Probing the fractal phase of a random matrix using replicas

Davide Venturelli

Journées de Physique Statistique, ENS Paris, 25 January 2023

LPTMC

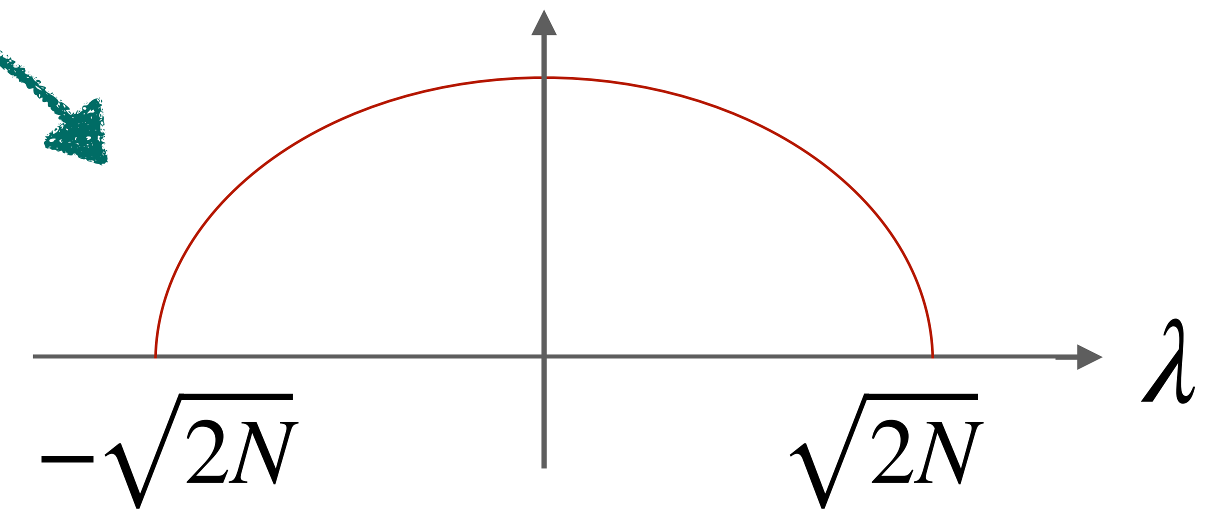
SISSA

LPTHE
LABORATOIRE DE PHYSIQUE THEORIQUE ET HAUTES ENERGIES

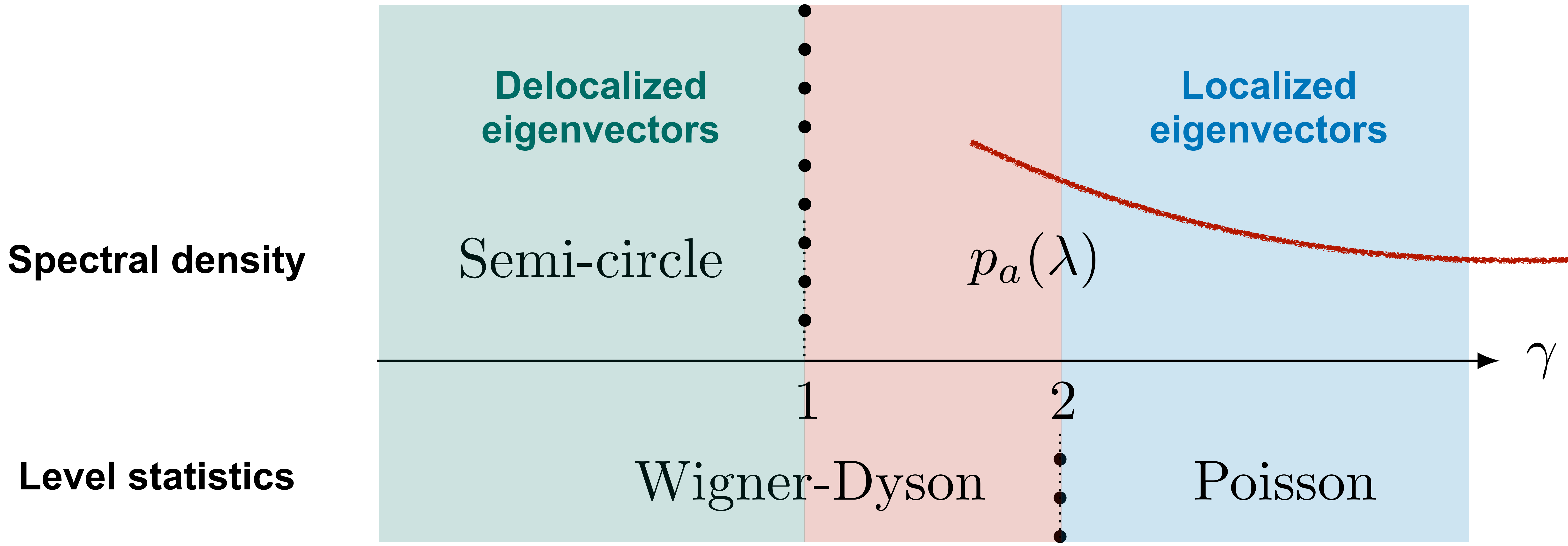
The generalized Rosenzweig-Porter model

$$H = \begin{pmatrix} a_{11} & & & 0 \\ & a_{22} & & \\ & & \ddots & \\ 0 & & & a_{NN} \end{pmatrix} + \frac{\nu}{N^{\gamma/2}} \underbrace{\begin{pmatrix} & & \\ & \mathbf{GOE} & \\ & & \end{pmatrix}}_{N}$$

$p_a(a_{ii})$



$$\rho_N(\lambda) = \frac{1}{N}\sum_{i=1}^{N}\delta(\lambda - \lambda_i)$$

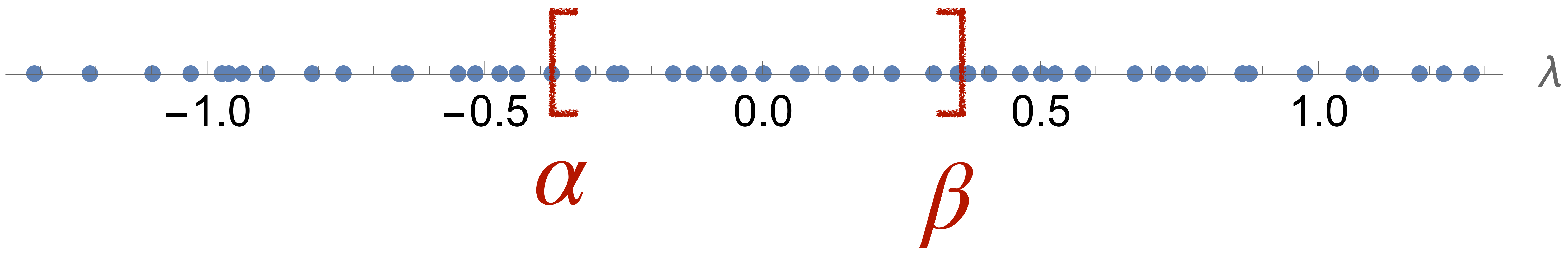


$1 < \gamma < 2$

Fractal phase: partially delocalized eigenvectors

Kravtsov et al., 2015 New J. Phys. 17 122002

Level statistics: # of eigenvalues in an interval



$$I_N[-\alpha,\beta] \equiv N \int_{\alpha}^{\beta} \mathrm{d}\lambda \, \rho_N(\lambda)$$

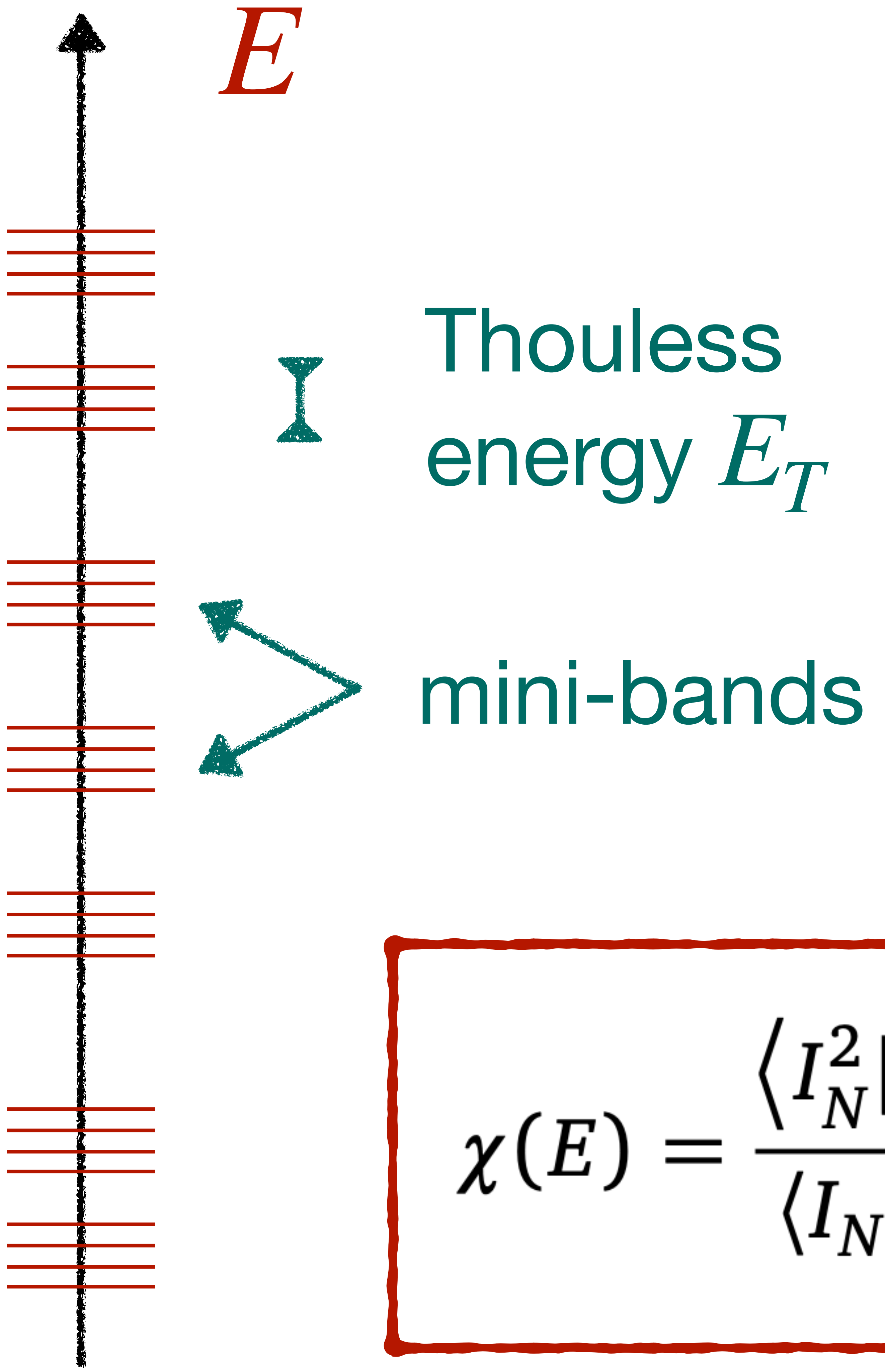
Cumulant generating function:

$$\mathcal{F}_{[-E,E]}(s) \equiv \lim_{N\to\infty} \frac{1}{N} \ln \left\langle e^{-s I_N[-E,E]} \right\rangle$$

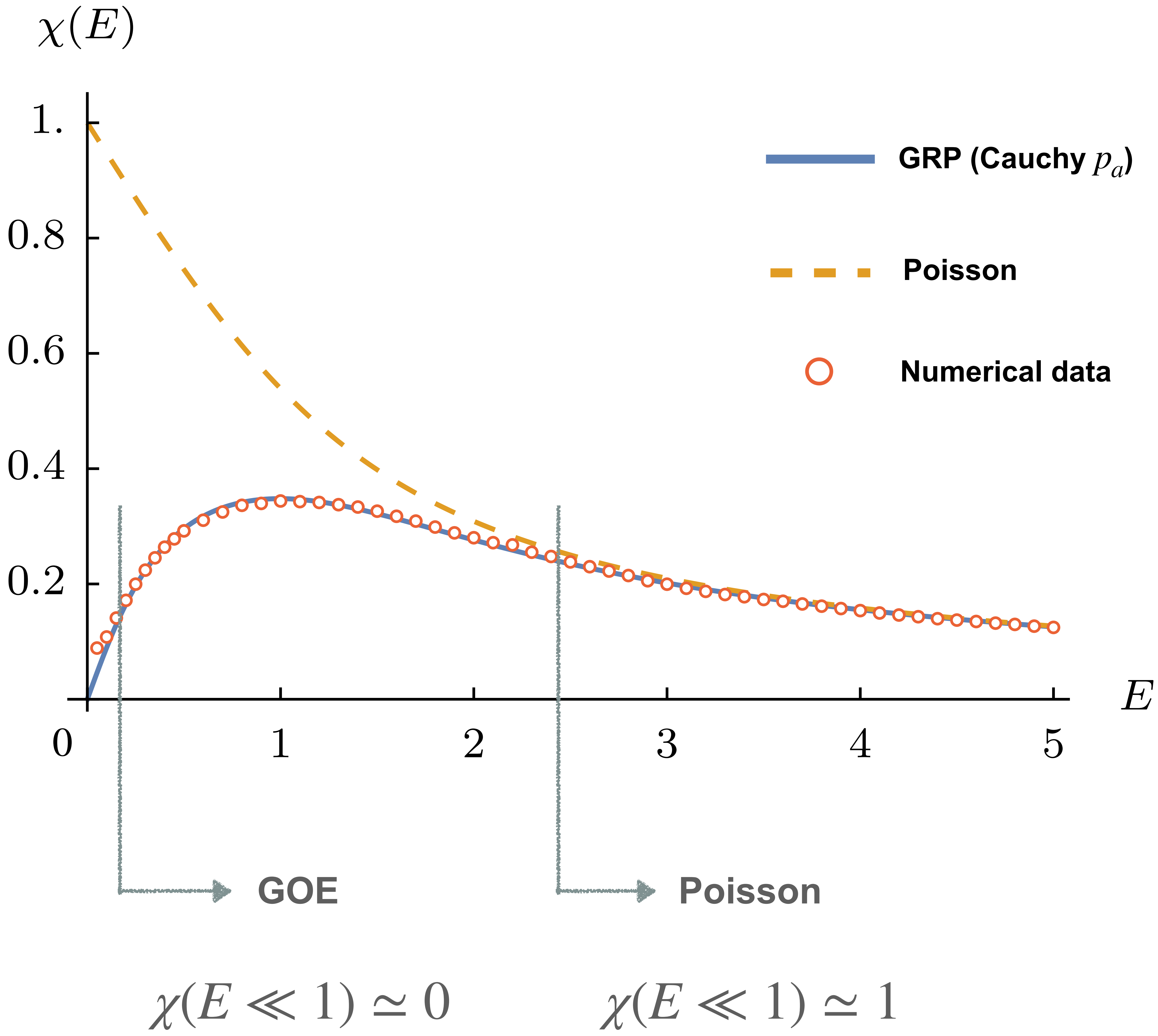
can be accessed using the **replica** method

Details of replica calculation

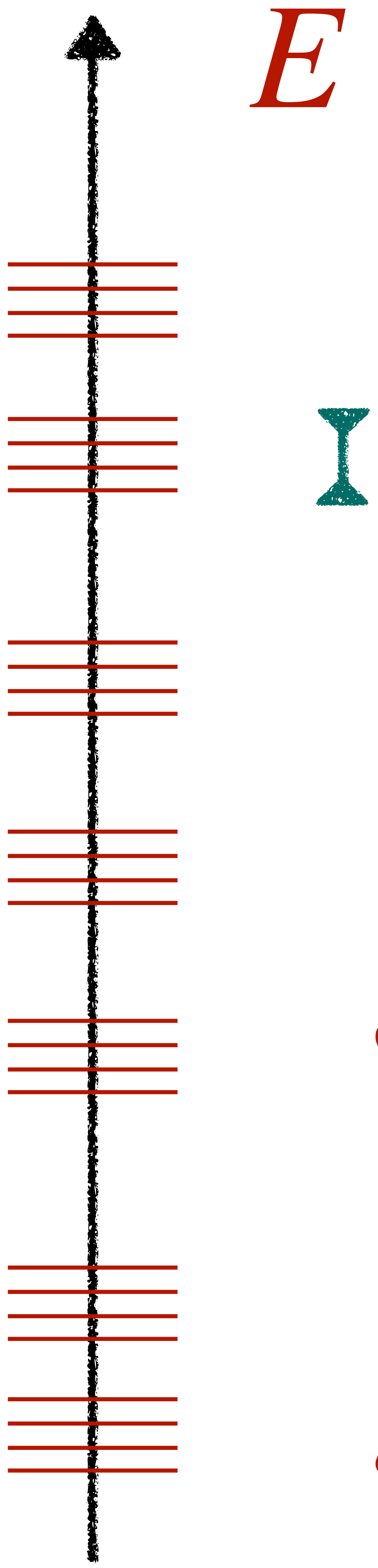
Level compressibility



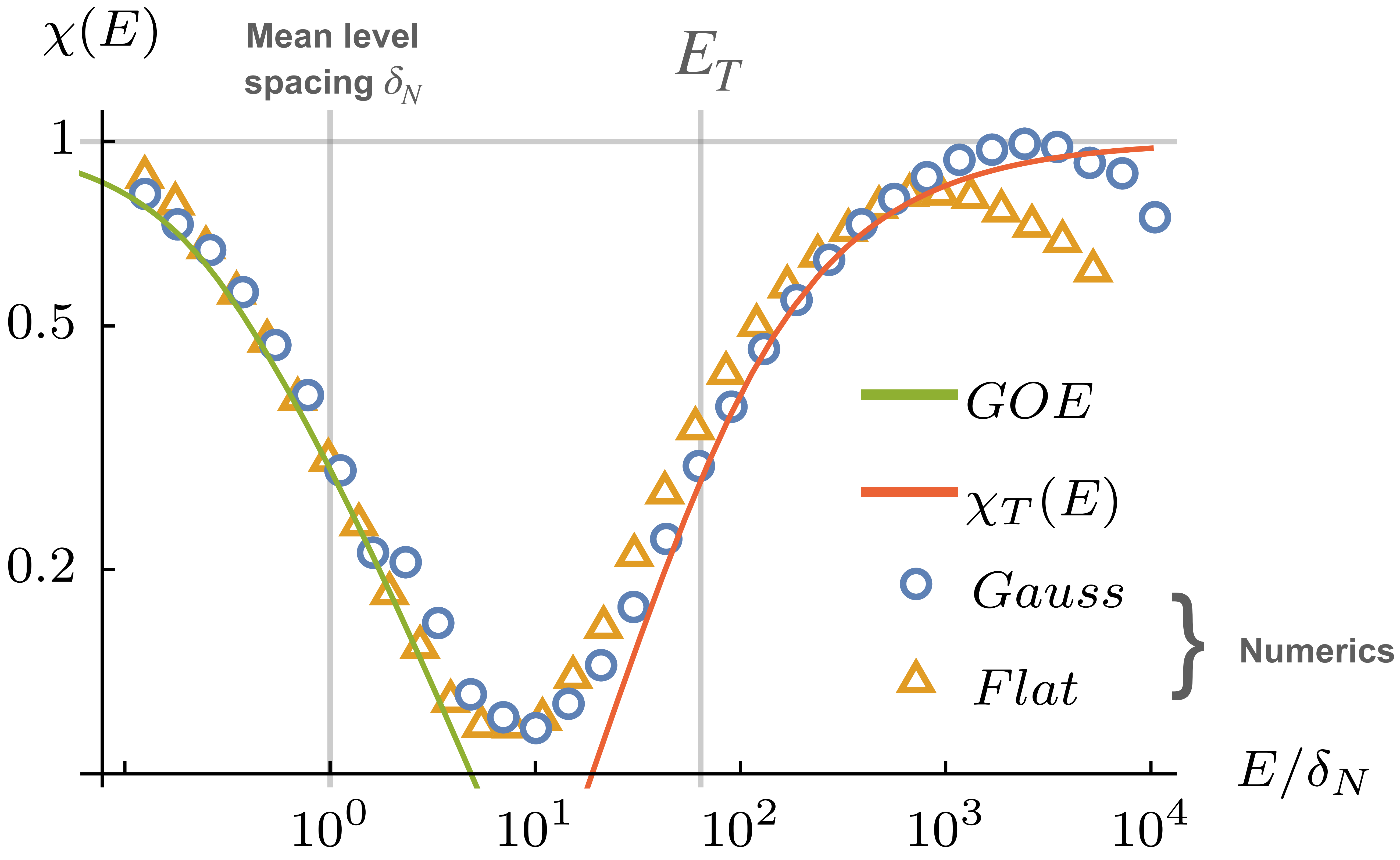
$$\boxed{\chi(E) = \frac{\left\langle I_N^2[-E,E]\right\rangle_c}{\langle I_N[-E,E]\rangle}}$$



Level compressibility



Thouless energy E_T



$$\boxed{\begin{aligned}\chi(E) &\simeq \chi_T\left(y = E/E_T\right)\\ &= \frac{1}{\pi y}\left[2y\arctan(y) - \ln(1+y^2)\right]\end{aligned}}$$

universal for $E \sim E_T$,
independent of $p_a(a_{ii})$

Venturelli, Cugliandolo, Schehr, Tarzia, SciPost Phys. 14, 110 (2023)