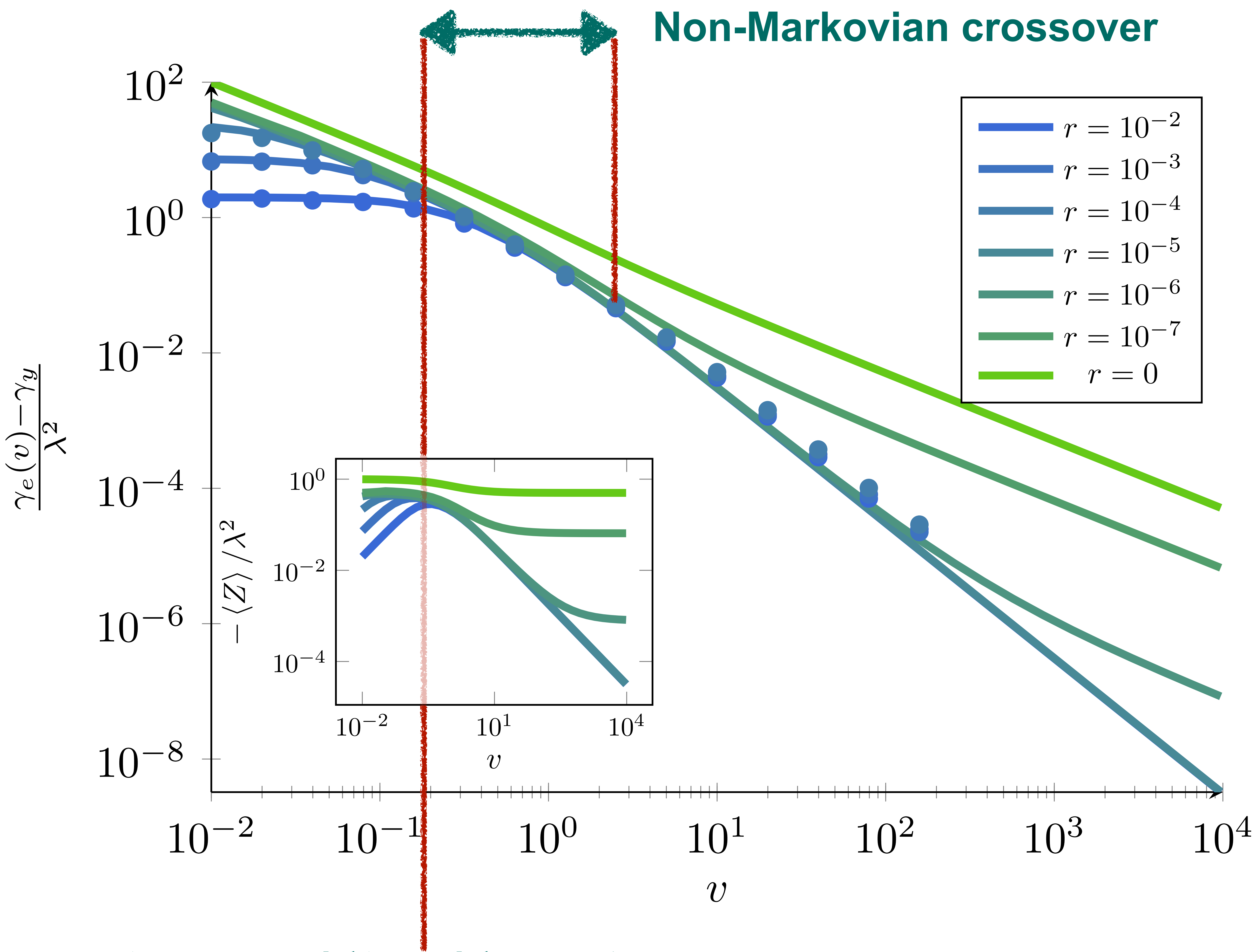
$$\dot{W} = -\kappa \mathbf{v} \cdot (\mathbf{Y} - \mathbf{v}t)$$

- Average value

$$\langle \dot{W} \rangle \equiv \gamma_e(v) v^2$$

- In the uncoupled case,

$$\gamma_e(v) \equiv \gamma_y$$




$$\mathrm{Wi}_\xi = \tau_\phi(q \sim \pi/\xi)\, v/\xi \equiv 1$$

[DV, S. M. Loos, B. Walter, É. Roldán,
A. Gambassi, arxiv:2305.16235]

# Stochastic thermodynamics

## Second law

$$\Delta S_{\text{tot}}[\{\mathbf{Y},\phi\}_{t_\text{i}}^{t_\text{f}}] = \ln \frac{\mathcal{P}[\{\mathbf{Y},\phi\}_{t_\text{i}}^{t_\text{f}}]}{\mathcal{P}^\text{R}[\{\mathbf{Y}^\text{R},\phi^\text{R}\}_{t_\text{i}}^{t_\text{f}}]} \geq 0$$

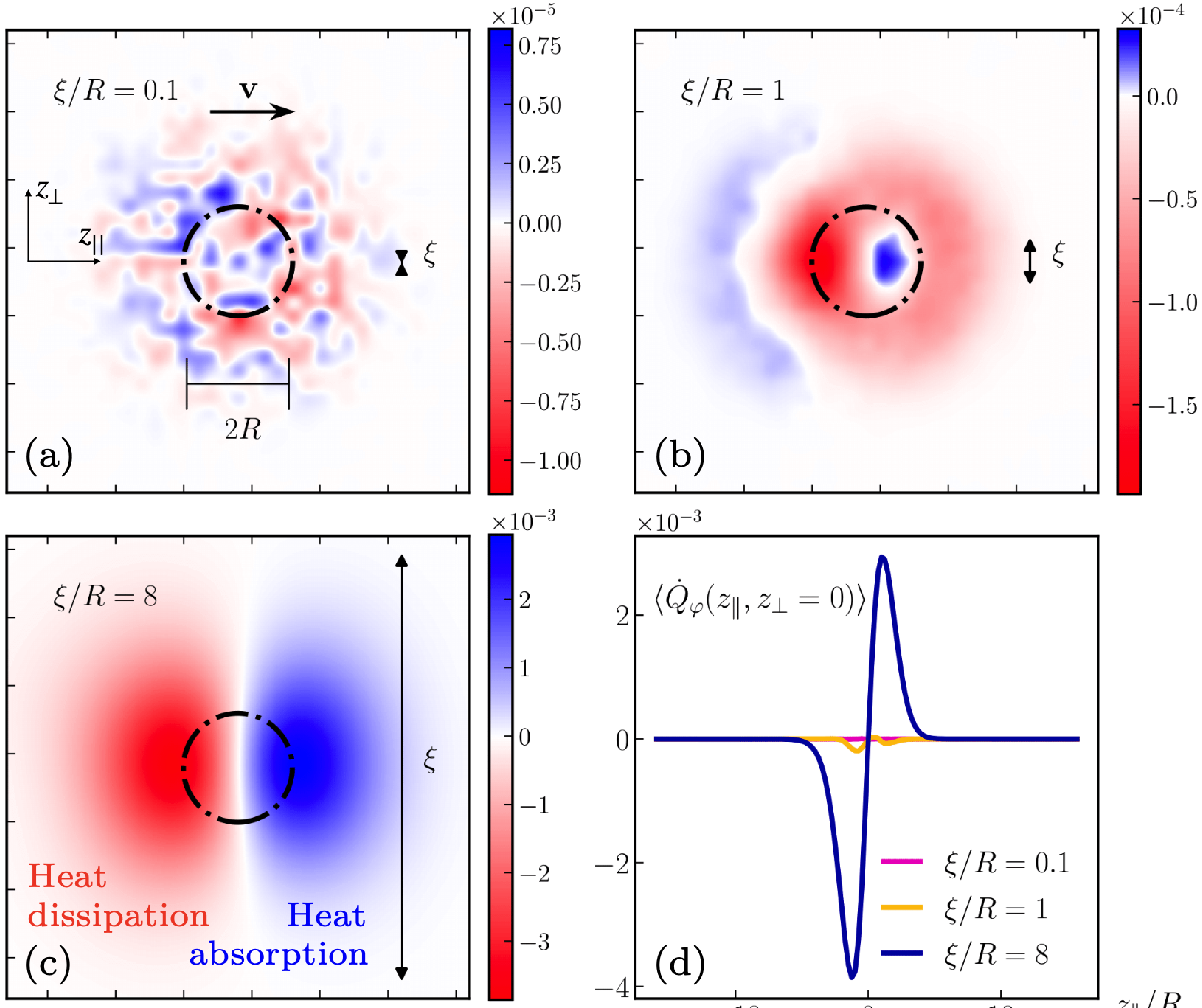
$$= -\frac{Q_y}{T} - \int \mathrm{d}^d\mathbf{x}\, \frac{Q_\phi(\mathbf{x})}{T} + \Delta S^{\text{sh}}_{y,\phi}$$

with

$$S^{\text{sh}}_{y,\phi} \propto -\ln \rho_{\mathbf{Y},\phi} \longleftarrow \text{Shannon entropy}$$

$$\text{đ}Q_y = (\boldsymbol{\xi} - \gamma_y \dot{\mathbf{Y}}) \circ \mathrm{d}\mathbf{Y}$$

$$\text{đ}Q_\phi(\mathbf{x}) = \left\{(-\nabla^{-2})^{\alpha/2}\left[\eta(\mathbf{x}) - \gamma_\phi \dot{\phi}(\mathbf{x})\right]\right\} \circ \mathrm{d}\phi(\mathbf{x})$$



# To sum up

- Particle + field with spatio-temporal correlations
- Memory-induced oscillations
- Spatially-resolved stochastic thermodynamics

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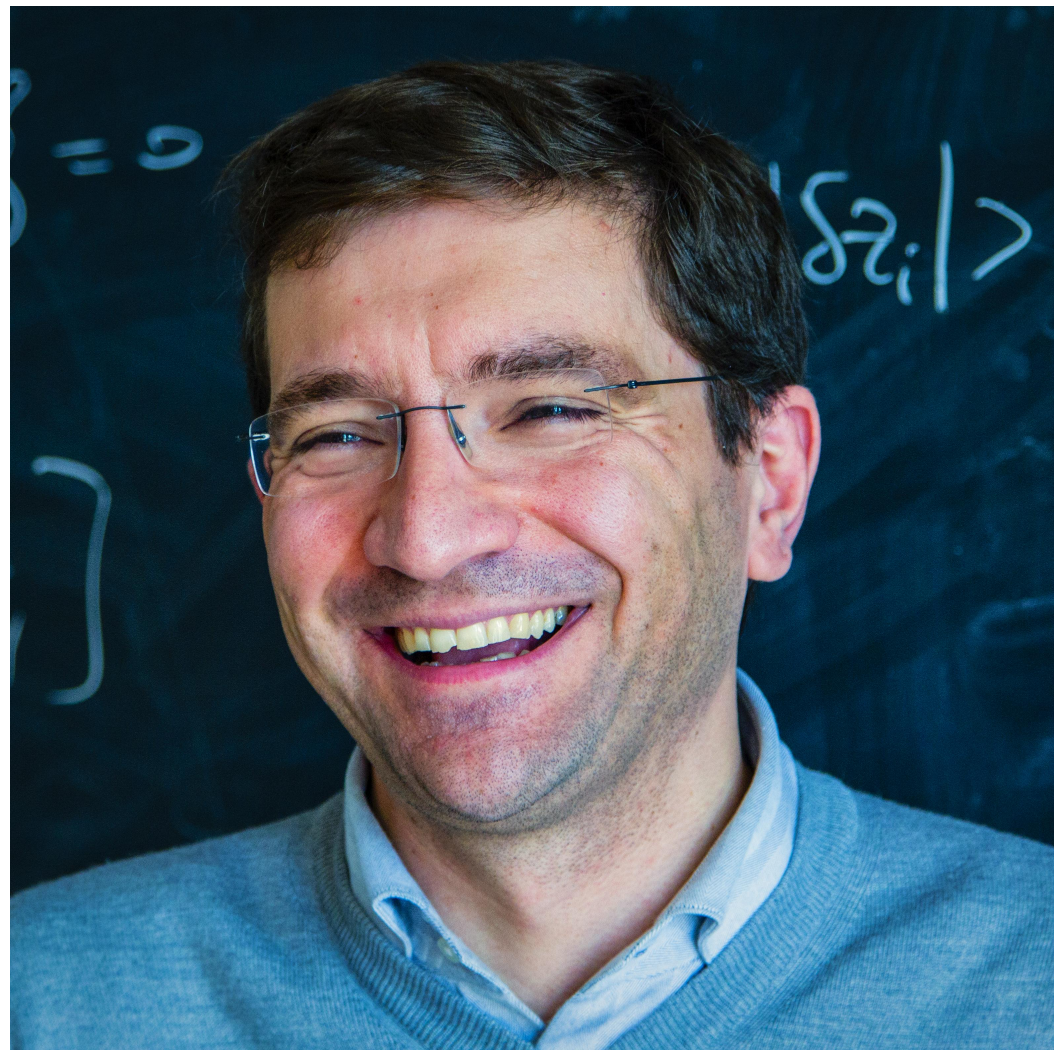

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