Tracer-bath correlations in d-dimensional interacting particle systems

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Work in collaboration with P. Illien, T. Berlioz, A. Grabsch, O. Bénichou

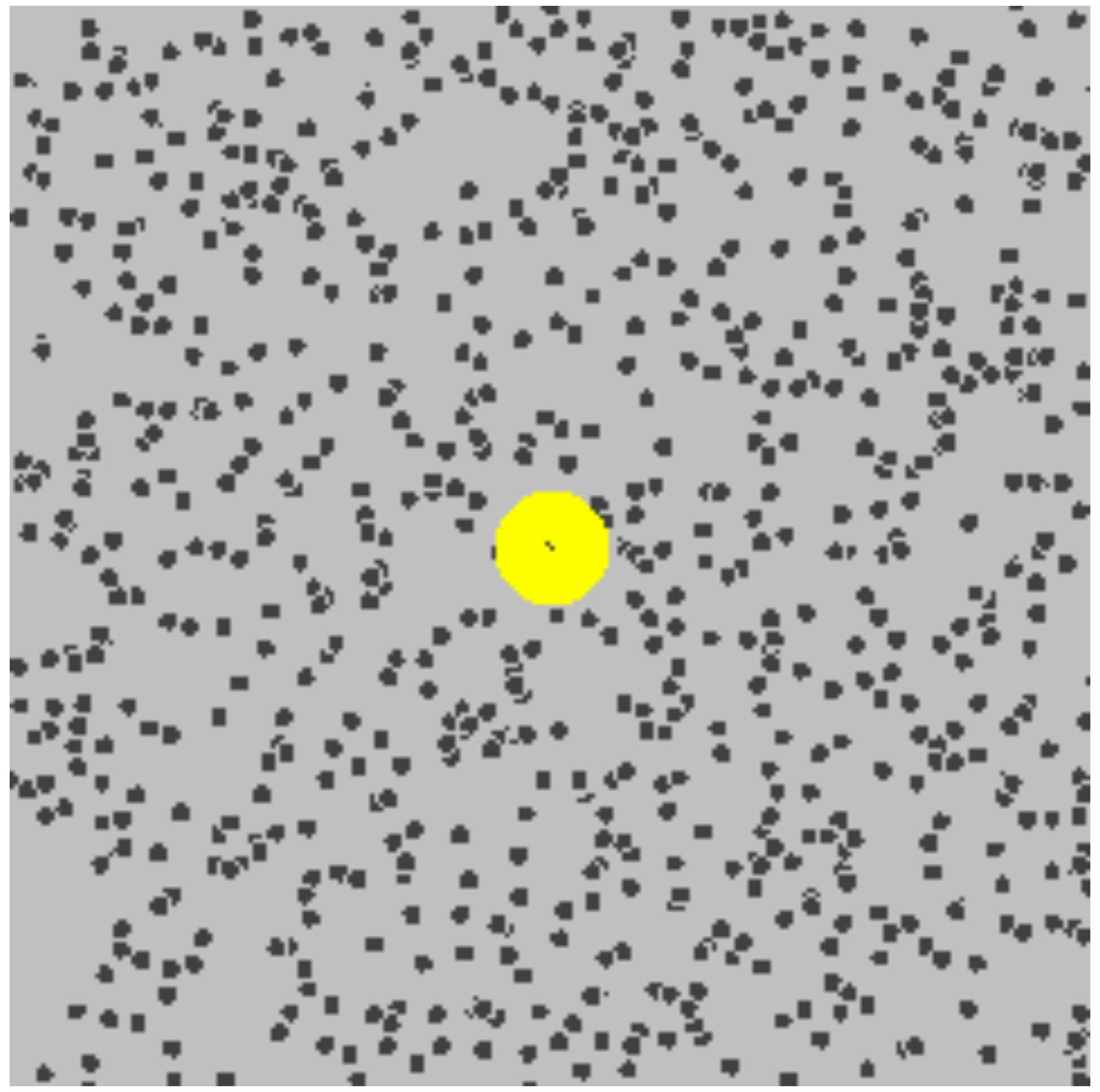
Tracer particle
in a thermal bath

$$m\,\ddot{X}(t) = -\gamma \dot{X}(t) + \zeta(t)$$
$$\langle \zeta(t)\zeta(t')\rangle = 2\gamma k_B T\,\delta(t-t')$$

- **Brownian motion**:
  bath in **equilibrium**, **structureless**,
  no **tracer-bath** correlations, **diffusive** behaviour

$$\langle X^n(t)\,\zeta(t)\rangle = 0, \qquad \left\langle X^2(t)\right\rangle \propto t$$

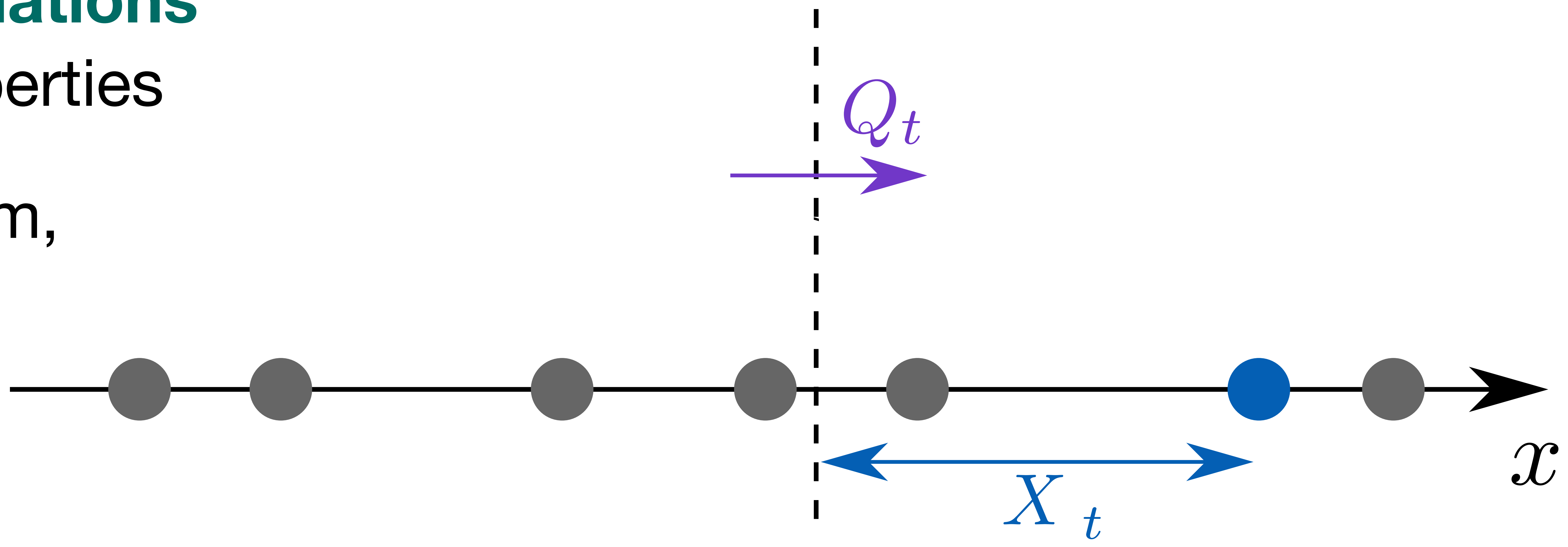
- What if particles have similar sizes?



Classical interacting particle systems

and why we still talk about them

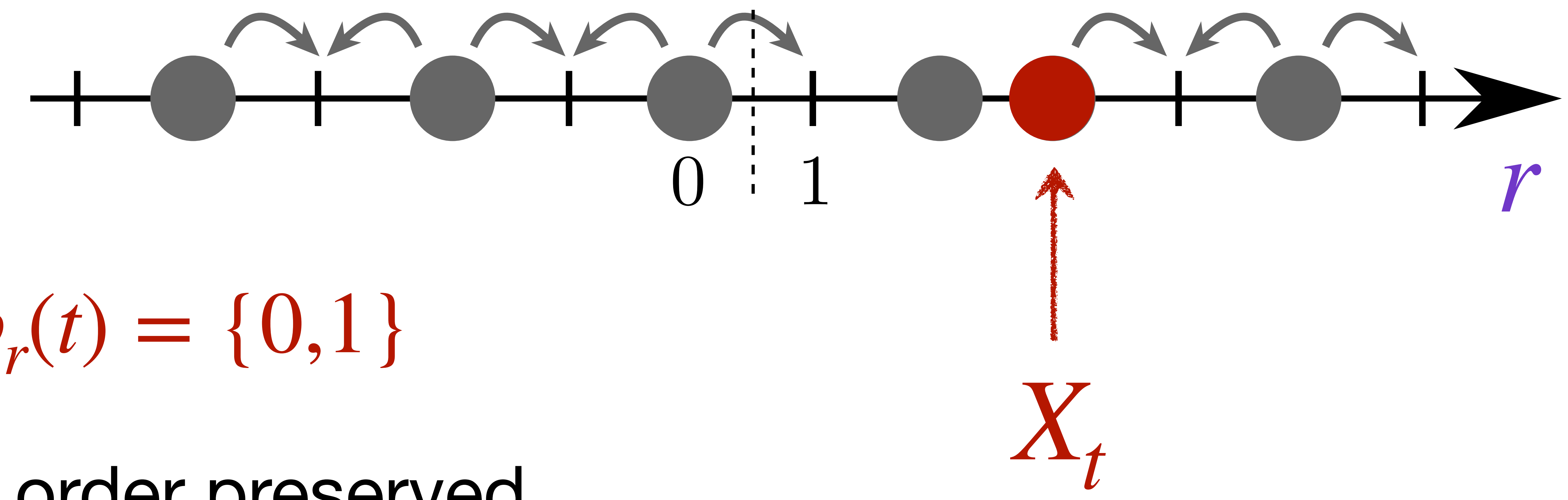
- Lattice gases, interacting Brownian particles, simple liquids…
- Dynamics, transport properties
- **Local observables**: integrated current Q_t , position X_t of tagged particle
- Random due to thermal **fluctuations**
  $\longrightarrow$ determine statistical properties
- Interacting many-body problem,
  out of equilibrium



Symmetric Exclusion Process

as paradigmatic diffusive system

- Particles on a lattice
  + random hoppings (equal rates),
  only if target site is empty



- **State of the system**: occupations $\rho_r(t) = \{0,1\}$

- In $1d$, **single-file** geometry $\rightarrow$ initial order preserved

- Subdiffusive behavior of tracer

$$\langle X_t^2 \rangle \propto \sqrt{t}$$

(zeolites, confined colloids, dipolar spheres…)

Lin, Meron, Cui, Rice, Diamant, Phys. Rev. Lett. **94** (21), 216001 (2005)

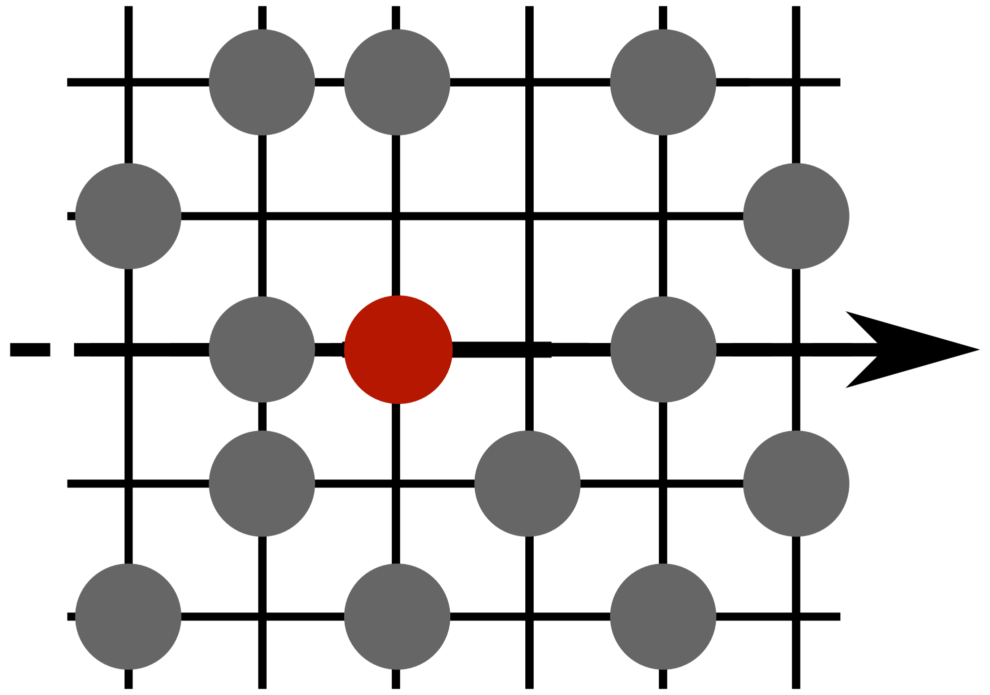
Wei, Bechinger, Leiderer, Science **287** (5453), 625-7 (2000)

Hahn, Kärger, Kukla, Phys. Rev. Lett. **76** (15), 2762-2765 (1996)

H. Spohn, *Large scale dynamics of interacting particles* (1991)

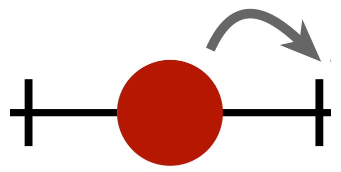
Role of correlations

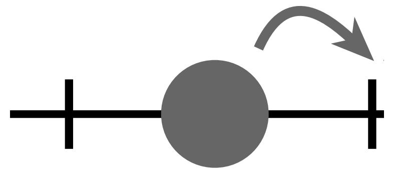
with surrounding bath



σ

- Master equation





$$\partial_t P(X,\underline{\rho},t) = \left[\mathscr{L}_{\textbf{tracer}} + \mathscr{L}_{\textbf{bath}}\right] P(X,\underline{\rho},t)$$

- Multiply by $\mathrm{e}^{\lambda\cdot X}$ and average,

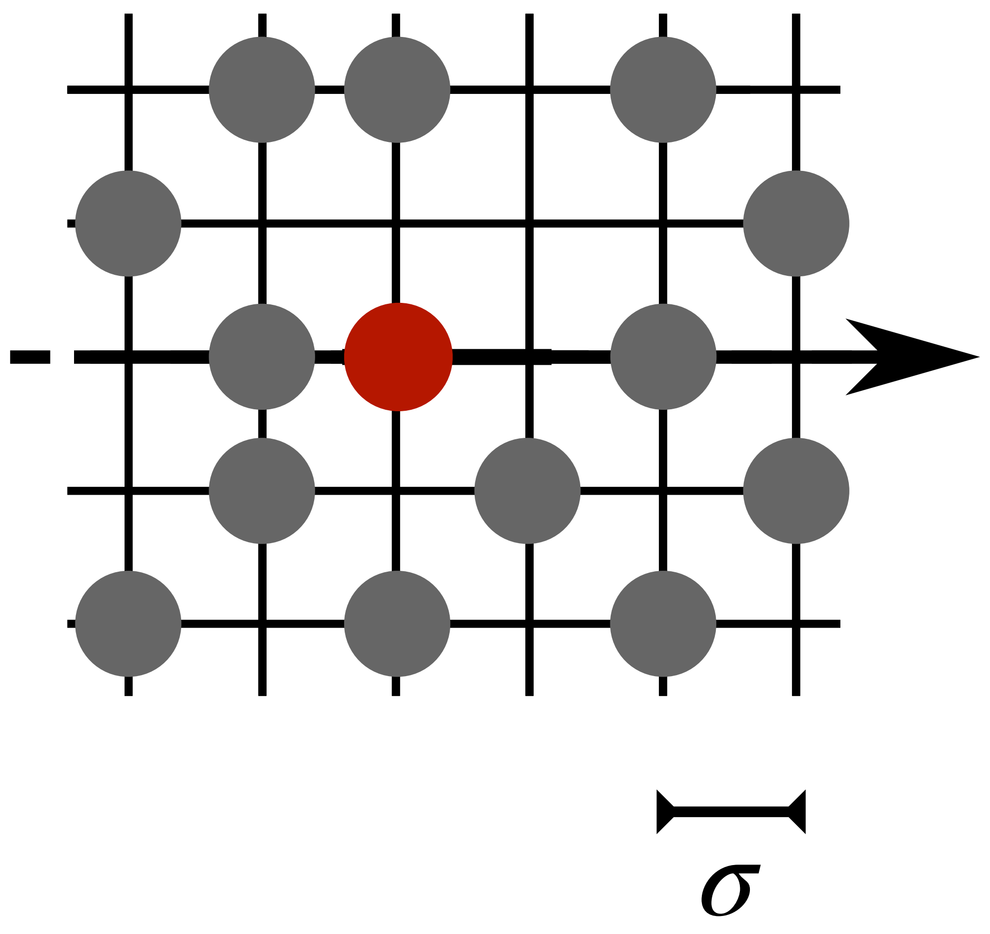
$$\partial_t \Psi(\lambda,t) = \frac{1}{2d\,\tau}\sum_{\mu=-d}^{d}\left(\mathrm{e}^{\sigma\lambda\cdot\hat{\mathbf{e}}_\mu}-1\right)\left[1-w_{\mathbf{e}_\mu}(\lambda,t)\right]$$

$$\Psi(\lambda,t) = \ln\left\langle \mathrm{e}^{\lambda\cdot X}\right\rangle = \sum_{n=1}^{\infty}\frac{\lambda^n}{n!}\langle X^n\rangle_c,$$

$$w_r(\lambda,t) = \frac{\left\langle \rho_{X+r}\,\mathrm{e}^{\lambda\cdot X}\right\rangle}{\left\langle \mathrm{e}^{\lambda\cdot X}\right\rangle} = \sum_{n=0}^{\infty}\frac{\lambda^n}{n!}\left\langle \rho_{X+r}X^n\right\rangle_c$$

Role of correlations

with surrounding bath

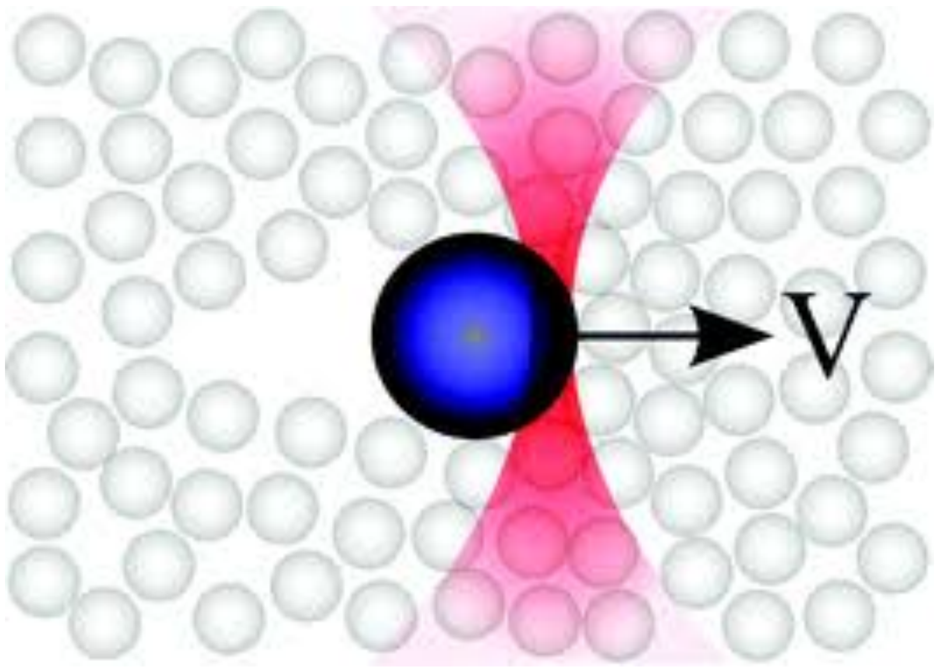


$$\partial_t \Psi(\boldsymbol{\lambda}, t) = \frac{1}{2d\,\tau} \sum_{\mu=-d}^{d} \left(e^{\sigma \boldsymbol{\lambda} \cdot \hat{\mathbf{e}}_\mu} - 1 \right) \left[1 - w_{\mathbf{e}_\mu}(\boldsymbol{\lambda}, t) \right]$$

- Knowing $w_{\mathbf{e}_\mu}(\boldsymbol{\lambda}, t)$ on neighbouring sites is enough to deduce $\Psi(\boldsymbol{\lambda}, t)$

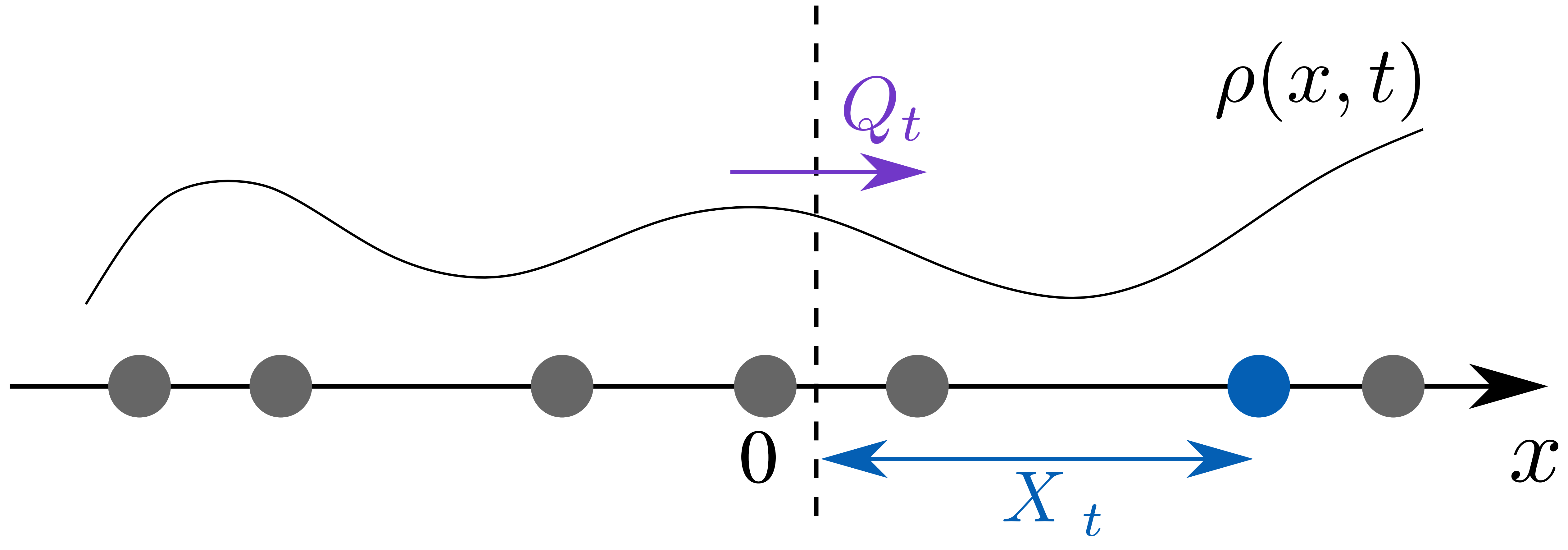
- $\partial_t w_{\mathbf{e}_\mu}(\boldsymbol{\lambda}, t) = \dots \left[w_{\mathbf{r}}(\boldsymbol{\lambda}, t) \right] \dots$
  generically depends on $w_{\mathbf{r}}(\boldsymbol{\lambda}, t)$ even from far away $\boldsymbol{r}$
  $\longrightarrow$ **strongly correlated**

- $w_{\mathbf{r}}(\boldsymbol{\lambda}, t)$ encodes the **response** of the bath



Integrated current Q_t

- Q_t = net # of particles crossing (0—1) $\rightarrow$ in $d = 1$, $\langle Q_t^2 \rangle \propto \sqrt{t}$
- A positive fluctuation of Q_t is correlated with an increase of $\rho_r(t)$ on its r.h.s.
  $\rightarrow$ **correlations dictate the subdiffusive behavior of** Q_t
- $\left\langle \rho_r(t)\, \mathrm{e}^{\lambda Q_t} \right\rangle$ encodes the **response** of the bath
- Fully understood in 1d SEP
- Open problem in $d > 1$

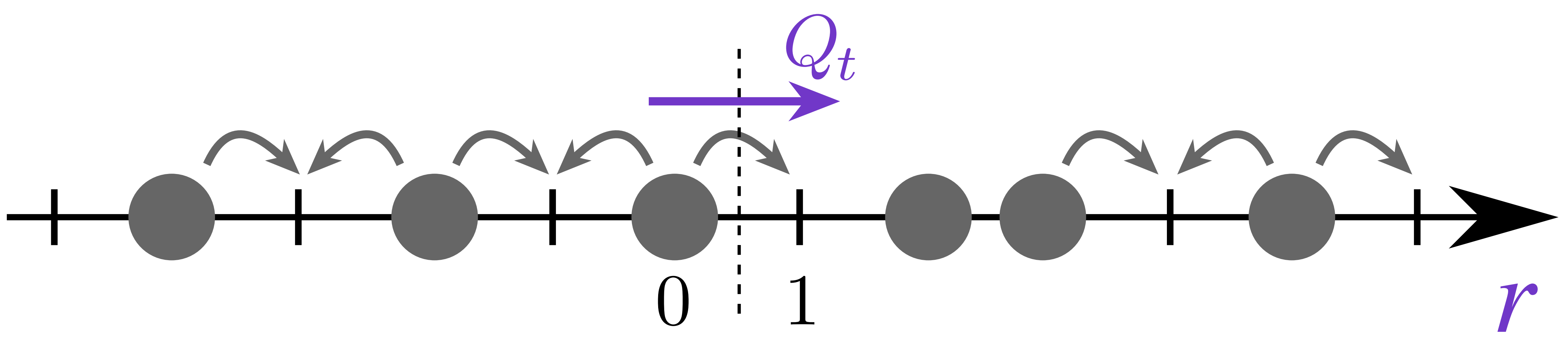


Grabsch, Poncet, Rizkallah, Illien, Bénichou, Sci. Adv. 8, eabm5043 (2022)

1. Current fluctuations

- As **first step**, focus on

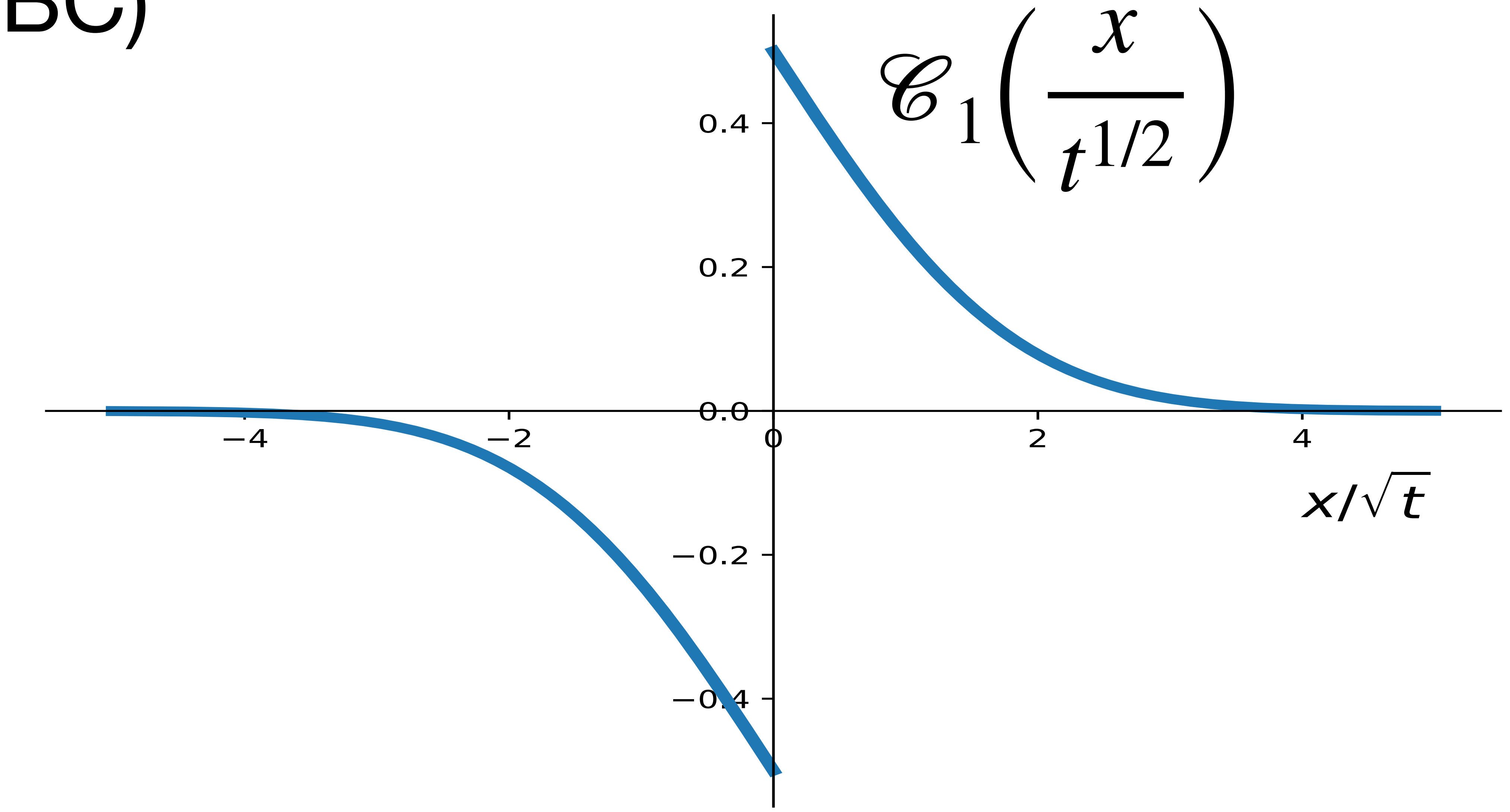
$$c_r(t) \equiv \langle Q_t \, \rho_r(t) \rangle$$



- **Fact 1:** infinite lattice $d = 1$ (no reservoirs, no PBC)

$$c_r(t) \xrightarrow[t \to \infty]{} \bar{\rho}(1-\bar{\rho})\, \mathscr{C}_1\left(\frac{r}{t^{1/2}}\right),$$

$$\langle Q_t^2 \rangle \propto \bar{\rho}(1-\bar{\rho})\, t^{1/2}$$



- **Fact 2:** finite systems (any spatial dimension d)

$$c_{\vec{r}}(t) \xrightarrow[t \to \infty]{} c_{\vec{r}}\,, \qquad \langle Q_t^2 \rangle \propto t$$

Recall $\partial_t \langle Q_t^2 \rangle = \dots \left[c_{\vec{r}}(t) \right] \dots$

infinite lattices in $d > 1$?

From $d = 1$ to higher d

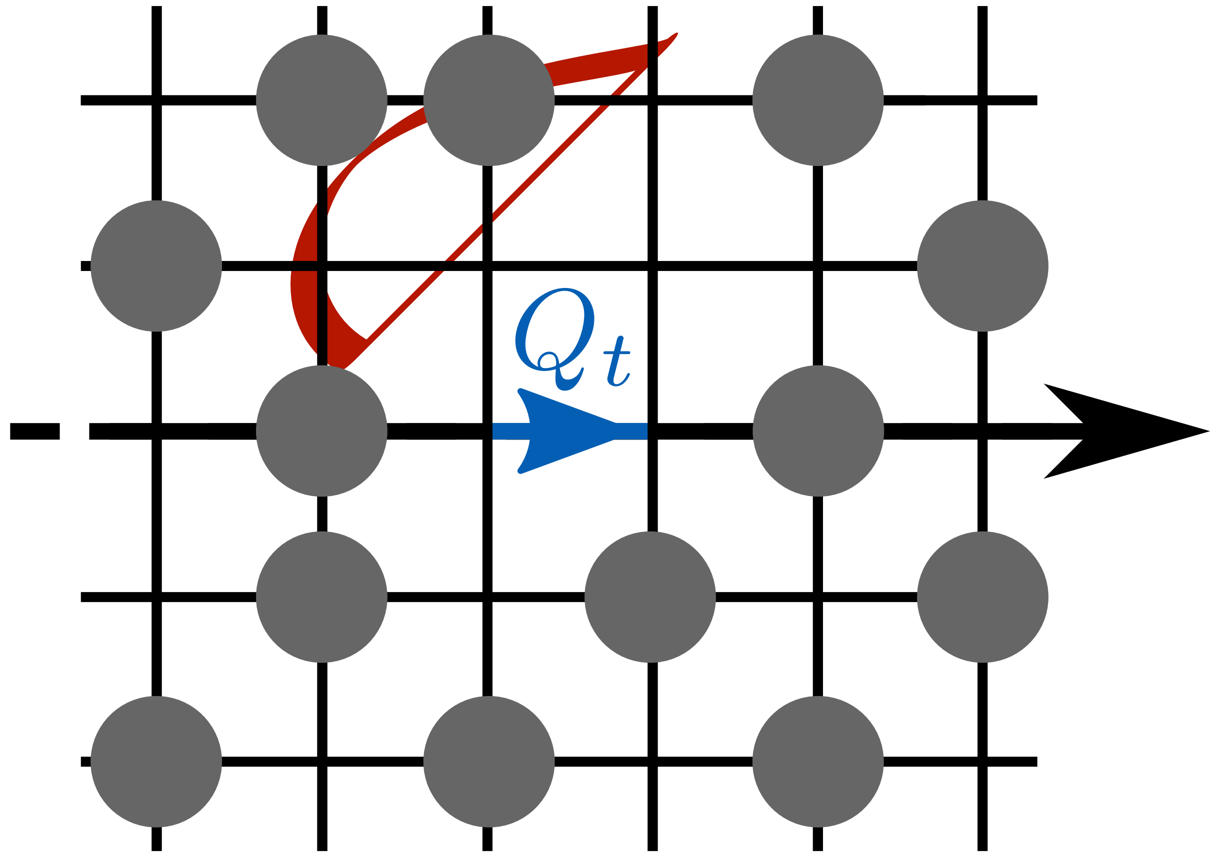
passing through the comb

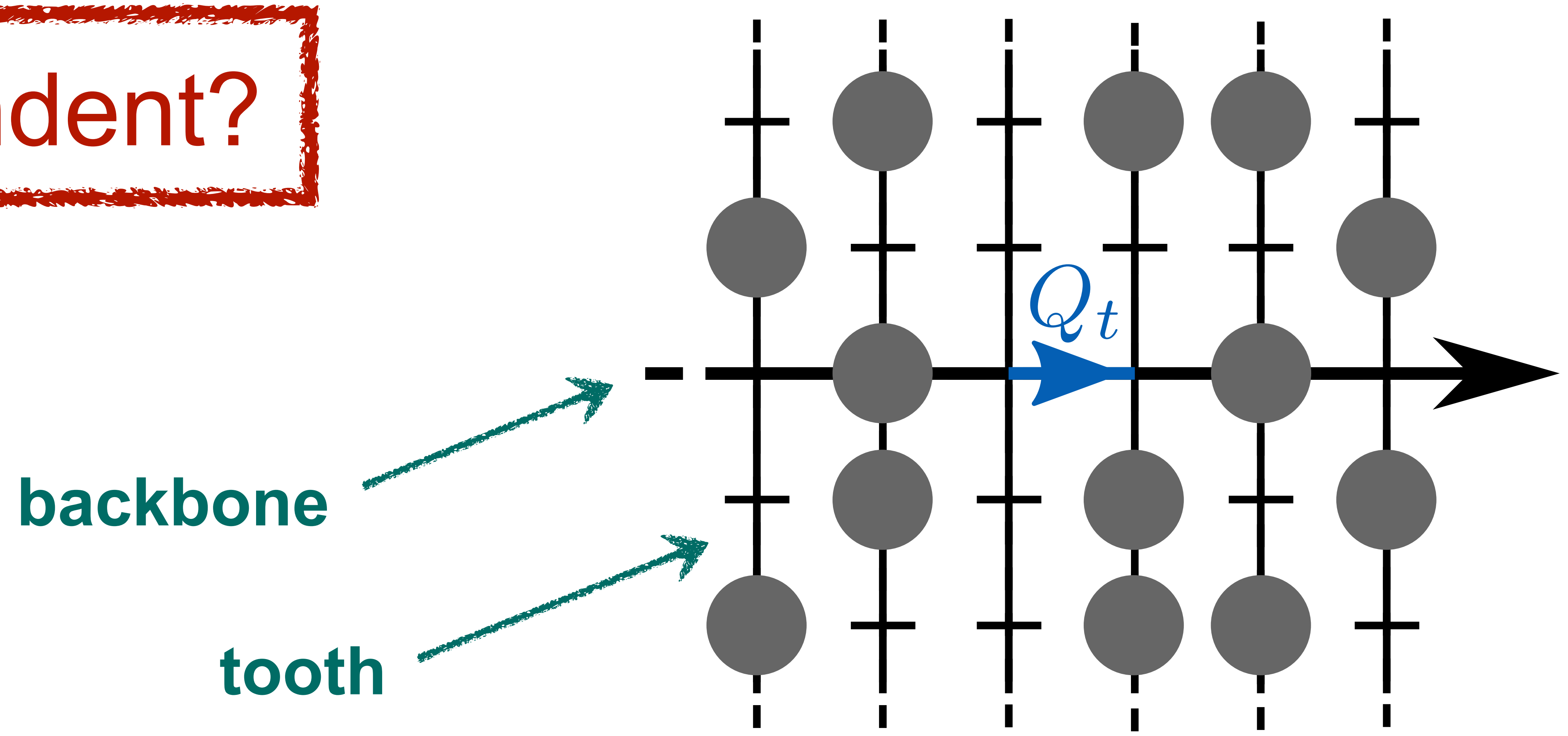
1. Order-preserving (single-file)
2. Tree-like (no loops!)

Do we need both, to make $c_r(t)$ time dependent?

- **Higher d** : ~~(1)~~ ~~(2)~~
- **Comb lattice** : ~~(1)~~ (2)

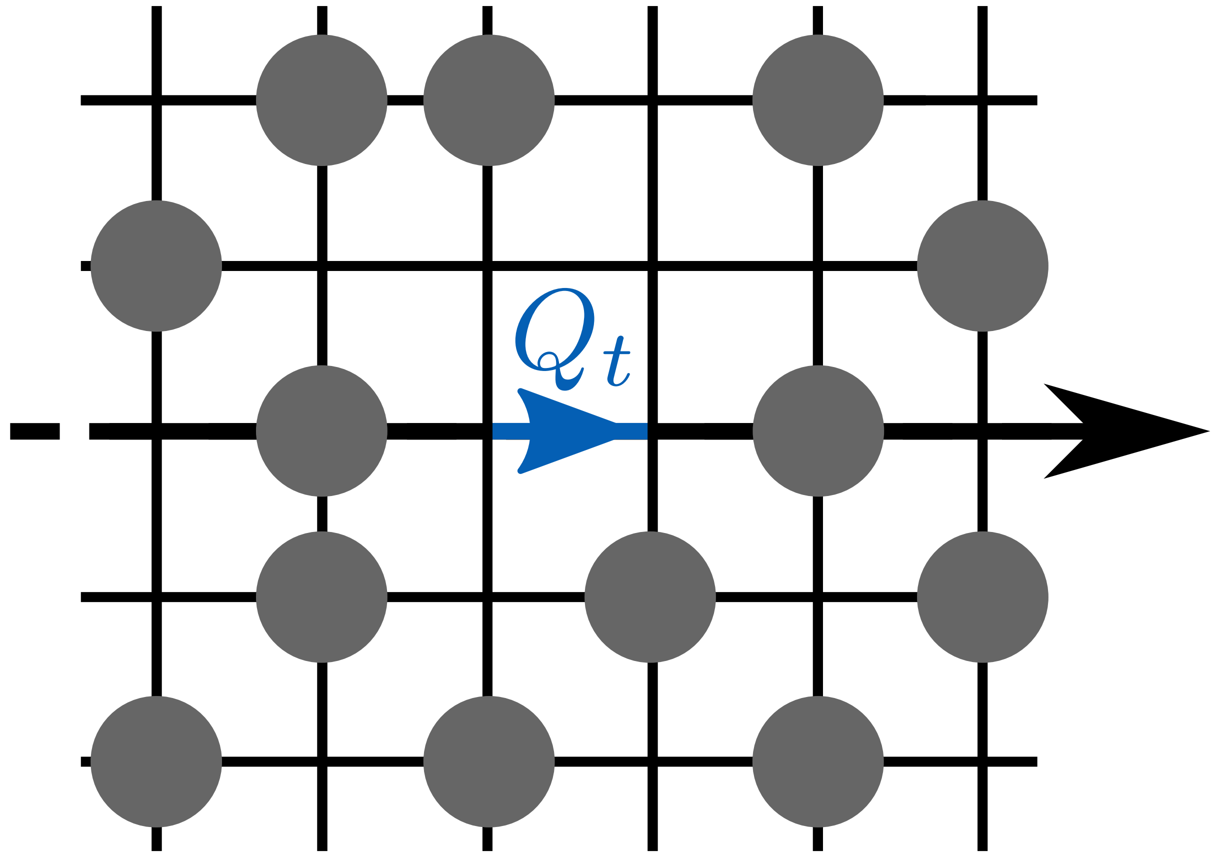
Transport in porous media,
subdiffusion (even single particle!)



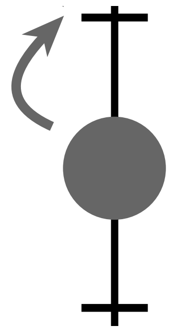


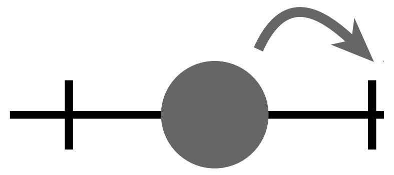
Bénichou, Illien, Oshanin, Sarracino, Voituriez, Phys. Rev. Lett. **115**, 220601 (2015)
Ben-Avraham, Havlin, *Diffusion and reactions in fractals and disordered systems* (2000)

Microscopic calculation

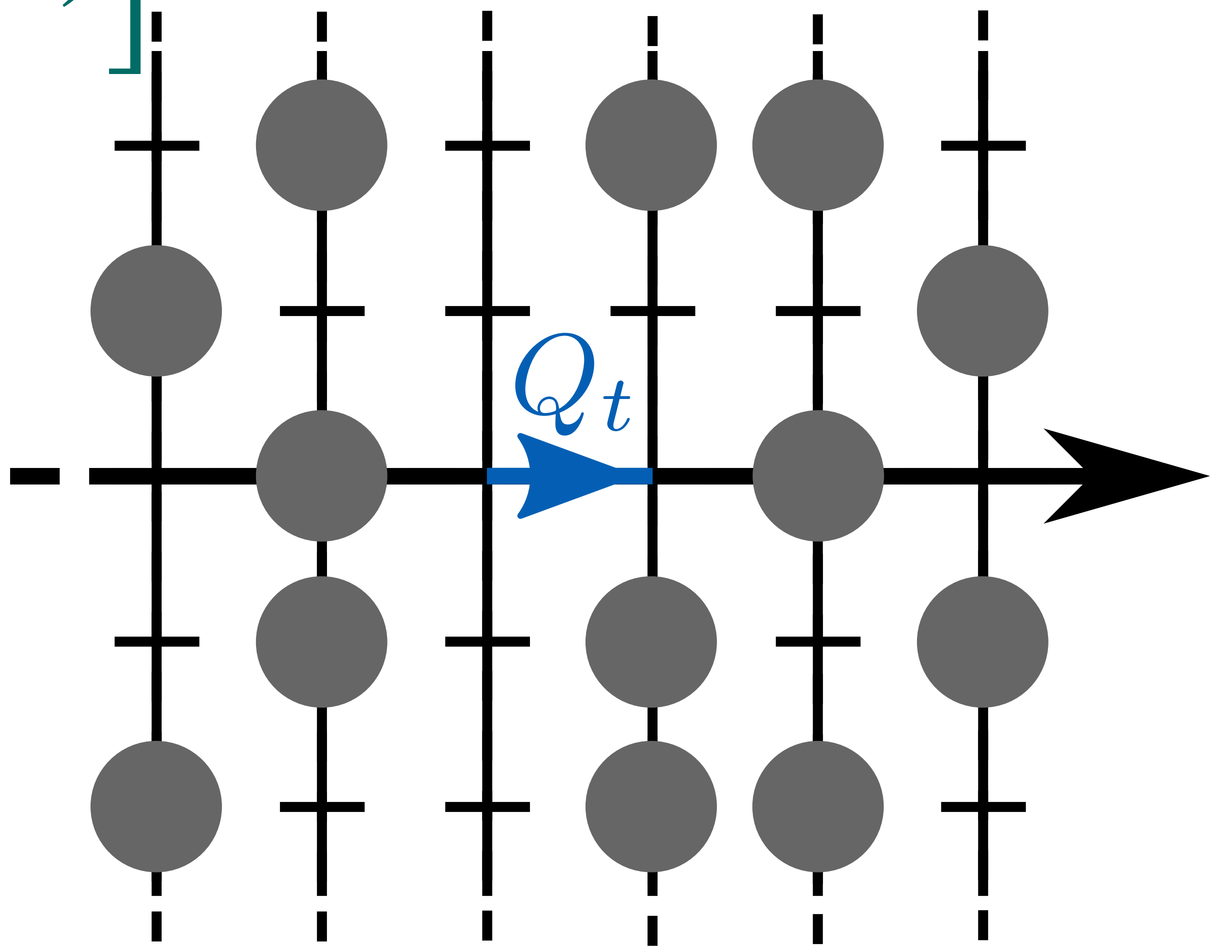


- Master equation





$$\partial_t P_t(\underline{\rho}) = \sum_{x,y} \left[P(\underline{\rho}^{x,y+}, t) - P(\underline{\rho}, t) \right] + \sum_{x,y} \left[P(\underline{\rho}^{x+,y}, t) - P(\underline{\rho}, t) \right]$$

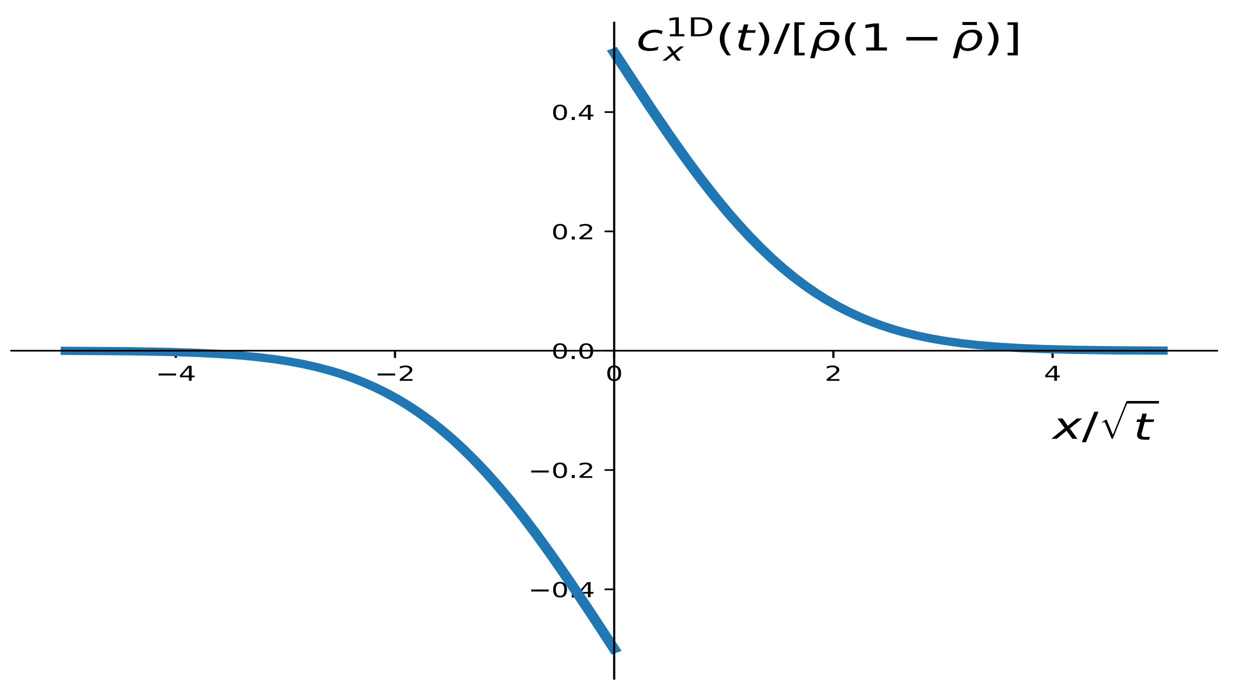


- Look for moments, e.g. $\partial_t \left\langle Q_t^2 \right\rangle = \ldots \; \left[c_{\vec{r}}(t) \right] \; \ldots$
- Find exact closed equations $\partial_t c_{\vec{r}}(t)$ for $c_{\vec{r}}(t) = \left\langle Q_t \, \rho_{\vec{r}}(t) \right\rangle$
- Solve them in Fourier-Laplace (self-consistently)

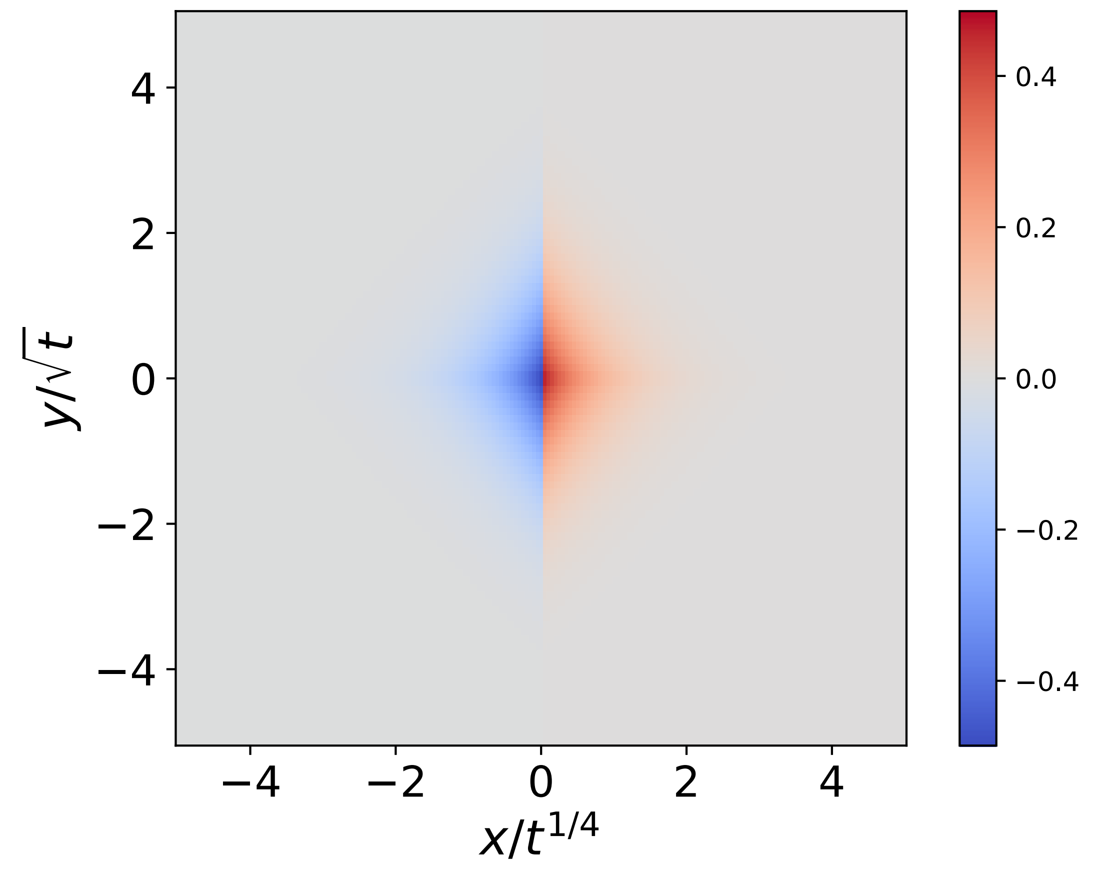
T. Berlioz, D. Venturelli, A. Grabsch, O. Bénichou, J. Stat. Mech. (2024) 113208

Results

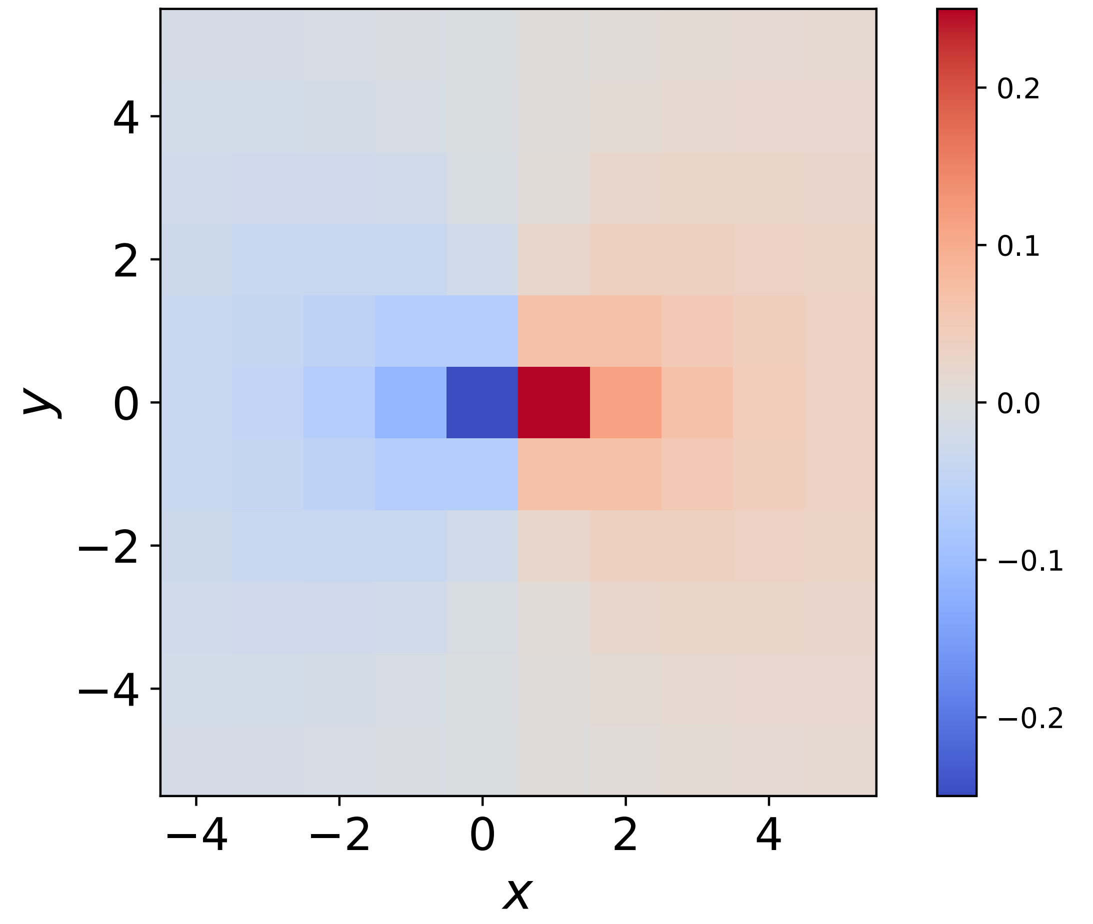
- $d = 1$ $$c_r(t) \xrightarrow[t \to \infty]{} \bar{\rho}(1-\bar{\rho})\,\mathscr{C}_1\!\left(\frac{r}{t^{1/2}}\right)$$ $$\langle Q_t^2 \rangle = n_1\,\bar{\rho}(1-\bar{\rho})\;t^{1/2}$$



- Comb $$c_{\vec{r}}(t) \xrightarrow[t \to \infty]{} \bar{\rho}(1-\bar{\rho})\,\mathscr{C}_c\!\left(\frac{x}{t^{1/4}}, \frac{y}{t^{1/2}}\right)$$ $$\langle Q_t^2 \rangle = n_c\,\bar{\rho}(1-\bar{\rho})\;t^{3/4}$$



- $d = 2$ $$c_{\vec{r}}(t) \xrightarrow[t \to \infty]{} \bar{\rho}(1-\bar{\rho})\,\mathscr{C}_2(\vec{r})$$ $$\langle Q_t^2 \rangle = n_2\,\bar{\rho}(1-\bar{\rho})\;t$$



$\boxed{\text{loops!}}$

Macroscopic Fluctuation Theory

Hydrodynamic description for the occupations,

$$\partial_t \left\langle \rho_{\vec{r}}(t) \right\rangle = \delta_{y,0}\, \Delta_x \left\langle \rho_{\vec{r}}(t) \right\rangle + \Delta_y \left\langle \rho_{\vec{r}}(t) \right\rangle$$

$$\left\langle \rho_{\vec{r}}(t) \right\rangle \simeq \rho\left(\frac{x}{T^{1/4}}, \frac{y}{T^{1/2}}, \frac{t}{T}\right) \longrightarrow$$

$$\partial_t \rho + \vec{\nabla} \cdot \vec{j} = 0$$

$$\vec{j} = -\mathbf{D}\, \vec{\nabla} \rho + \vec{\nu}$$

$$\mathbf{D} = \begin{pmatrix} \delta(y) & 0 \\ 0 & 1 \end{pmatrix}$$

Add noise:

$$\left\langle \nu_i(x, y, t) \nu_j(x', y', t') \right\rangle = \mathbf{\Sigma}_{i,j}(\rho(x, y, t))\, \delta(x - x')\delta(y - y')\delta(t - t')$$

$$\mathbf{\Sigma}(\rho) = 2\rho(1 - \rho)\begin{pmatrix} \delta(y) & 0 \\ 0 & 1 \end{pmatrix}$$

T. Berlioz, D. Venturelli, A. Grabsch, O. Bénichou, J. Stat. Mech. (2024) 113208

Macroscopic Fluctuation Theory

on the comb

$$\partial_t \rho(\vec{r}, t) = \overrightarrow{\nabla} \cdot [\mathbf{D}\, \overrightarrow{\nabla} \rho + \ \vec{\nu}] \longrightarrow P[\rho] = \int \mathscr{D}H\, e^{-T^{3/4}\, S[\rho,H]}$$

- Integrated current fluctuations

$$Q_T \simeq T^{3/4} \int_0^\infty dx \int_{-\infty}^\infty dy\, [\rho(x,y,1) - \rho(x,y,0)] \qquad \left\langle e^{\lambda Q_T} \right\rangle = \int \mathscr{D}\rho\, \mathscr{D}H\, e^{-T^{3/4}\, S[\rho,H] + \lambda Q_T}$$

- Saddle point

correlation profile

optimal density

$$\begin{cases} \rho = \rho^{(0)} + \lambda \rho^{(1)} + \dots \\ H = H^{(0)} + \lambda H^{(1)} + \dots \end{cases}$$

$$\frac{\left\langle \rho_{\vec{r}=(x,y)}(T)\, e^{\lambda Q_T} \right\rangle}{\left\langle e^{\lambda Q_T} \right\rangle} \simeq \rho^*(x, y, 1)$$

Macroscopic Fluctuation Theory

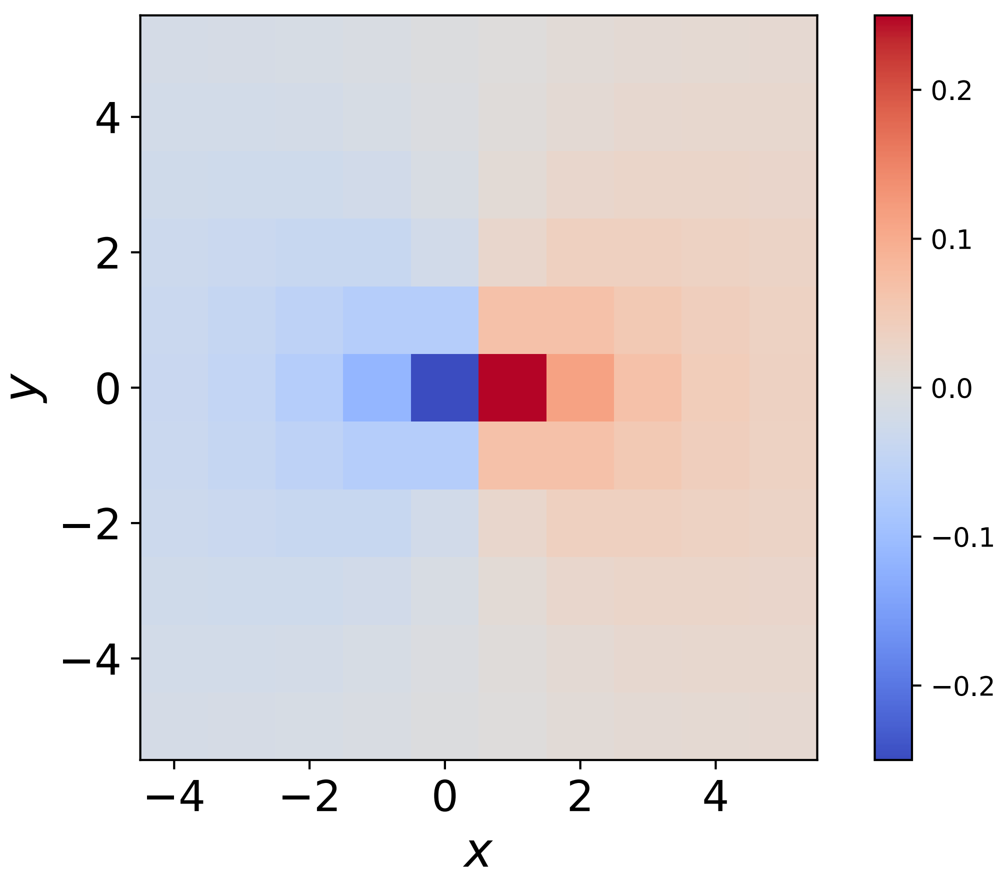
towards higher moments and general diffusive processes

- Recover $\left\langle \rho_r(T)\, Q_T \right\rangle,\ \left\langle Q_T^2 \right\rangle$ + in principle higher moments/correlations
- Can be extended to other **models**
- First application of MFT to an **inhomogeneous** system (**comb**)

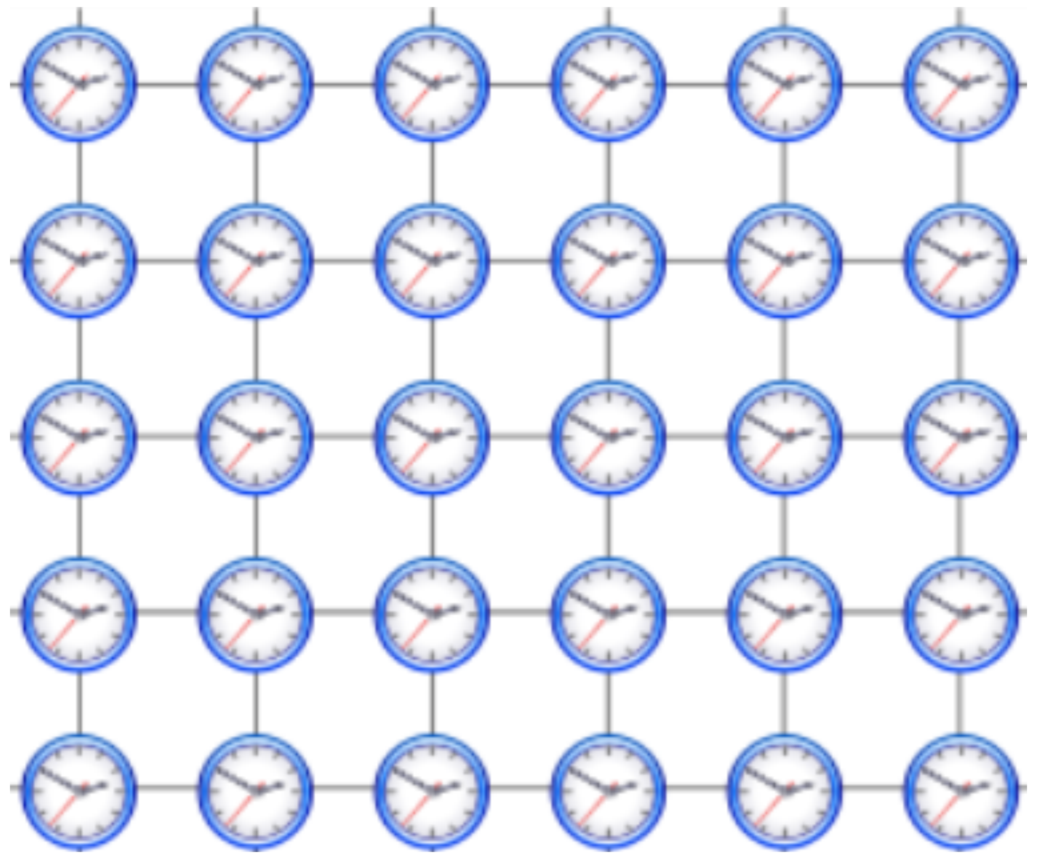
T. Berlioz, D. Venturelli, A. Grabsch, O. Bénichou, J. Stat. Mech. (2024) 113208

What about higher d ?

- In $1d$ & comb, correlations vary slowly at the lattice scale
- In higher d correlations become stationary, no scaling limit!



Microscopic path-integral representation



$$\rho_{\vec r}(t+dt) - \rho_{\vec r}(t) = dt \sum_{\nu} \left(\vec j_{\vec r - \vec \nu}(t) - \vec j_{\vec r}(t)\right)\cdot \vec\nu \,,$$

$$\vec j_{\vec r}(t)\,dt = \sum_{\vec \nu} \left[\rho_{\vec r}(1-\rho_{\vec r+\vec\nu})\,\xi_{\vec r,\vec\nu}(t) - \rho_{\vec r+\vec\nu}(1-\rho_{\vec r})\,\xi_{\vec r+\vec\nu,-\vec\nu}(t)\right]\vec\nu \,,$$

equivalent to the M.E. if Poissonian noise is $\xi_{\vec r,\vec\mu}(t) = \begin{cases}1 & \textbf{prob. } \gamma\,dt\,,\\ 0 & \textbf{prob. } 1-\gamma\,dt\,.\end{cases}$

Usual MSR machinery gives

$$\left\langle e^{\lambda Q_T}\right\rangle = \int \mathscr{D}\theta_{\vec r}\,\mathscr{D}\vec\varphi_{\vec r}\, e^{-S[\rho_{\vec r},\vec j_{\vec r},\theta_{\vec r},\vec\varphi_{\vec r}]+\lambda Q_T} \longrightarrow \quad Q_T = \int_0^T dt \left(\vec j_{\vec r=\vec 0}(t)\right)_1$$

A. Lefèvre, G. Biroli, J. Stat. Mech. (2007) P07024

Microscopic path-integral representation

- Saddle-point eqs are difference equations for $\rho_{\vec{r}}, \vec{j}_{\vec{r}}, \theta_{\vec{r}}, \overrightarrow{\varphi}_{\vec{r}}$
- Turn out to relax to a stationary limit,

$$\mathcal{S} = \int_0^T dt\, \mathscr{L}[\{\rho, \vec{j}, \theta, \overrightarrow{\varphi}\}] \simeq T\, \mathscr{L}[\{\rho^*, \vec{j}^*, \theta^*, \overrightarrow{\varphi}^*\}]$$

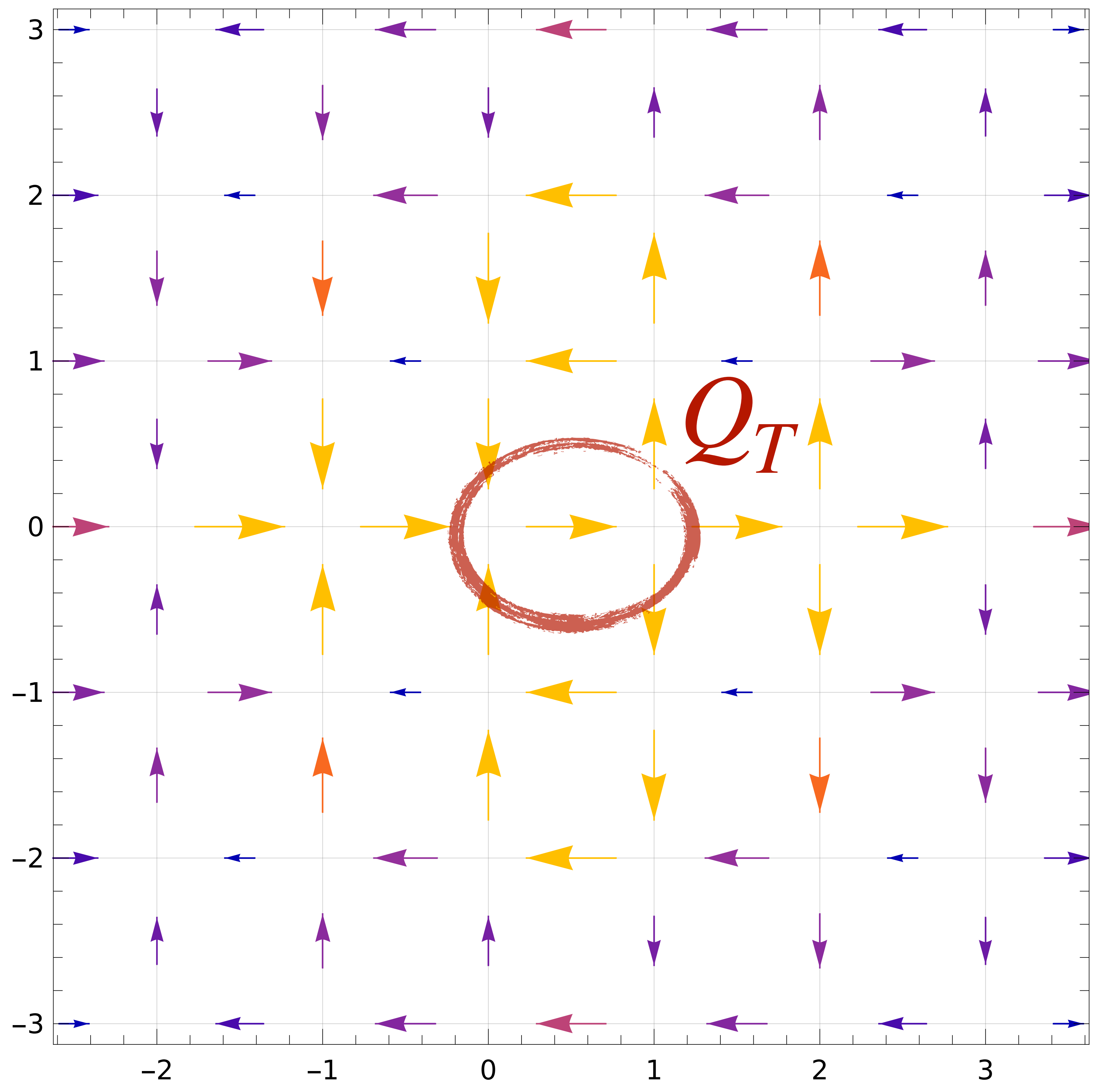
- Can be used to recover

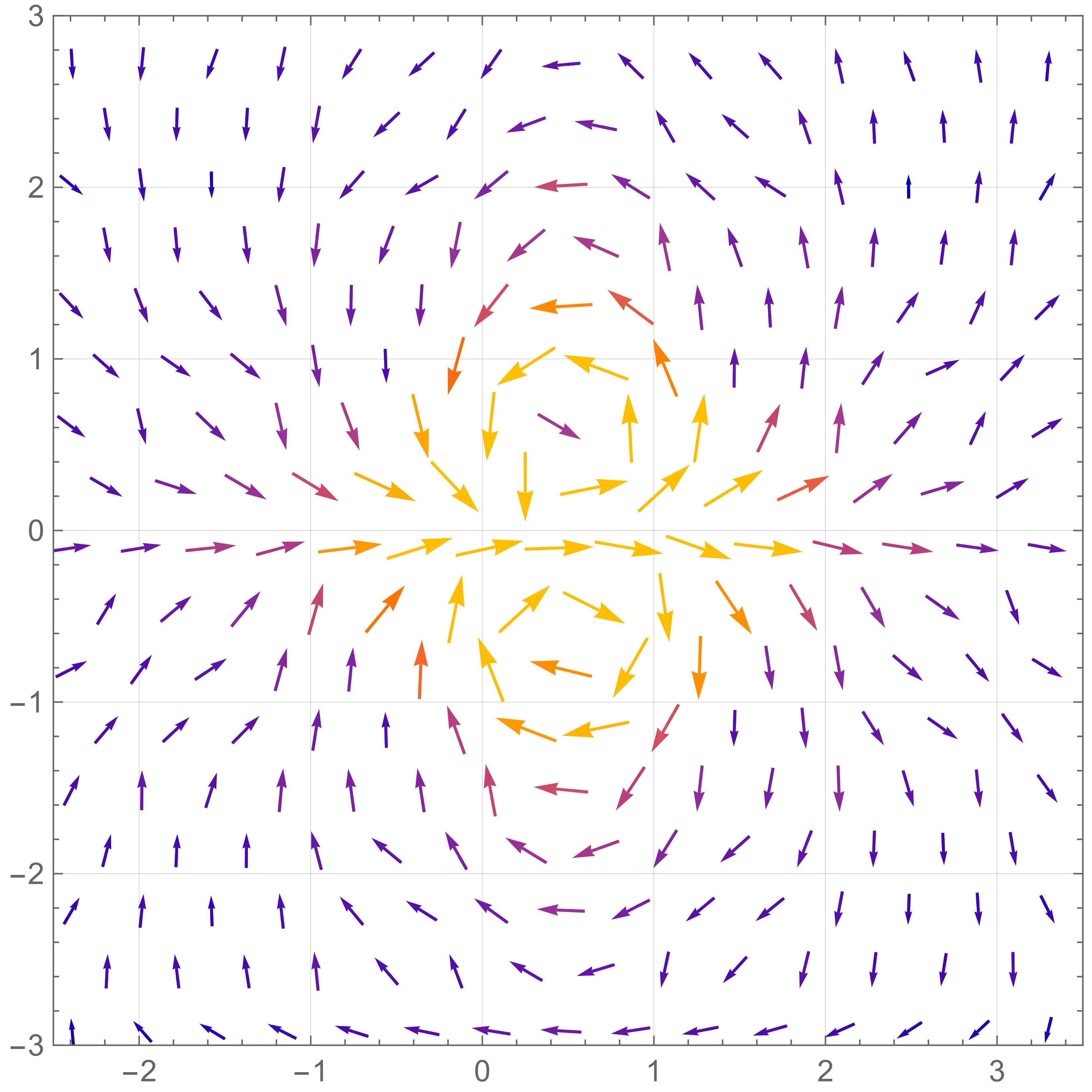
$$\left\langle e^{\lambda Q_T} \right\rangle \simeq \exp\{-T[\mathscr{L}^* - \lambda(\vec{j}^*_{\vec{r}=\vec{0}})_1]\} \longrightarrow \left\langle Q_t^2 \right\rangle = 2\gamma \left(1 - \frac{1}{d}\right) \bar{\rho}(1-\bar{\rho})\, t$$

Role of loops

$$\frac{\left\langle \vec{j}_{\vec{r}}(t)\, e^{\lambda Q_T} \right\rangle}{\left\langle e^{\lambda Q_T} \right\rangle} \simeq \vec{j}^{\,*}_{\vec{r}}(t) \longrightarrow \left\langle \vec{j}_{\vec{r}}(t)\, Q_T \right\rangle$$

$\vec{j}^{\,*}_{\vec{r}}$





looped structure of the lattice allows for **vortex** configurations, and thus for stationary $c_{\vec{r}}$

2. Tracer-bath correlations

Hard-core lattice gas

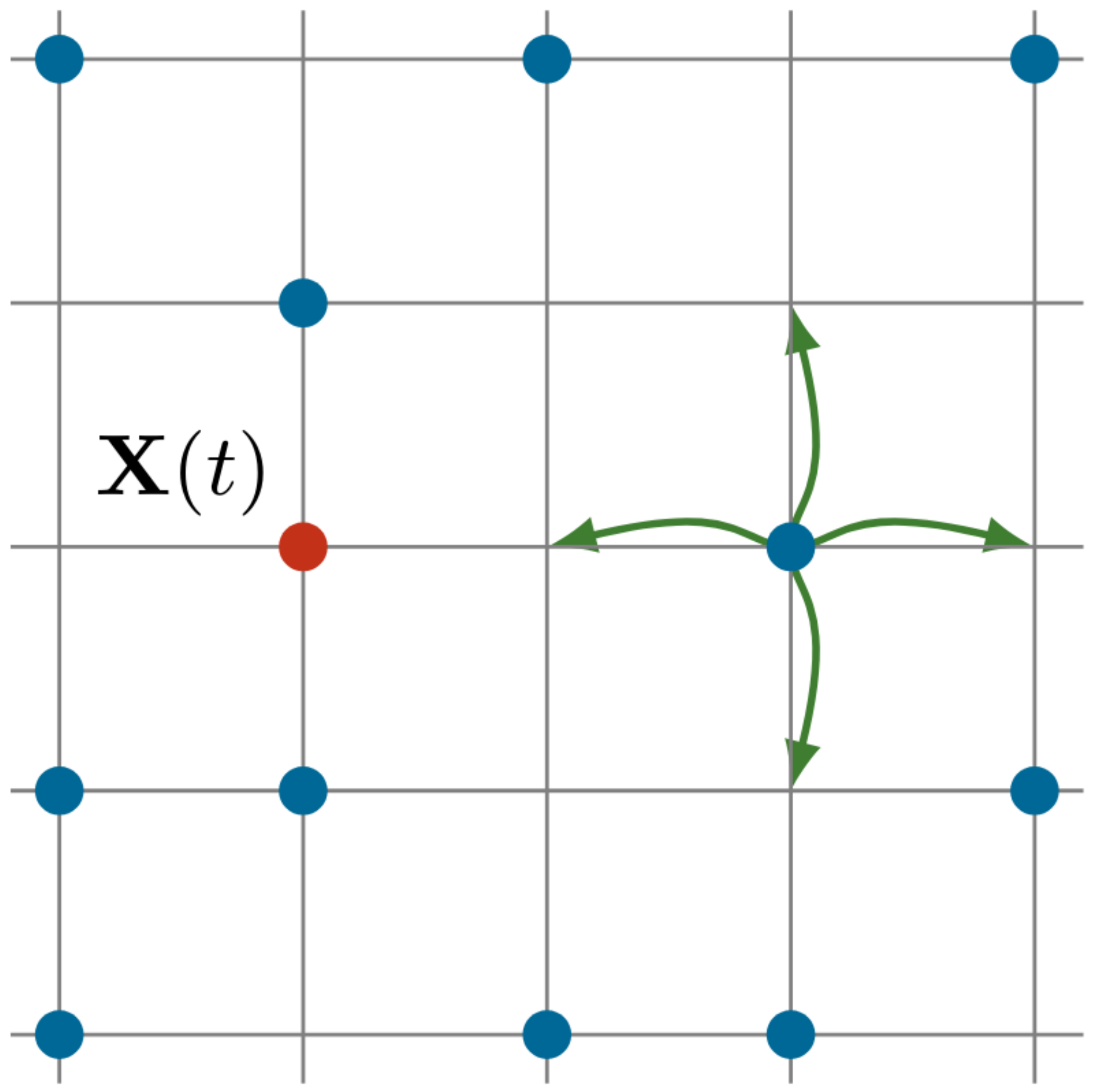
in d dimensions

- Tagged tracer at X_t, bath occupations $\rho_r(t) = \{0,1\}$
- Using the ME, get an equation for $g_r(t) = \langle X_t\, \rho_{X+r}(t) \rangle$ → **not closed!**
- $\partial_t g_r(t) = (\ldots)$ can be closed upon ***decoupling***

$$\langle \rho_{X+r} \rho_{X+r'} \rangle \simeq \langle \rho_{X+r} \rangle \langle \rho_{X+r'} \rangle$$

$$\langle X_t \rho_{X+r} \rho_{X+r'} \rangle \simeq \langle X_t\, \rho_{X+r} \rangle \langle \rho_{X+r'} \rangle + \langle X_t\, \rho_{X+r'} \rangle \langle \rho_{X+r} \rangle$$

(exact for large/small $\bar{\rho}$)



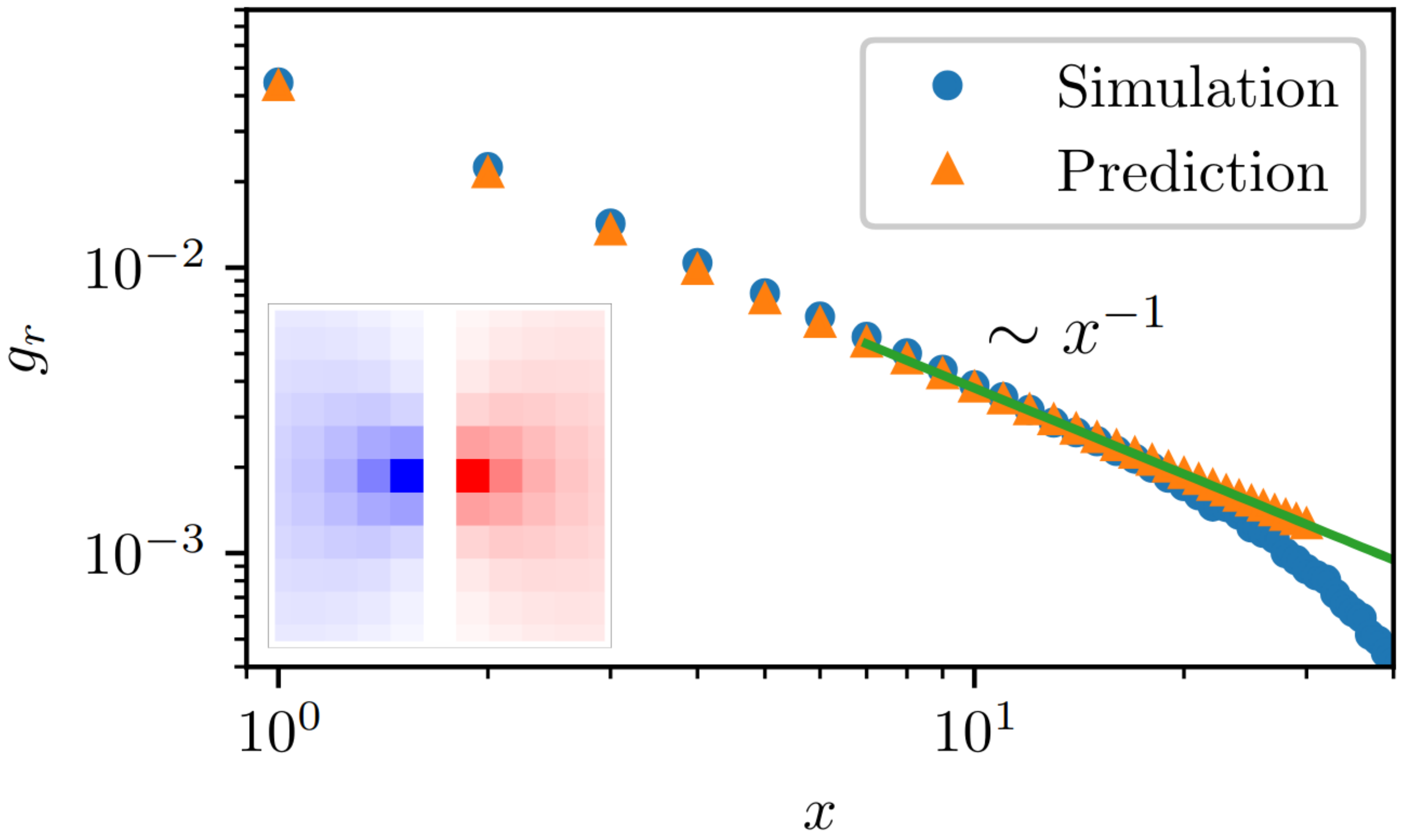
Bénichou, Illien, Oshanin, Sarracino, Voituriez, Phys. Rev. Lett. **115**, 220601 (2015)

Hard-core lattice gas

in d dimensions

- Self-consistent *difference* eq. for $g_r(t) = \langle X_t \, \rho_r(t) \rangle$
- Assume $g_r(t) \to g_r$ at long t
- For large $x = \mathbf{r} \cdot \hat{\mathbf{e}}_1$,

$$g_x \sim x^{1-d}$$



D. Venturelli, P. Illien, A. Grabsch, O. Bénichou, arXiv:2411.09326 (2024)

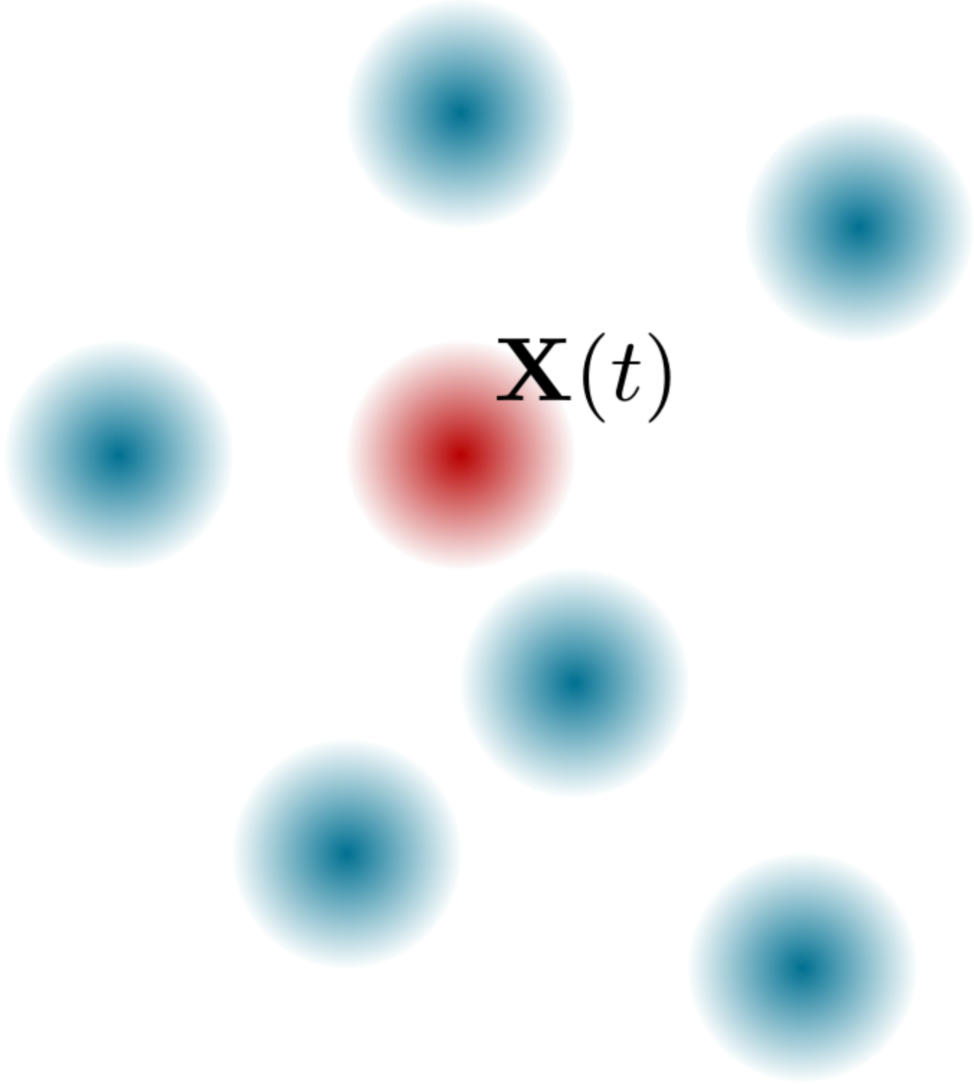
Soft interacting particles

in d dimensions

- Overdamped Brownian particles, $i = 0, \ldots, N$

$$\dot{\mathbf{X}}_i(t) = -\mu \sum_{j \neq i} \nabla_i U\left(\mathbf{X}_i(t) - \mathbf{X}_j(t)\right) + \boldsymbol{\eta}_i(t), \qquad \langle \boldsymbol{\eta}_i(t)^T \boldsymbol{\eta}_j(t') \rangle = 2\mu T \delta_{ij} \delta(t - t') I_d$$

- “soft” potentials $U(\mathbf{x})$, e.g. Gaussian core
- **Tracer** $i = 0$ (omitted)
- **Correlation profiles**?



Coarse-grained dynamics

with a tracer

- Dean-Kawasaki equation for $\rho(x,t) = \sum_{i=1}^{N} \delta\left(x - X_i(t)\right)$,

$$\partial_t \mathbf{X}(t) = -\mu \nabla_{\mathbf{X}} \mathscr{F}[\rho, \mathbf{X}] + \boldsymbol{\eta}_0(t),$$

$$\partial_t \rho(\mathbf{x},t) = \mu \nabla \cdot \left[\rho(\mathbf{x},t) \nabla \frac{\delta \mathscr{F}}{\delta \rho(\mathbf{x},t)}\right] + \nabla \cdot \left[\rho^{\frac{1}{2}}(\mathbf{x},t)\, \boldsymbol{\xi}(\mathbf{x},t)\right],$$

$$\mathscr{F}[\rho, \mathbf{X}] = T \int \mathrm{d}\mathbf{x}\, \rho(\mathbf{x}) \log\left(\frac{\rho(\mathbf{x})}{\rho_0}\right) + \frac{1}{2}\int \mathrm{d}\mathbf{x}\, \mathrm{d}\mathbf{y}\, \rho(\mathbf{x}) U(\mathbf{x}-\mathbf{y}) \rho(\mathbf{y}) + \int \mathrm{d}\mathbf{y}\, \rho(\mathbf{y}) U(\mathbf{y} - \mathbf{X})$$

FIG. 1. David Dean and Kyozi Kawasaki, who are very happy that their equation is being used for the zillionth time.

- Linearize about $\rho(\mathbf{x},t) = \rho_0 + \sqrt{\rho_0}\,\phi(\mathbf{x},t)$, assuming $\phi/\sqrt{\rho_0} \ll 1$.

V. Démery et al., New J. Phys. **16** (2014) 053032

Tracer statistics
from correlation profiles

$$\boxed{\partial_t \Psi(\boldsymbol{\lambda}, t) = \frac{1}{2d\,\tau} \sum_{\mu=-d}^{d} \left(\mathrm{e}^{\sigma \boldsymbol{\lambda} \cdot \hat{\mathbf{e}}_\mu} - 1 \right) \left[1 - w_{\mathbf{e}_\mu}(\boldsymbol{\lambda}, t) \right]}$$

- $\rho(\mathbf{x}, t) = \rho_0 + \sqrt{\rho_0}\,\phi(\mathbf{x}, t) \to$ coupled eqs for $\mathbf{X}(t), \phi(\mathbf{x}, t)$ (linear in ϕ)

- How does $\Psi(\lambda, t) = \ln \left\langle \mathrm{e}^{\lambda \cdot \mathbf{X}(t)} \right\rangle$ evolve?

$$\partial_t \Psi(\boldsymbol{\lambda}, t) = \lambda^2 \mu T + \sqrt{\rho_0}\,\mu\,\boldsymbol{\lambda} \cdot \int \mathrm{d}^d x\, U(\mathbf{x})\, \nabla_{\mathbf{x}}\, w(\mathbf{x}, \boldsymbol{\lambda}, t),$$

with the profile

average density profile

$$w(\mathbf{x}, \lambda, t) = \frac{\left\langle \phi(\mathbf{x} + \mathbf{X}(t), t)\, \mathrm{e}^{\lambda \cdot \mathbf{X}(t)} \right\rangle}{\left\langle \mathrm{e}^{\lambda \cdot \mathbf{X}(t)} \right\rangle} = \langle \phi(\mathbf{x} + \mathbf{X}(t), t) \rangle + \lambda \cdot \langle \mathbf{X}(t)\, \phi(\mathbf{x} + \mathbf{X}(t), t) \rangle + \mathcal{O}(\lambda^2)$$

correlation profile $\mathbf{g}(x, t)$

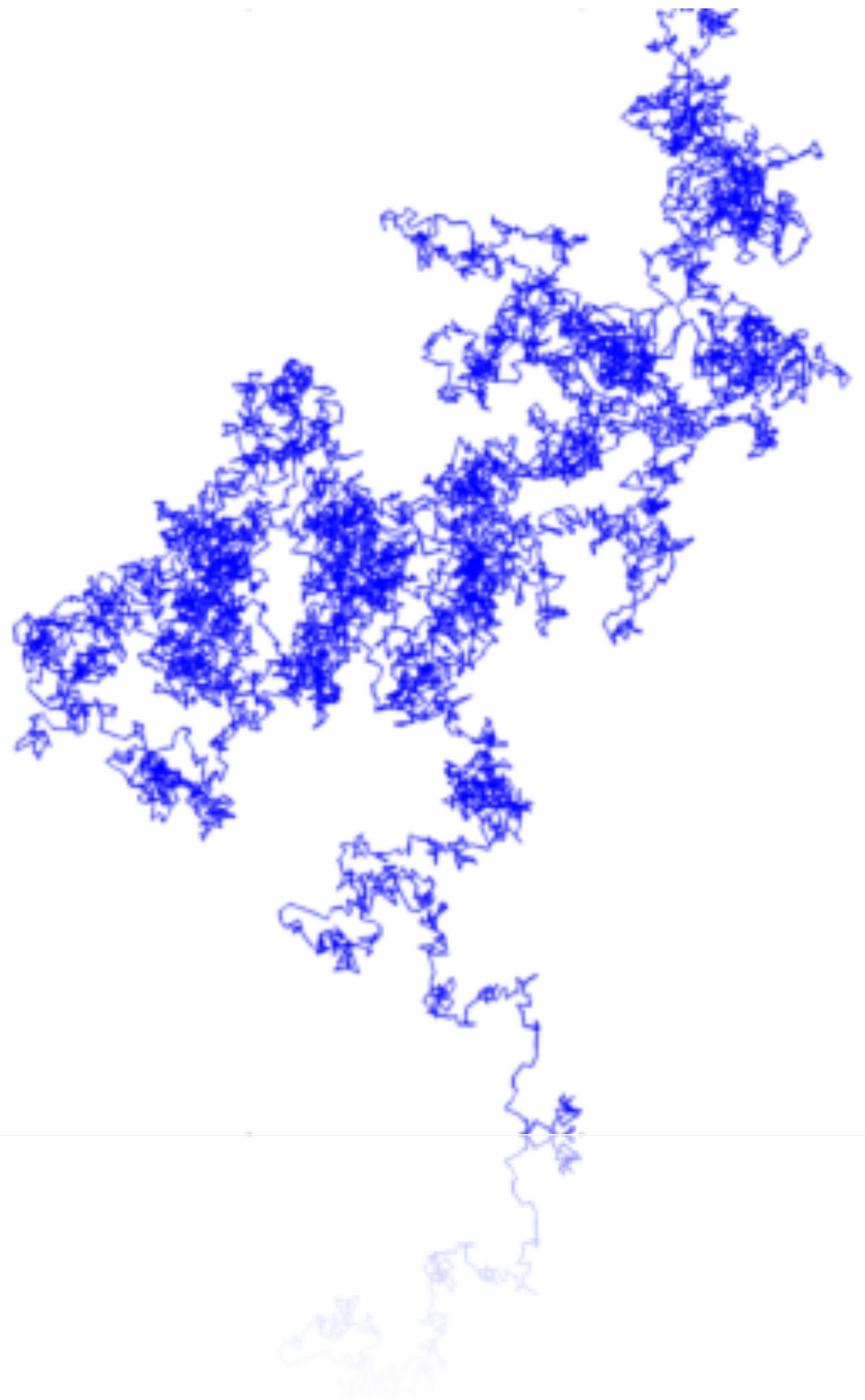
Beware

of zero modes

- In the theory of fluids, stationary quantities are computed as

$$\langle \mathbf{X}\,\rho(\mathbf{x}+\mathbf{X})\rangle \propto \int \mathscr{D}\rho \int \mathrm{d}\mathbf{X}\, \mathrm{e}^{-\frac{1}{T}\mathscr{F}[\rho,\mathbf{X}]} \left[\mathbf{X}\,\rho(\mathbf{x}+\mathbf{X})\right] \quad = 0$$

upon defining $\rho'(\mathbf{x}) = \rho(\mathbf{x}+\mathbf{X})$.



- But thermodynamic quantities only depend on $|\,\mathbf{X}_i - \mathbf{X}_j\,|$, whereas $\left[\mathbf{X}\,\rho(\mathbf{x}+\mathbf{X})\right]$ depends also on **COM**

- Trivial zero! → Need to use EOM to predict stationary profiles.

Stationary correlation profile

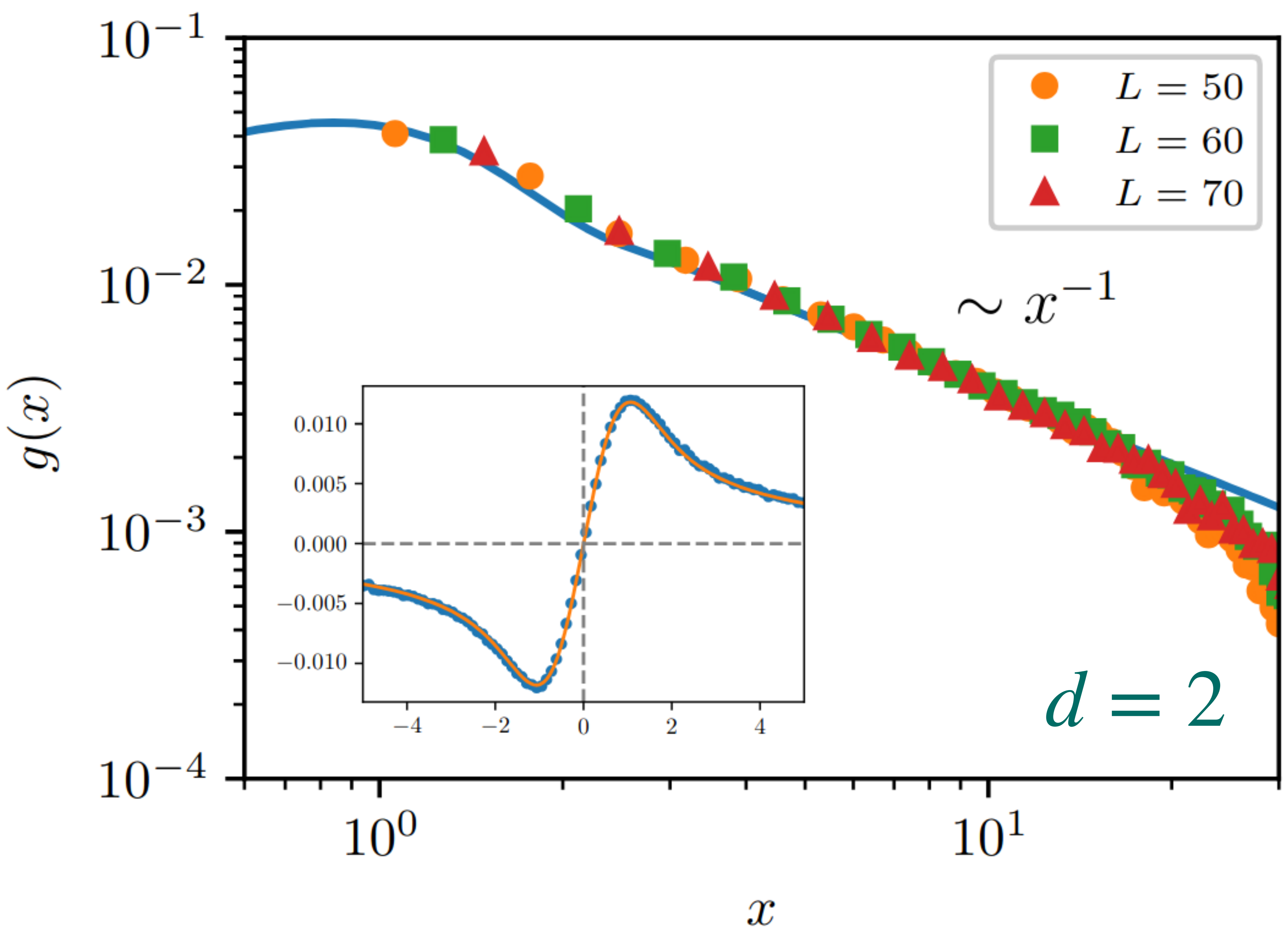
large-distance behavior

- From coupled EOM for $\partial_t \mathbf{X}(t)$ and $\partial_t \phi(\mathbf{x}, t)$, write one for

$$g(\mathbf{x}, t) = \hat{\mathbf{e}}_1 \cdot \langle \mathbf{X}(t) \phi(\mathbf{x} + \mathbf{X}(t), t) \rangle$$

- Compute $g(\mathbf{x})$ perturbatively
- At large distance $x = \mathbf{x} \cdot \hat{\mathbf{e}}_1$,

$$g(x) \sim x^{1-d}$$

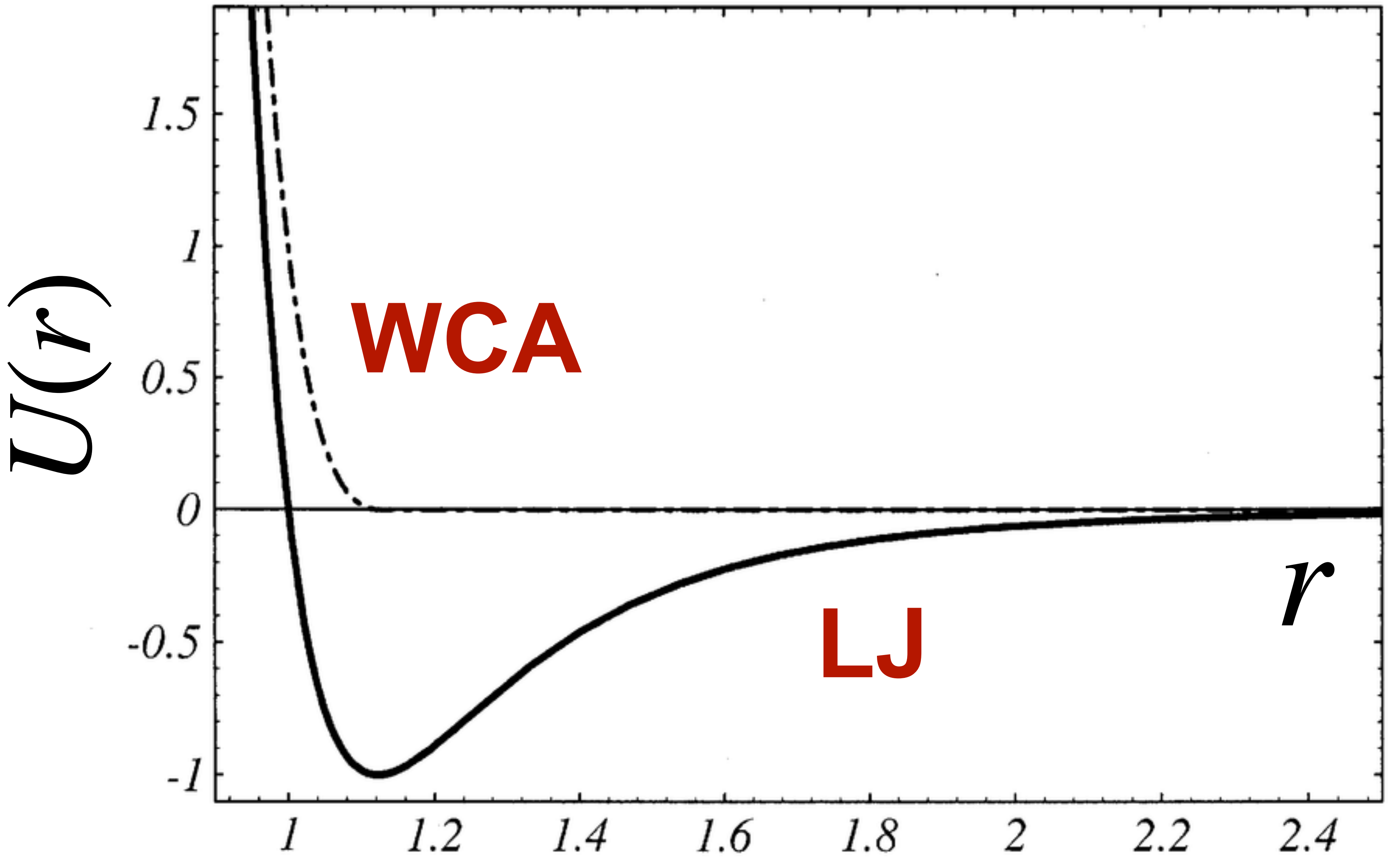


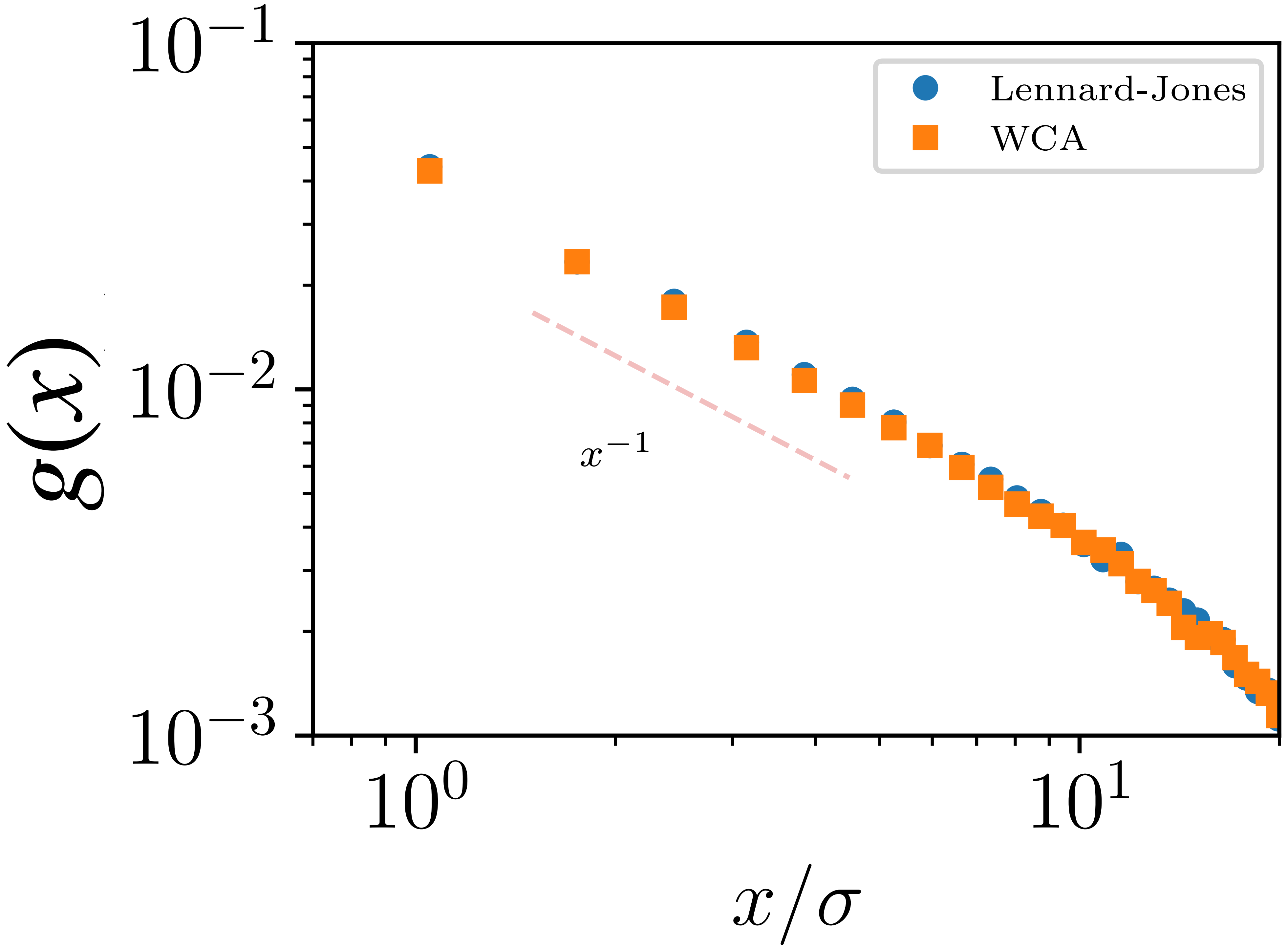
D. Venturelli, P. Illien, A. Grabsch, O. Bénichou, arXiv:2411.09326 (2024)

Lennard-Jones fluids
stationary correlation profile

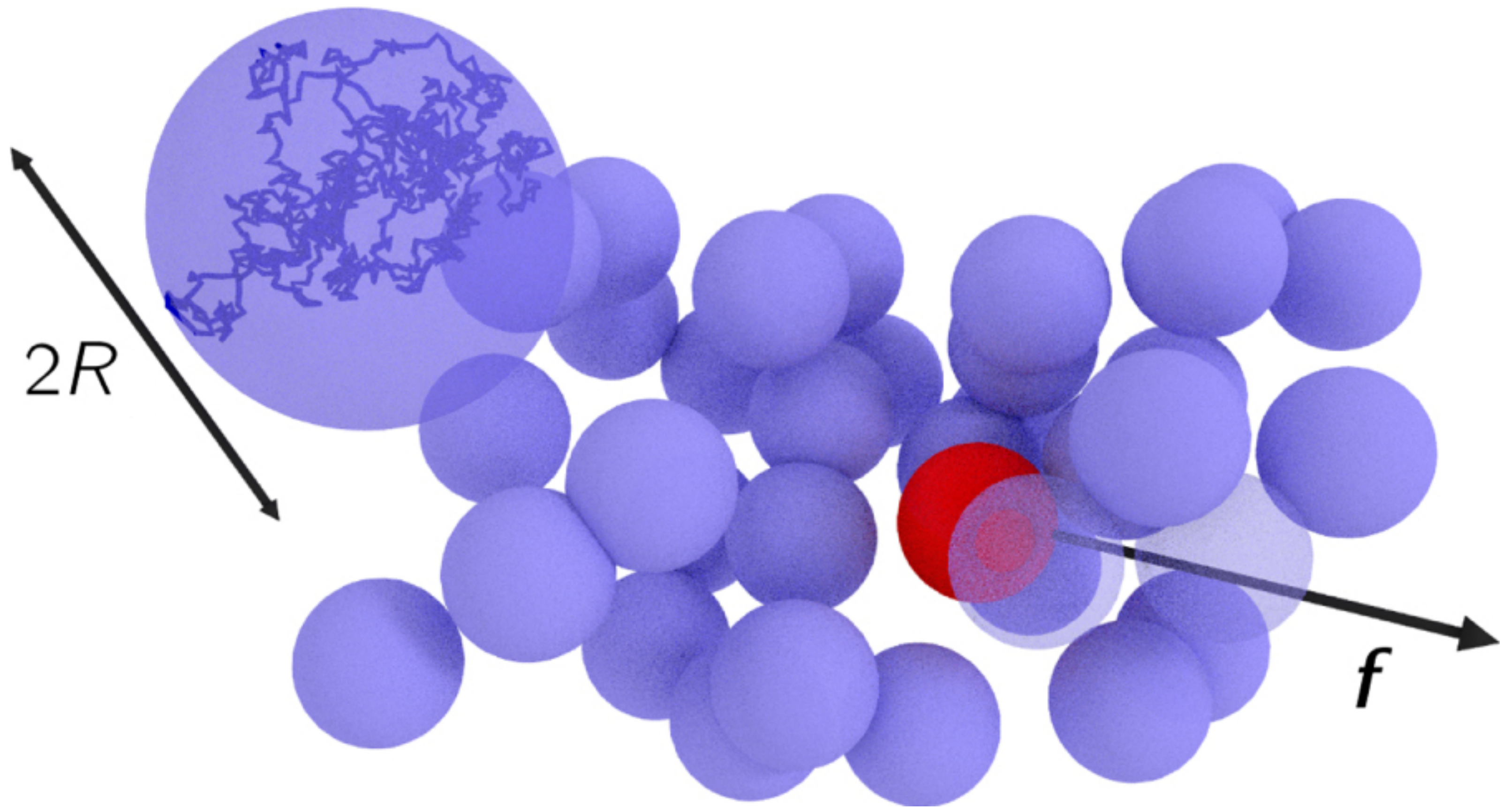
Are short-distance details of interactions irrelevant for large-distance behaviour of the bath response?

- Strong repulsion beyond linearised D-K theory
- Simulate LJ and WCA suspensions





Average density profile
under steady driving

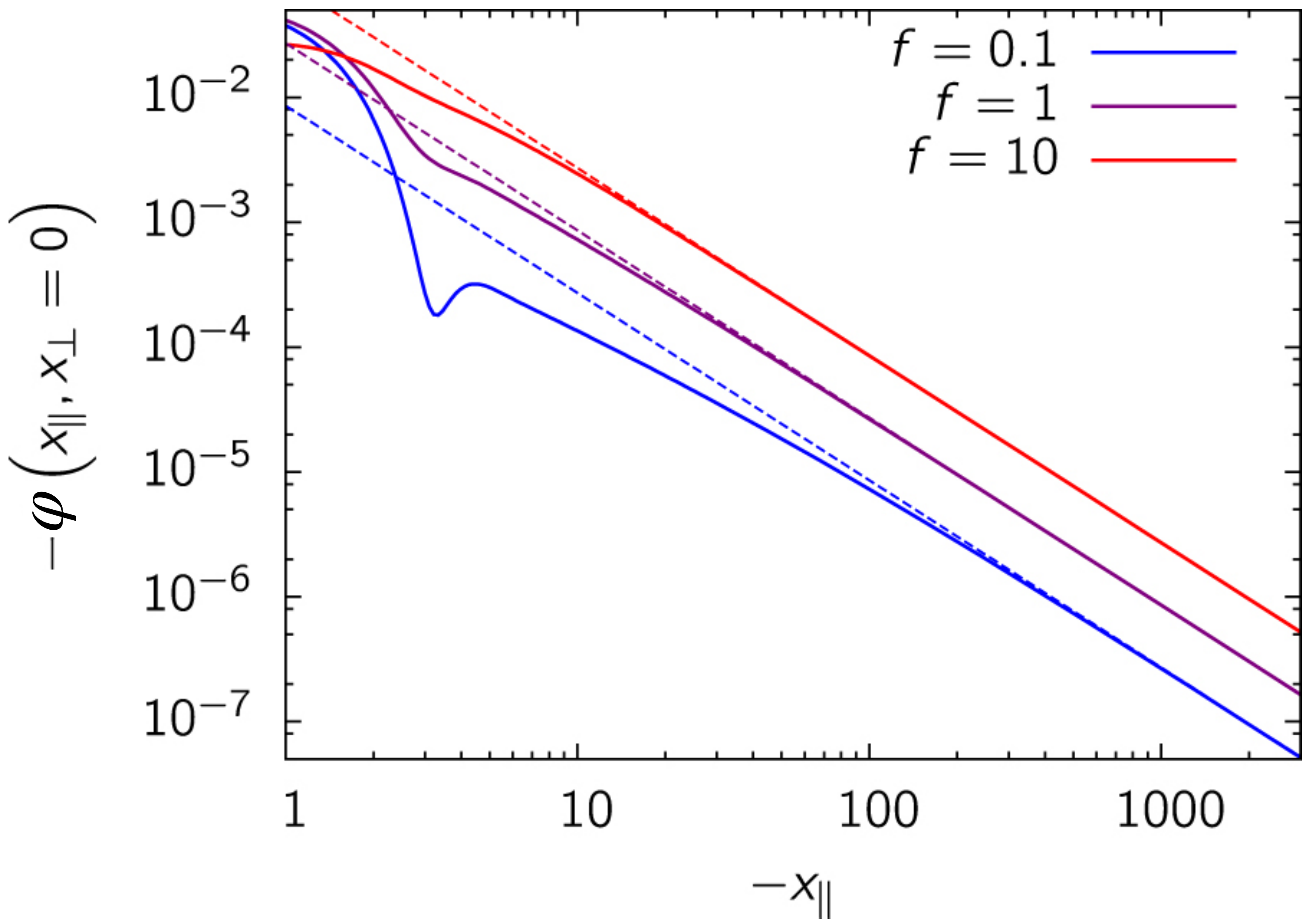


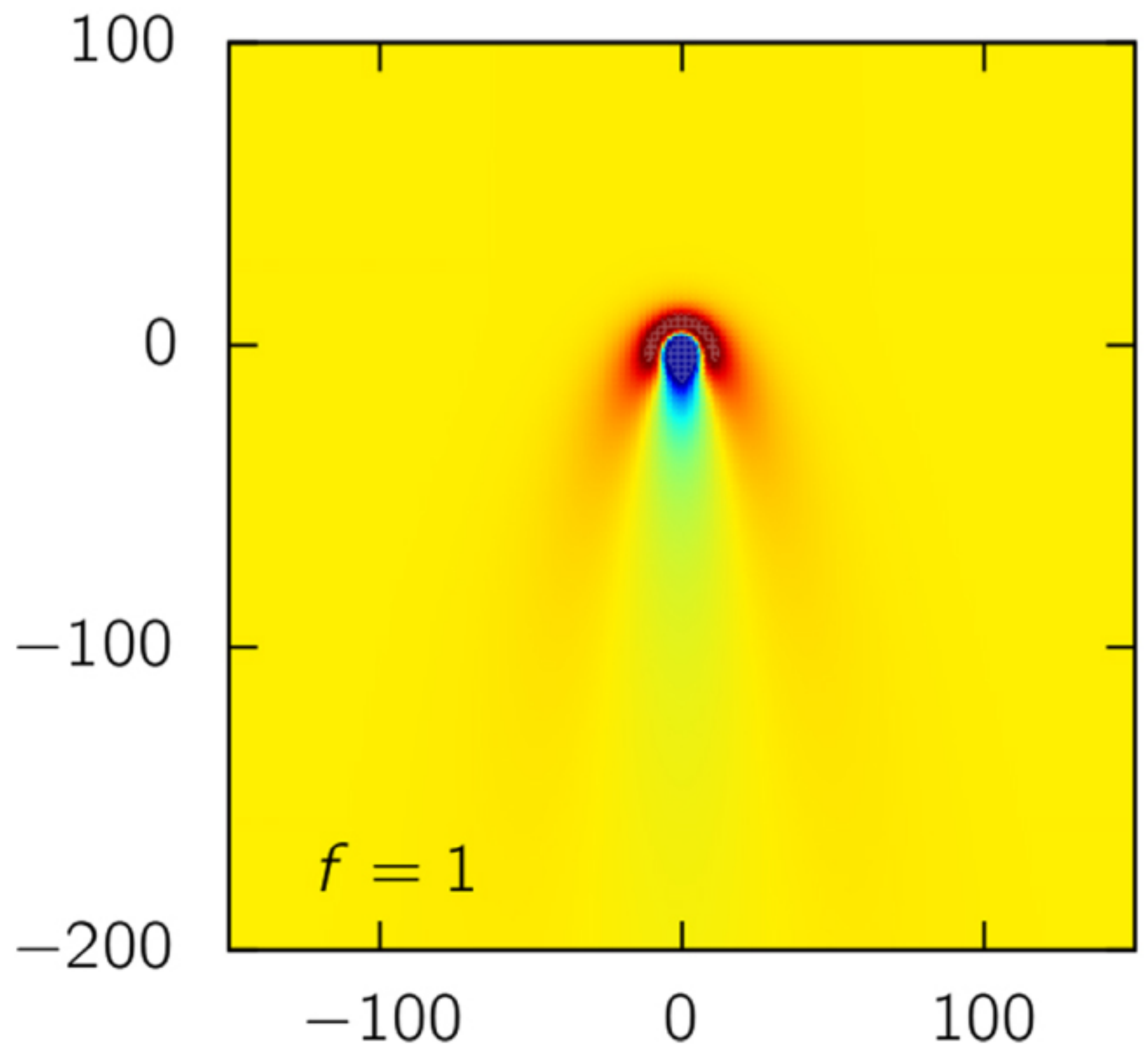
- Stationary density profile in the frame of the tracer,

$$\varphi(\mathbf{x}, t) = \langle \phi(\mathbf{x} + \mathbf{X}(t), t) \rangle$$

- For $x_{||} \to -\infty$,

$$\varphi(x_{||}, \mathbf{x}_\perp = \mathbf{0}) \sim -|x_{||}|^{-\frac{1+d}{2}}$$

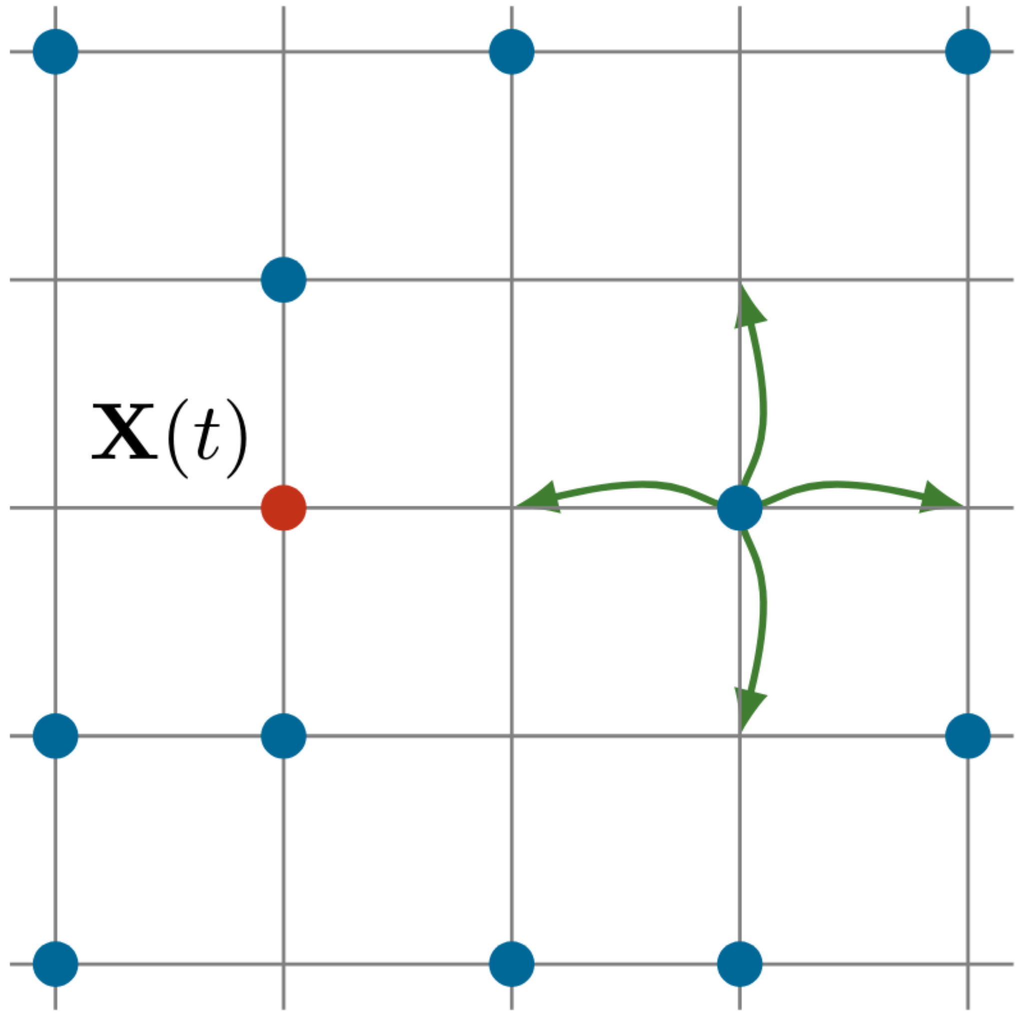




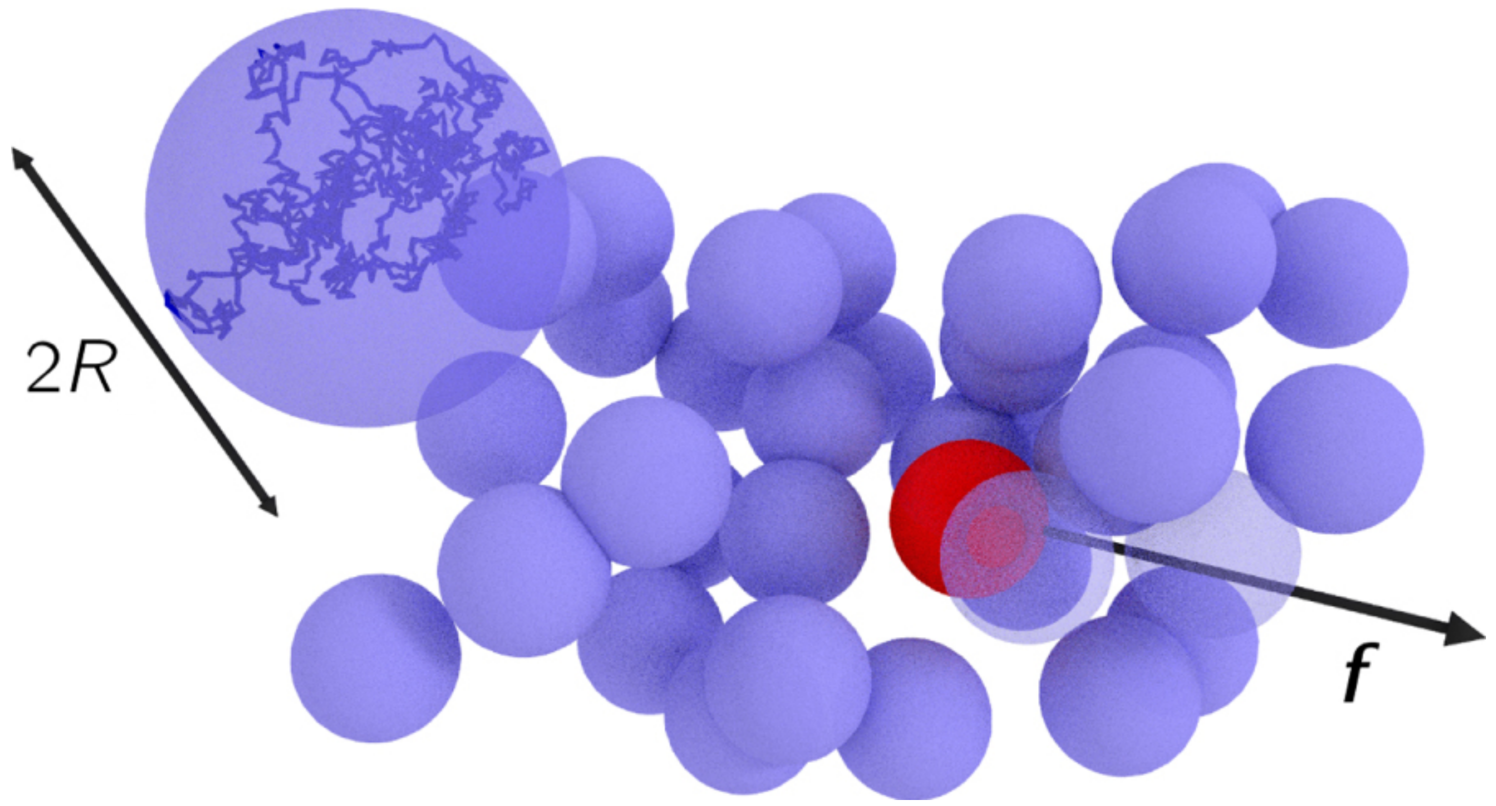
V. Démery et al., New J. Phys. **16** (2014) 053032

O. Bénichou et al., Phys. Rev. Lett. **84**, 511 (2000)

Universality
in diffusive systems



large-distance behaviour
of tracer-bath correlations



Fluctuating hydrodynamics,
macroscopic fluctuation theory

??

Dean-Kawasaki theory

Summing up

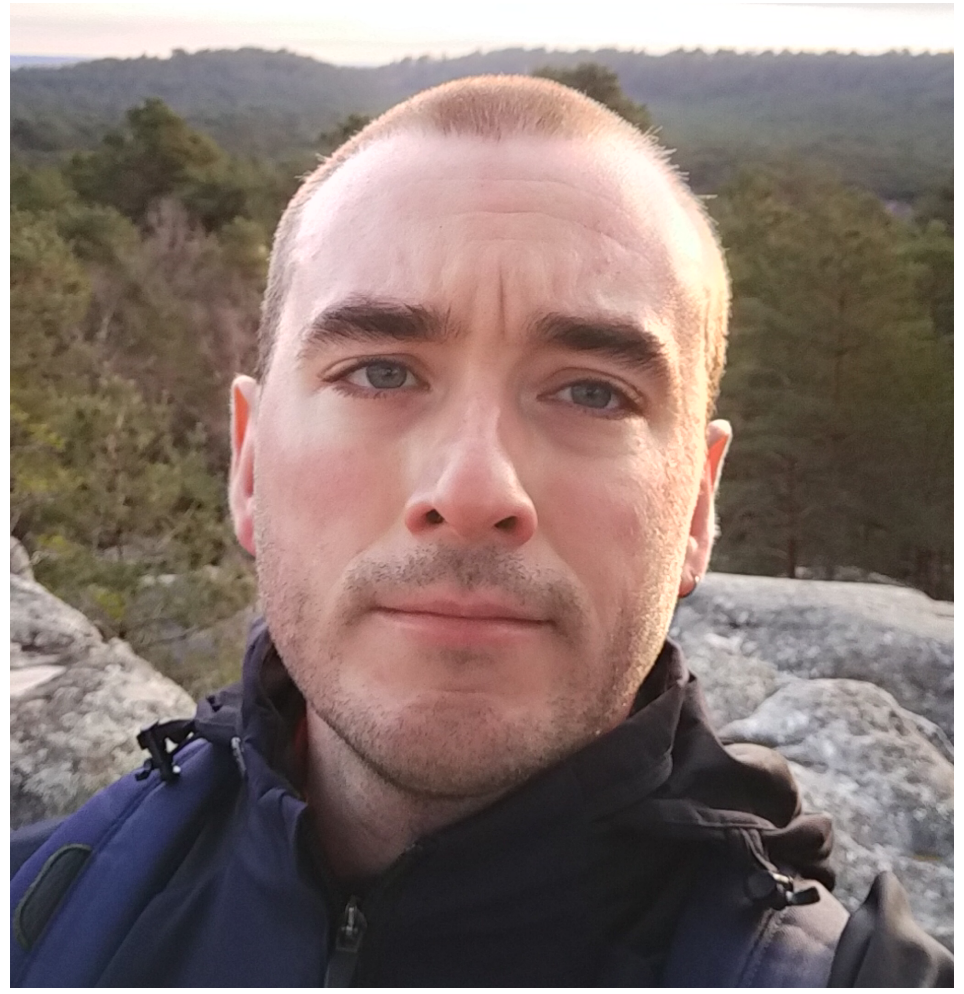
Spatial correlation profiles are worth checking out!

- ☑ $\langle Q_t \, \rho_r(t) \rangle$ gives info on **response** of the bath (e.g. role of **loops**),
- ☑ and gives access to **moments**, e.g. $\langle Q_t^2 \rangle$ on infinite lattices in $d > 1$.
- ☑ $\langle X_t \, \rho_r(t) \rangle \sim r^{1-d}$ for large r in a hard-core lattice gas,
- ☑ and for Brownian suspensions with both **weak**/**strong** repulsion.
- ☐ Why?

T. Berlioz, D. Venturelli, A. Grabsch, O. Bénichou, J. Stat. Mech. (2024) 113208

D. Venturelli, P. Illien, A. Grabsch, O. Bénichou, arXiv:2411.09326 (2024)

Thanks!



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