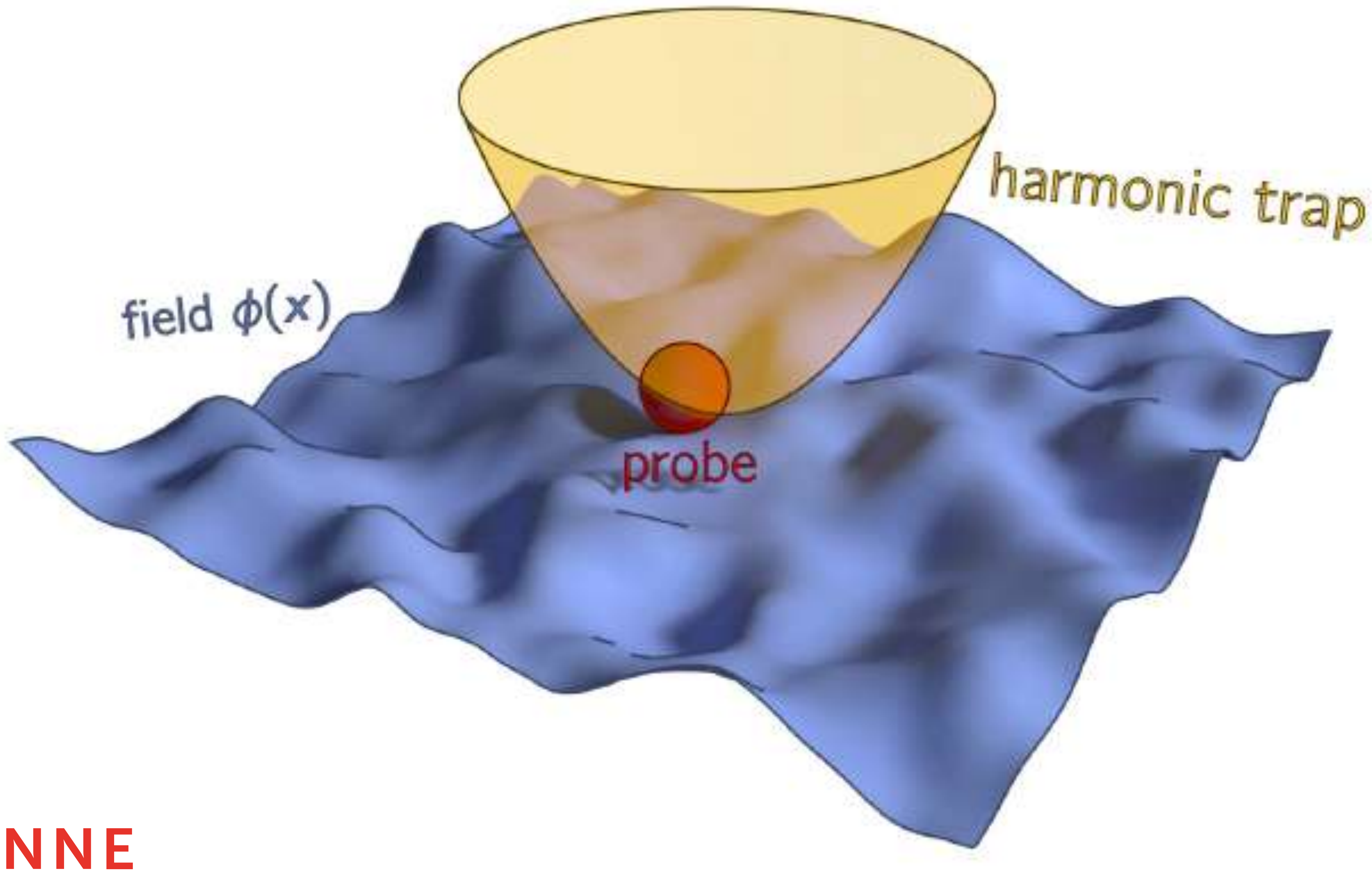
Dynamical signatures of field-mediated interactions

Davide Venturelli
LPTMC, Sorbonne Université

Seminar @ LPTMC
Sorbonne Université, 11 Mars 2025

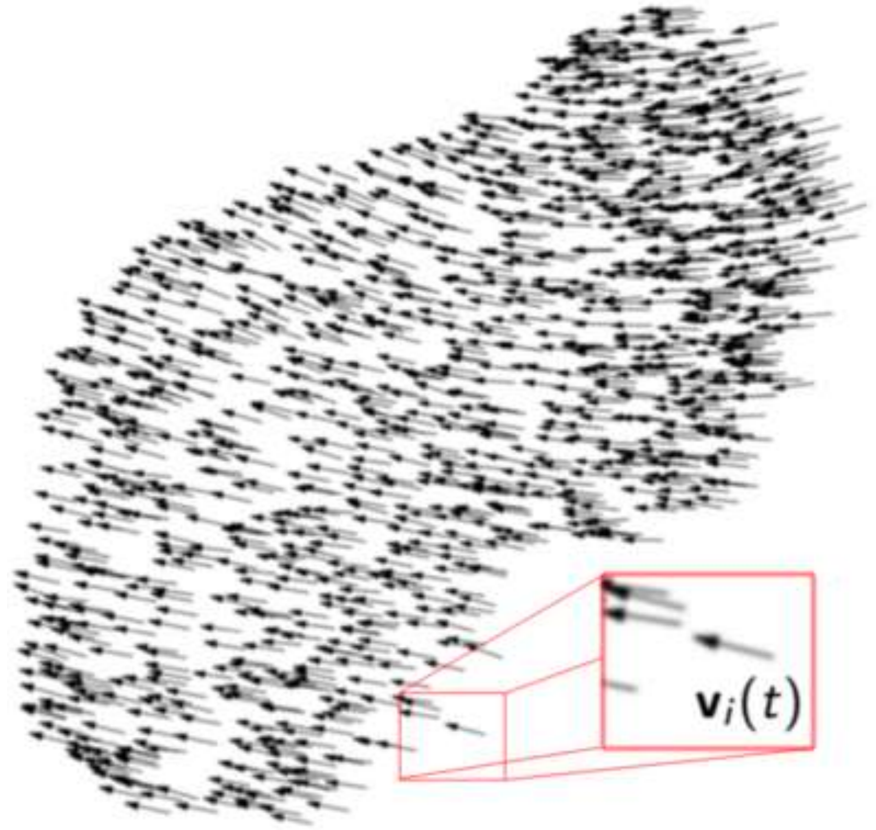


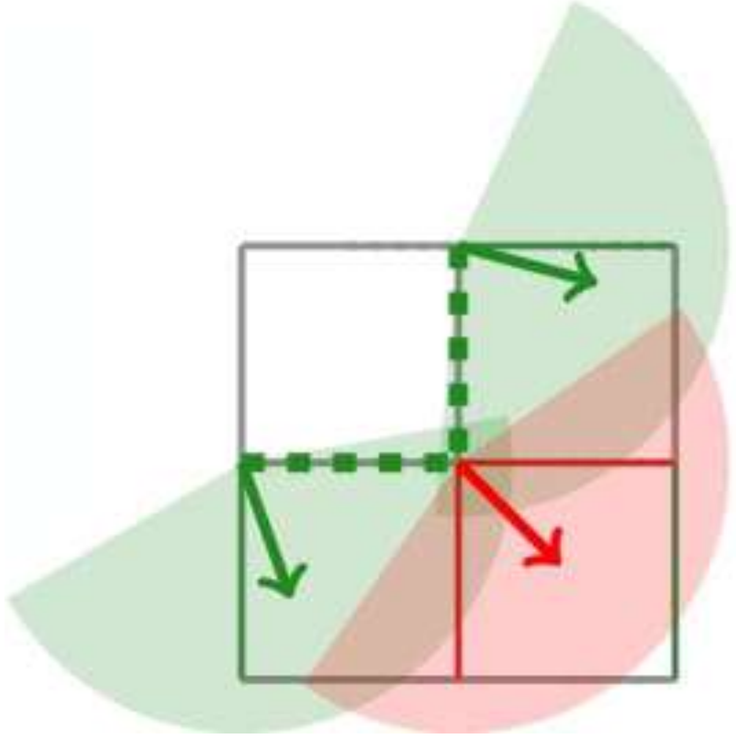
Besides this talk

- **Before LPTMC:** Sapienza (MSc, Rome), SISSA (PhD, Trieste) + visiting LPTHE
- **Active matter & collective behavior**
  - Dynamical **response** to a local **perturbation** in the Inertial Spin Model

    *with I. Giardina and E. Loffredo, 2023 Phys. Biol. **20** 035003*
  - LRO *without* non-reciprocal interactions in **XY** model + **vision cones**

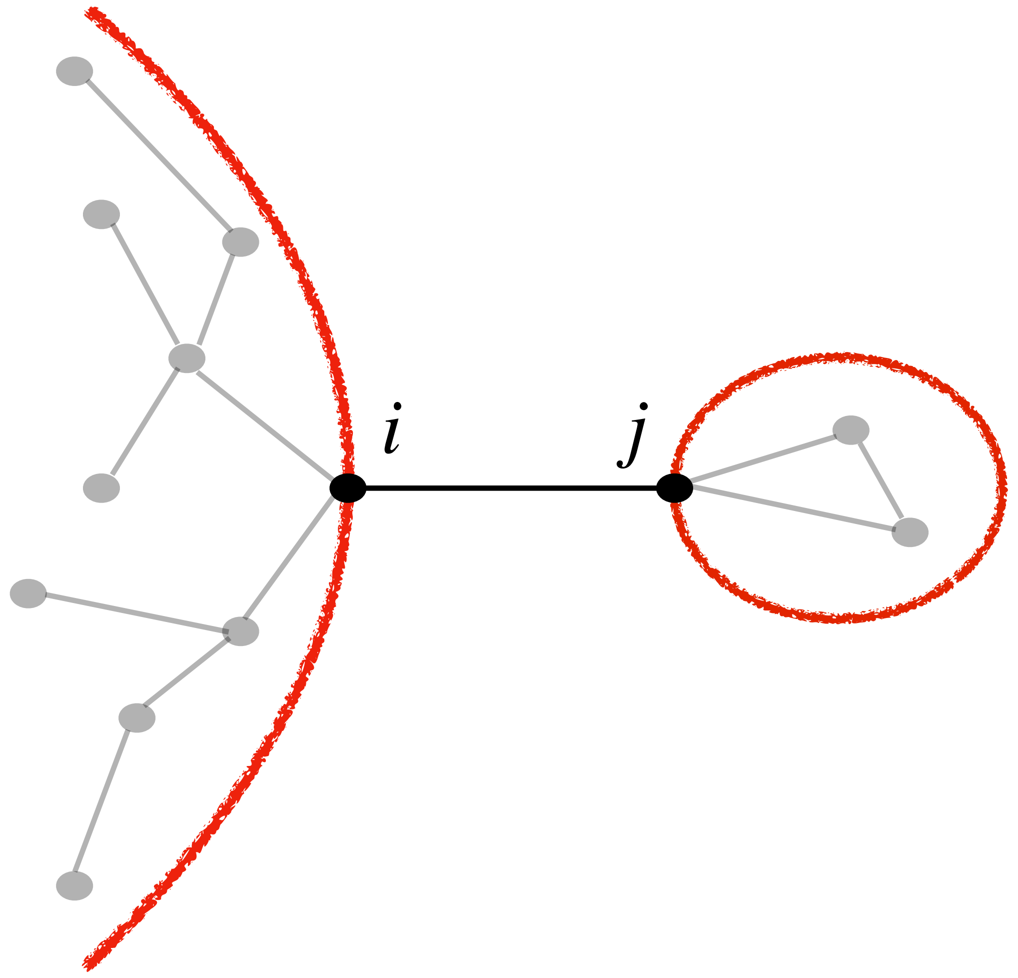
    with G. Bandini, S. A. M. Loos, A. Gambassi, A. Jelic, arXiv:2412.19297



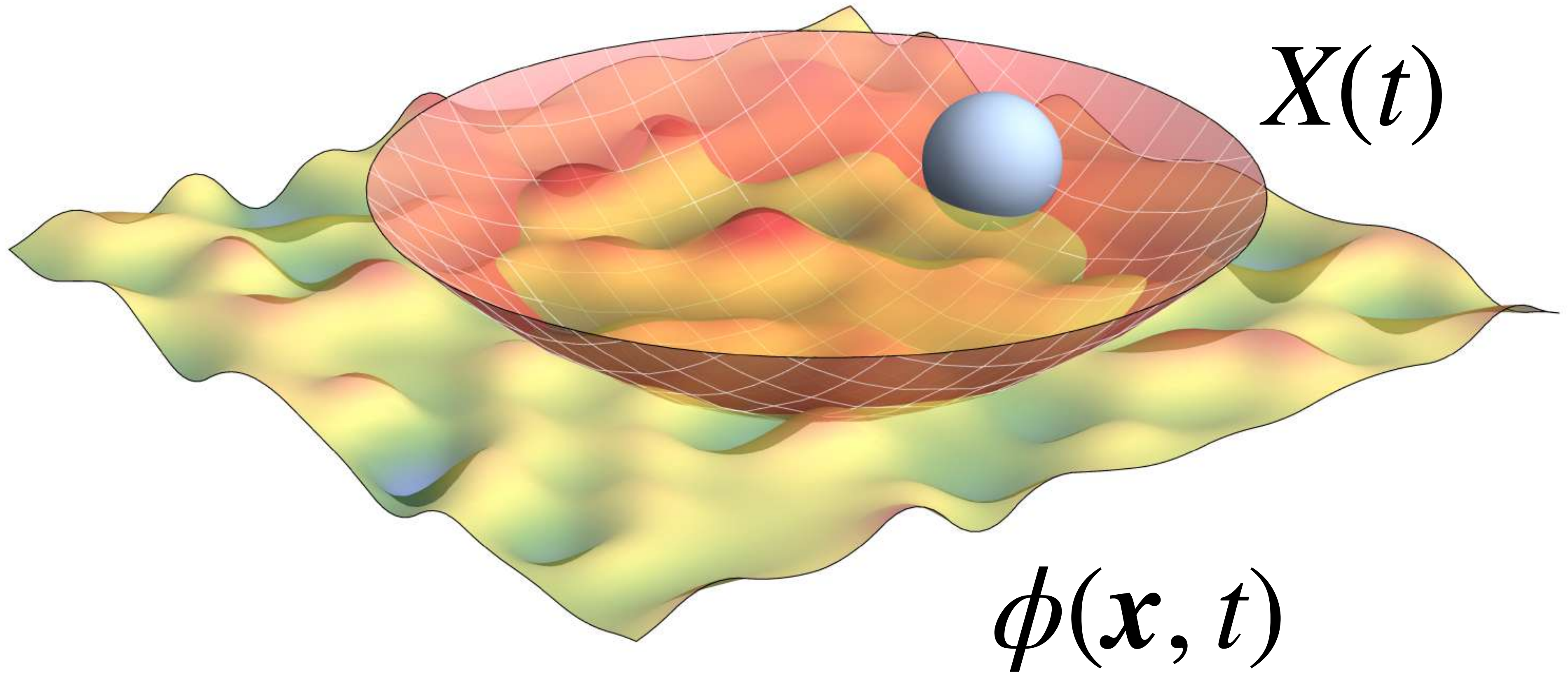


- **Random matrix theory & localization**
  - Partial wavefunction delocalization (**GRP** model, **Erdös-Rényi** graph)
  - Spectral and correlation properties (**replica field theory**, cavity)

with M. Tarzia, L. Cugliandolo, G. Schehr, V. Delapalme, P. Vivo, P. Le Doussal
*SciPost Phys. **14**, 110 (2023); Phys. Rev. B **110**, 174202 (2024)*



A minimal model



$$\dot{\mathbf{X}}(t) = -\nu \boldsymbol{\nabla}_X \mathcal{H} + \boldsymbol{\xi}(t)$$

$$\partial_t \phi(\mathbf{x}, t) = -D(i\boldsymbol{\nabla})^\alpha \frac{\delta \mathcal{H}}{\delta \phi(\mathbf{x}, t)} + \zeta(\mathbf{x}, t)$$

$$\boxed{\mathcal{H}[\phi, \mathbf{X}] = \mathcal{H}_\phi + \mathcal{U}(\mathbf{X}) - \lambda \mathcal{H}_{\text{int}}}$$

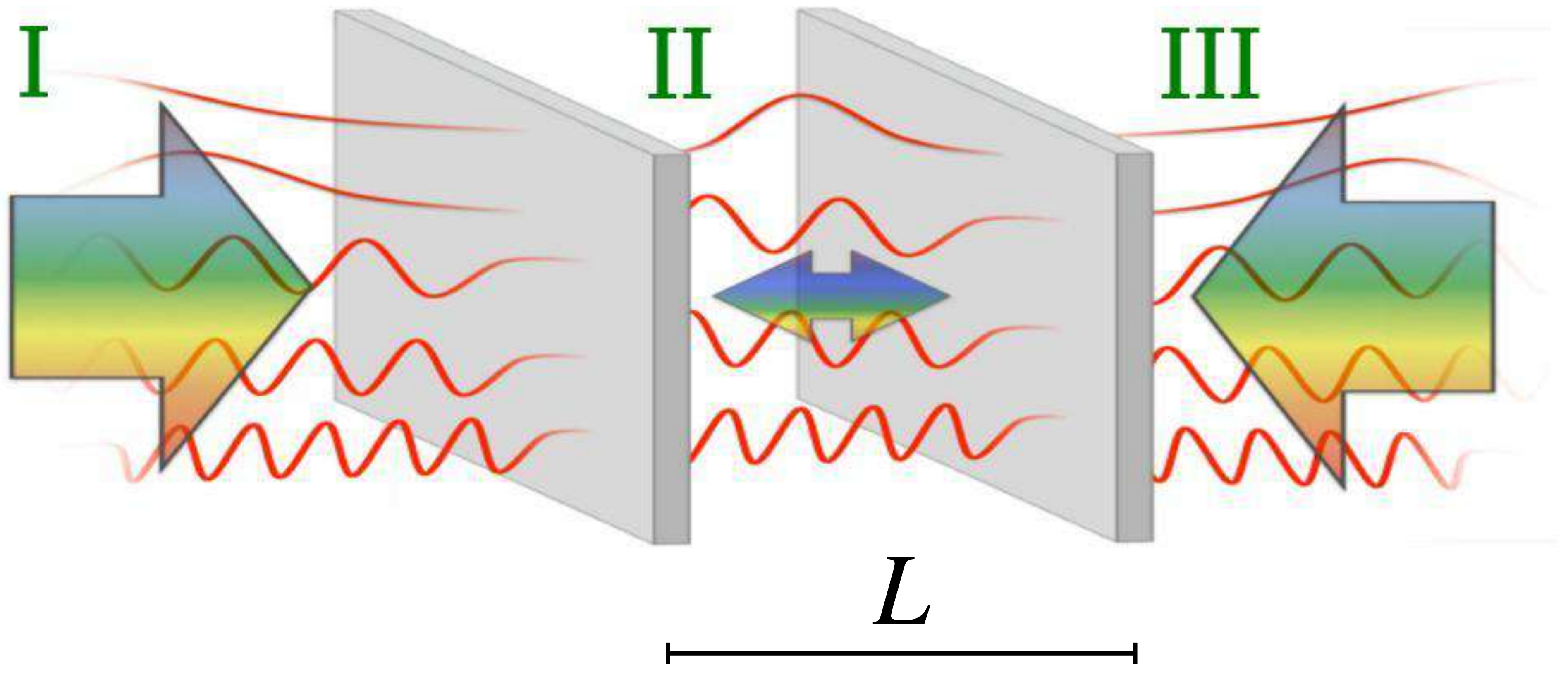
- **For what?** Effective dynamics of tracer(s) in complex fluctuating media, e.g.

1. Near-critical media
2. Viscoelastic media
3. Interacting particle systems

} spatial and temporal correlations

1. Near-critical media

Casimir forces
and thermal fluctuations



- In QED, boundary conditions on EM field

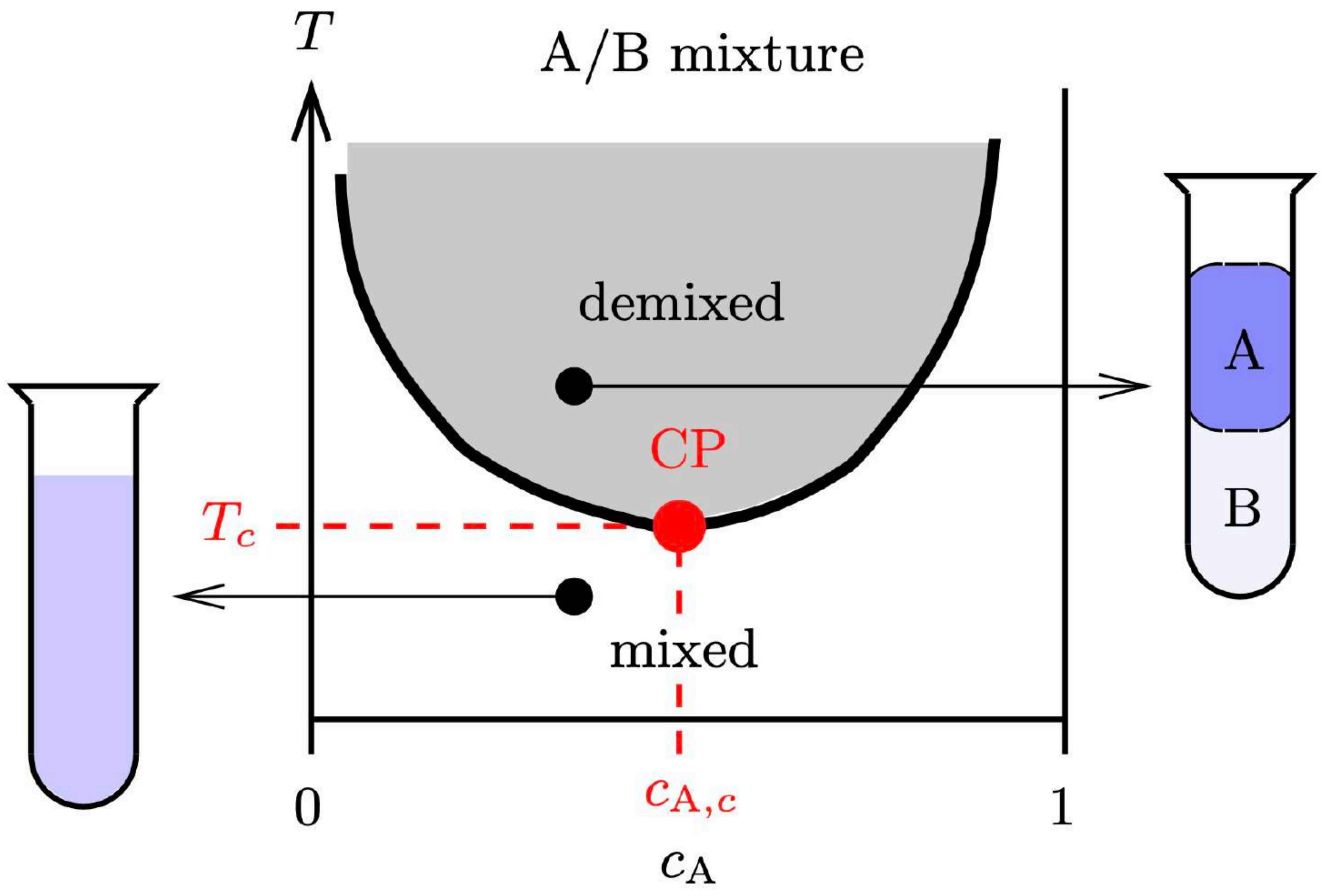
  energy $E = E(L) \implies$ force $\propto \hbar c$

- In a binary liquid mixture, order parameter

$$\phi(\boldsymbol{x}) = c_A(\boldsymbol{x}) - c_B(\boldsymbol{x}), \quad \langle \phi(\boldsymbol{x})\phi(\boldsymbol{x}') \rangle \propto \mathrm{e}^{-|\boldsymbol{x}-\boldsymbol{x}'|/\xi}$$

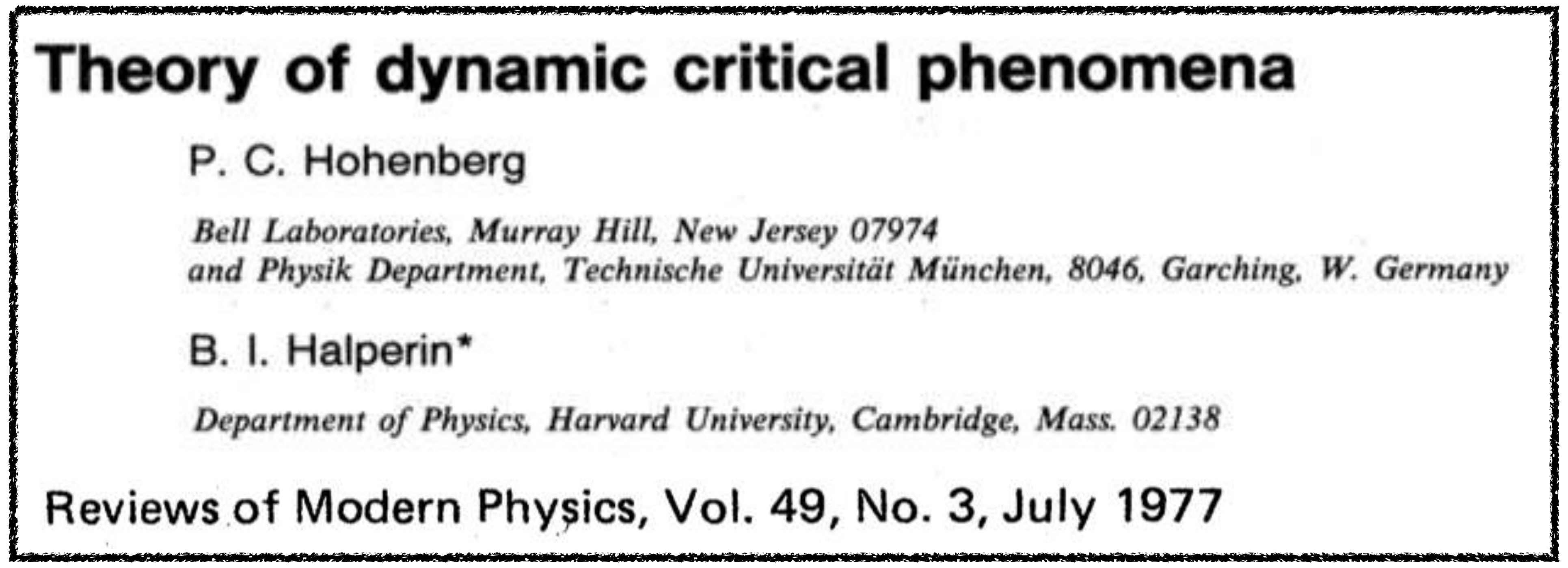
  with $\xi \to \infty$ close to CP

- **Preferential absorption** of A or B → constraint on ϕ
  - forces from free energy $\propto k_B T$, range $\propto \xi$
  - strong enough for colloidal particles   Hertlein *et al.*, Nature (2008)



Field-mediated forces

in non-static scenarios?



Theory of dynamic critical phenomena

P. C. Hohenberg

Bell Laboratories, Murray Hill, New Jersey 07974
and Physik Department, Technische Universität München, 8046, Garching, W. Germany

B. I. Halperin*

Department of Physics, Harvard University, Cambridge, Mass. 02138

Reviews of Modern Physics, Vol. 49, No. 3, July 1977

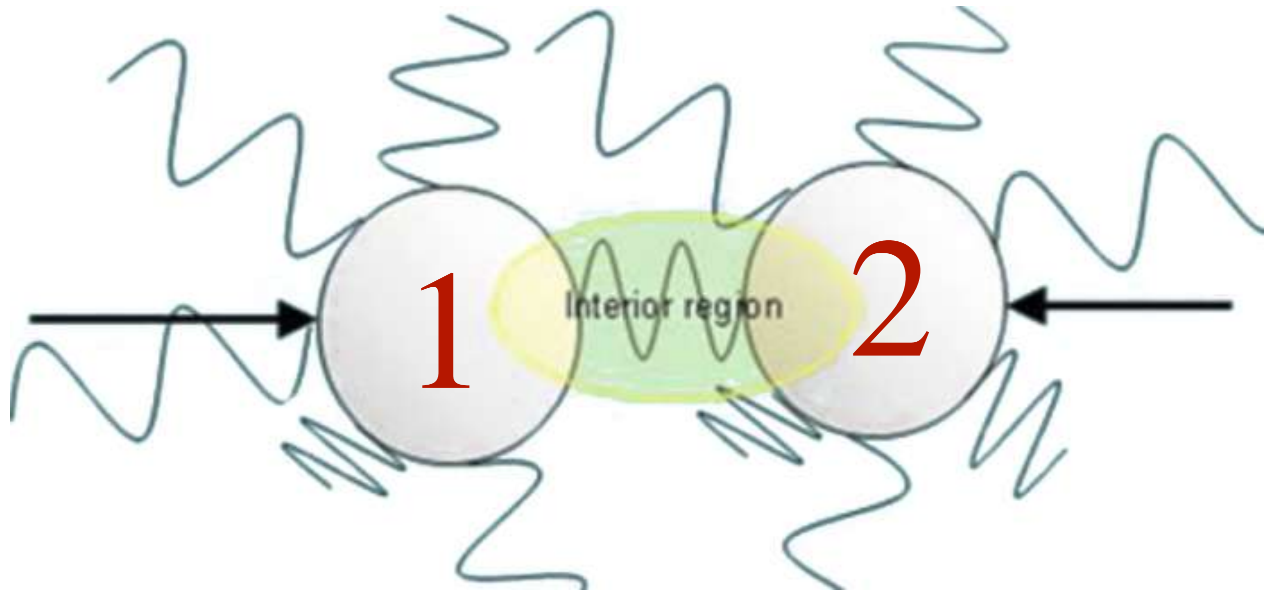
- Close to a continuous PT, **universality**
  $\rightarrow$ few relevant slow coarse-grained d.o.f. (order parameter, conserved densities)

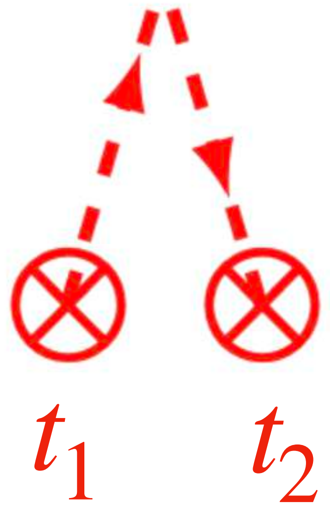
- Critical dynamics, e.g. **model A/B** (relaxation towards $\mathscr{P}[\phi] \propto \mathrm{e}^{-\beta \mathscr{H}_{\mathrm{LG}}[\phi]}$)

A $$\partial_t \phi(\boldsymbol{x}, t) = -D \frac{\delta \mathscr{H}_{\mathrm{LG}}[\phi]}{\delta \phi(\boldsymbol{x}, t)} + \zeta(\boldsymbol{x}, t)$$

conserved

B $$\partial_t \phi(\boldsymbol{x}, t) = D \nabla^2 \frac{\delta \mathscr{H}_{\mathrm{LG}}[\phi]}{\delta \phi(\boldsymbol{x}, t)} + \zeta(\boldsymbol{x}, t) = -\nabla \cdot \boldsymbol{J}(\boldsymbol{x}, t)$$





- $\tau \sim \xi^z$, critical **slowing down** $\rightarrow$ forces do not propagate **instantaneously**

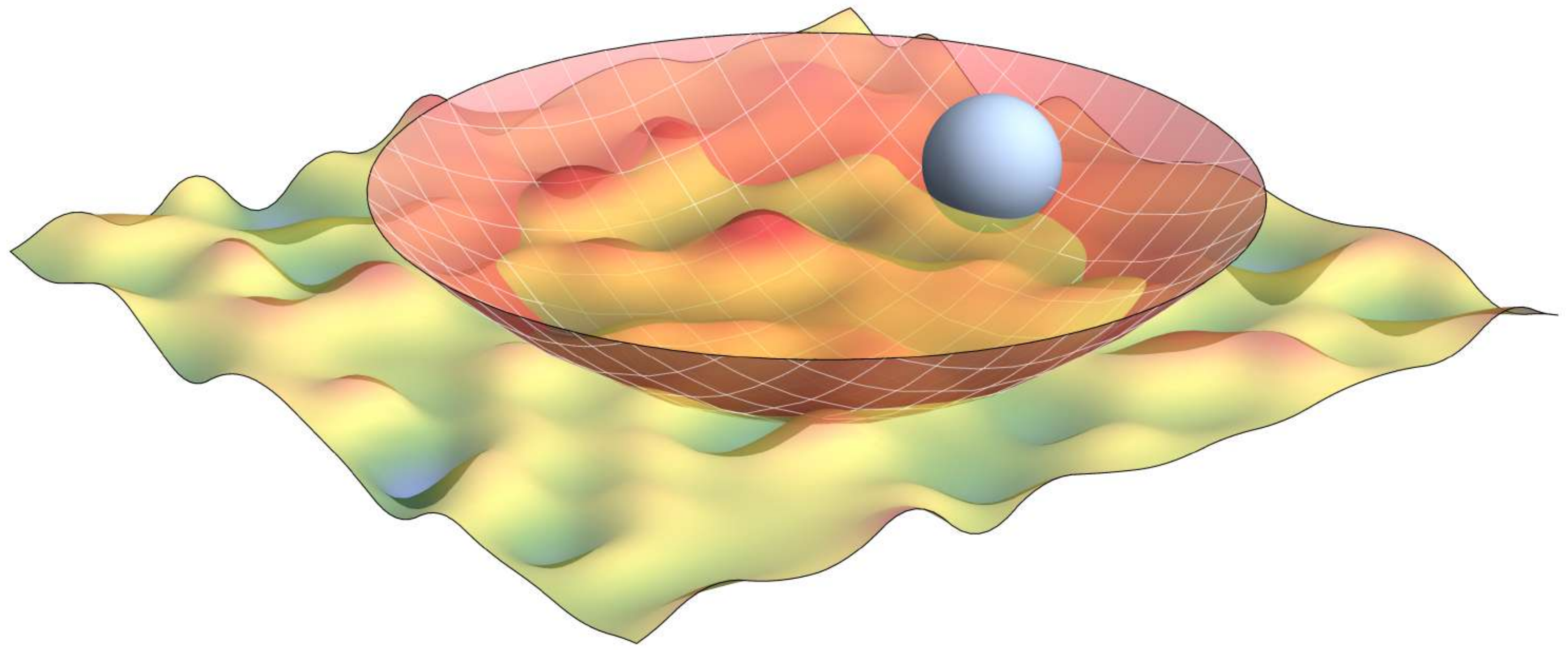
- **Dynamic** field-mediated forces (+ self-interaction) are an <u>open problem</u>, relevant for experiments

Hertlein *et al.*, Nature (2008); A. Magazzù *et al.*, Soft Matter (2018)

Minimal model

$\xi = r^{-1/2}$ **sets the range of spatial correlations of** $\phi(x,t)$

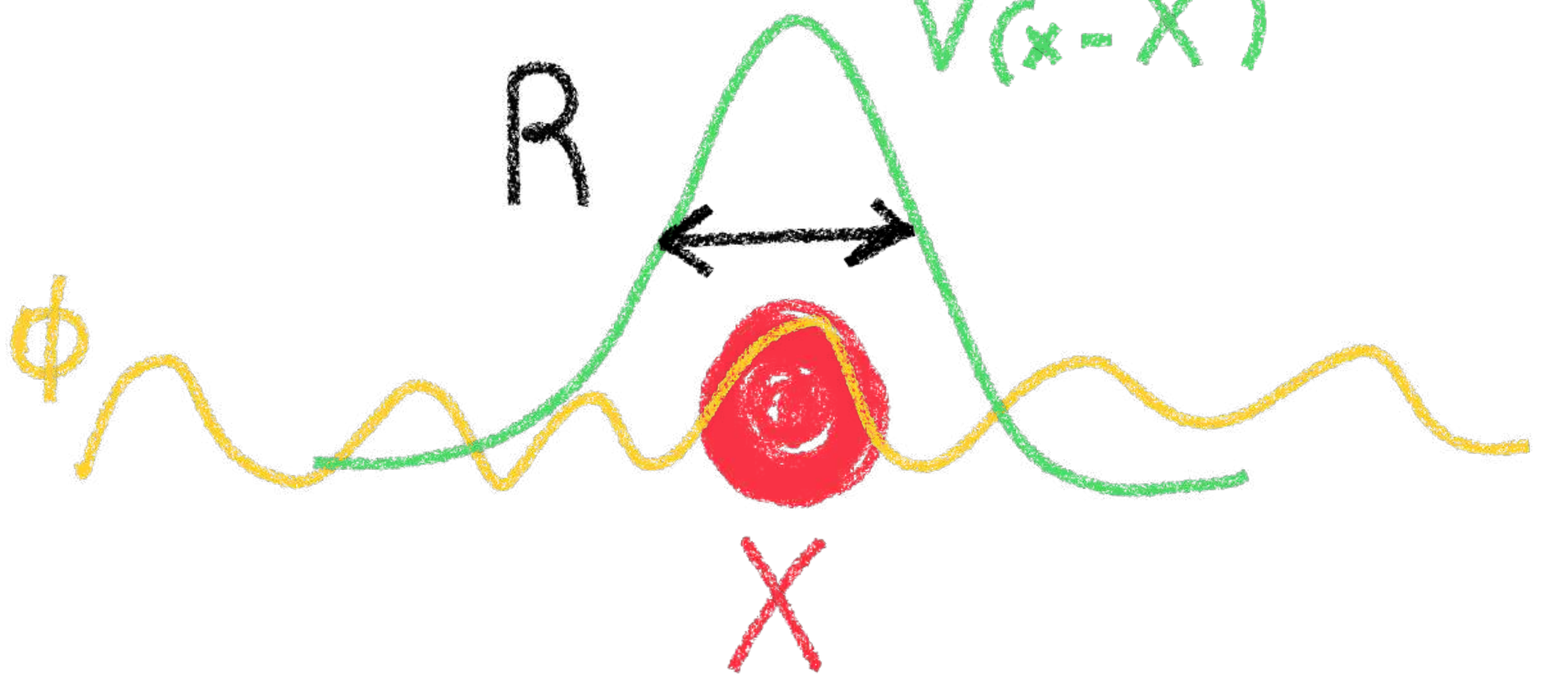
$$\mathcal{H}[\phi, \mathbf{X}] = \mathcal{H}_\phi + \mathcal{U}(\mathbf{X}) - \lambda \mathcal{H}_{\text{int}}$$



$$\mathcal{H}_\phi = \int \mathrm{d}^d\mathbf{x} \left[\frac{1}{2}(\nabla\phi)^2 + \frac{1}{2} r \phi^2 \right]$$

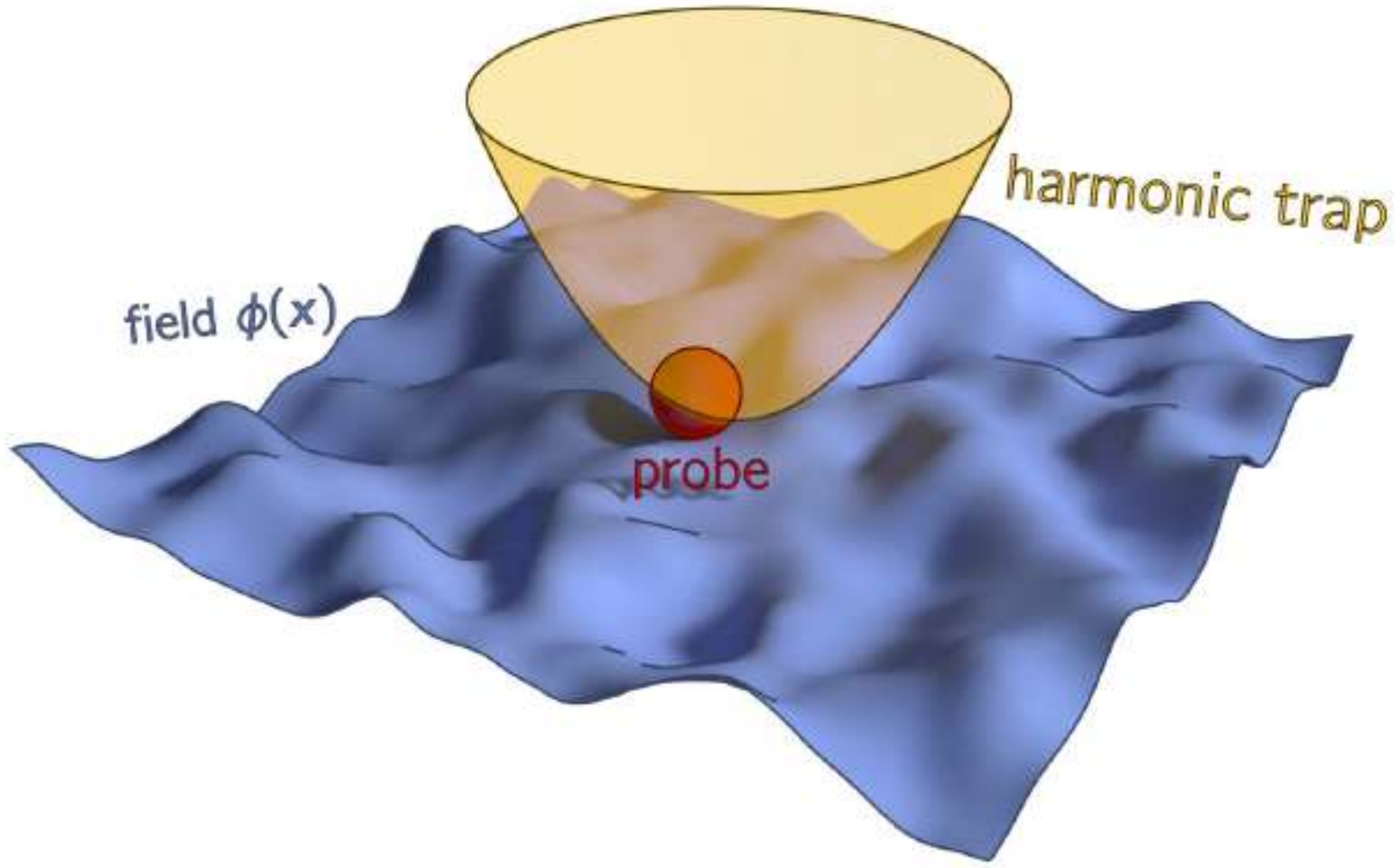
$$\mathcal{U}(\mathbf{X}) = \frac{\kappa}{2} X^2$$

$$\mathcal{H}_{\text{int}} = \int \mathrm{d}^d\mathbf{x}\, \phi(\mathbf{x}) V(\mathbf{x} - \mathbf{X})$$



AIM: effective dynamics of $X(t)$,
in out-of-equilibrium settings

Dynamics



ϕ, X influence each other:

$$\dot{\mathbf{X}}(t) = -\nu \nabla_X \mathcal{H} + \boldsymbol{\xi}(t)$$

$$\partial_t \phi(\mathbf{x}, t) = -D(i\nabla)^\alpha \frac{\delta \mathcal{H}}{\delta \phi(\mathbf{x}, t)} + \zeta(\mathbf{x}, t)$$

$\alpha = 0{,}2$

$$\langle \xi_i(t) \xi_j(t') \rangle = 2\nu T \delta_{ij} \delta(t - t')$$

$$\langle \zeta(\mathbf{x}, t) \zeta(\mathbf{x}', t') \rangle = 2DT(i\nabla)^\alpha \delta^d(\mathbf{x} - \mathbf{x}') \delta(t - t')$$

Go in **Fourier**:

$$\dot{\mathbf{X}}(t) = -\nu\kappa \mathbf{X}(t) + \lambda\nu \int_{\mathbb{R}^d} \frac{\mathrm{d}^d q}{(2\pi)^d} i\mathbf{q} V_{-q}\, \phi_q(t)\, \mathrm{e}^{i\mathbf{q}\cdot\mathbf{X}(t)} + \boldsymbol{\xi}(t)$$

slow for small q, and $r \to 0$

$$\partial_t \phi_q = -Dq^\alpha(q^2 + r)\phi_q + \lambda Dq^\alpha V_q\, e^{-i\mathbf{q}\cdot\mathbf{X}} + \zeta_q \longrightarrow \tau_\phi^{-1}(q) \equiv Dq^\alpha(q^2 + r)$$

Relaxation towards equilibrium

Perturbation theory

@ long times, **non-exponential** relaxation:

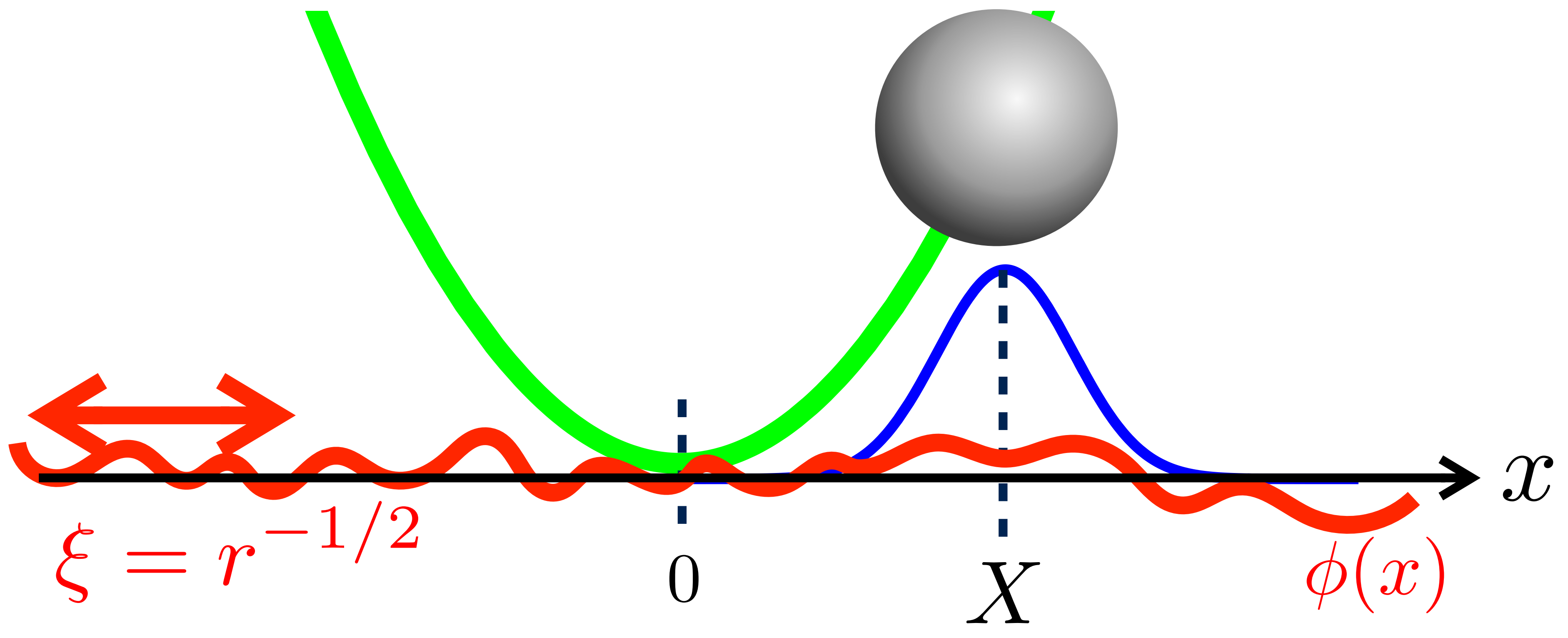
$$\langle X(t)\rangle \sim \begin{cases} t^{-\left(1+\frac{d}{2}\right)}, & \text{Model A}, \; r=0 \\ t^{-\left(1+\frac{d}{4}\right)}, & \text{Model B}, \; r=0 \\ t^{-\left(2+\frac{d}{2}\right)}, & \text{Model B}, \; r>0 \end{cases}$$

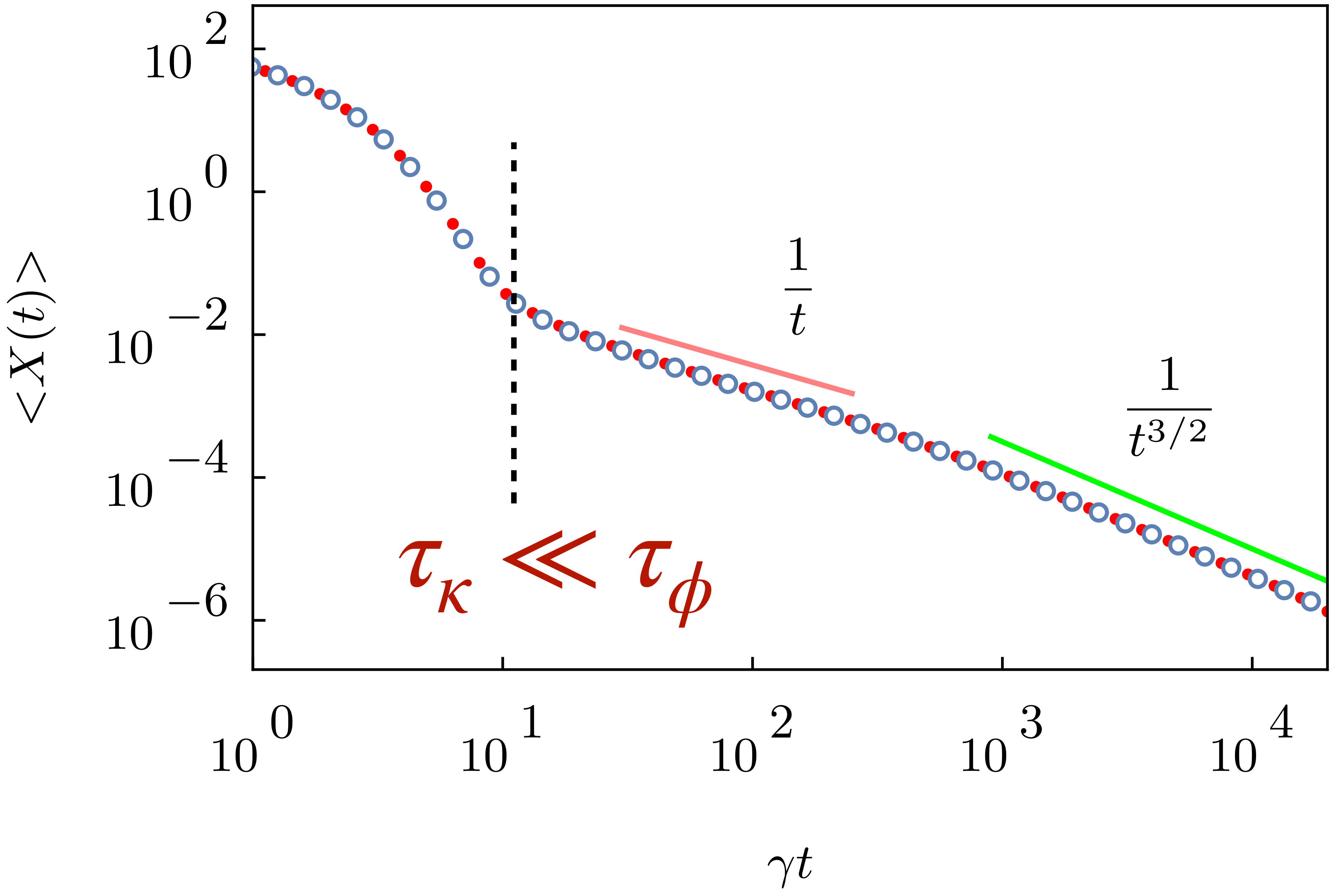
At the critical point,

$$\langle X(t)\rangle \sim t^{-\left(1+d/z\right)}$$

dynamical critical exponent of ϕ

link long-t behavior of $X(t)$ to critical properties of ϕ





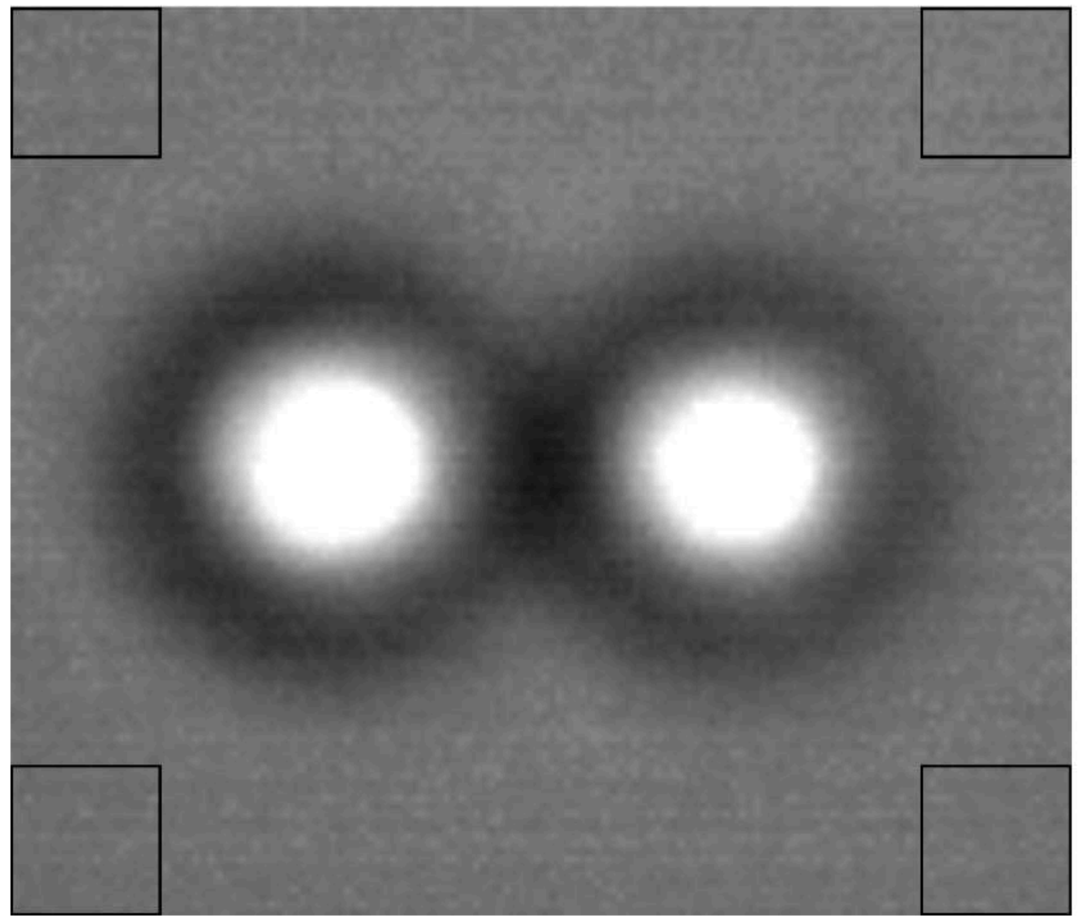
[D. Venturelli, F. Ferraro, A. Gambassi, PRE **105**, 054125 (2022)]

Energy Transfer between Colloids via Critical Interactions

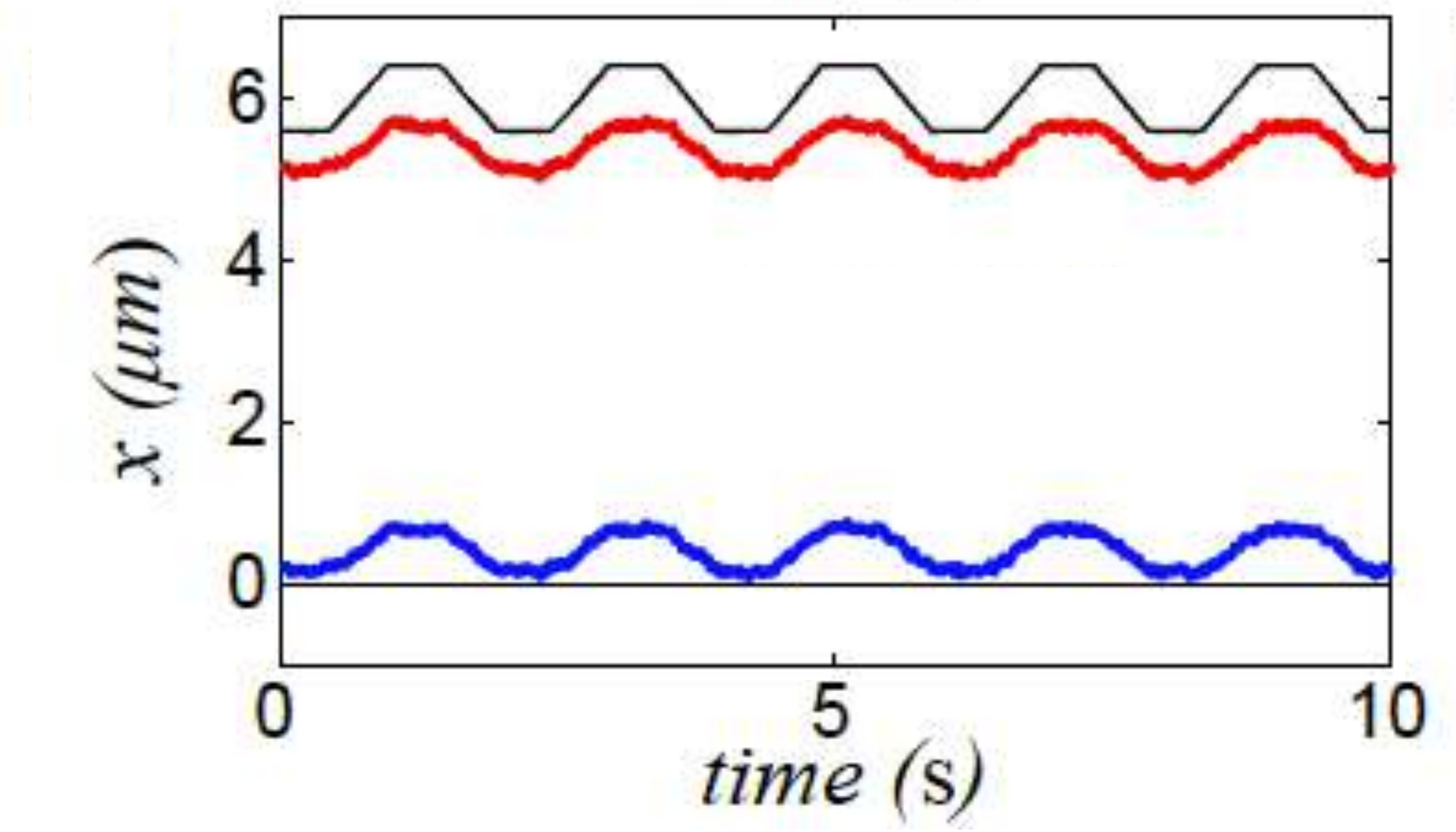
Ignacio A. Martínez [1,2,*], Clemence Devailly [1,3], Artyom Petrosyan [1] and Sergio Ciliberto [1,*]

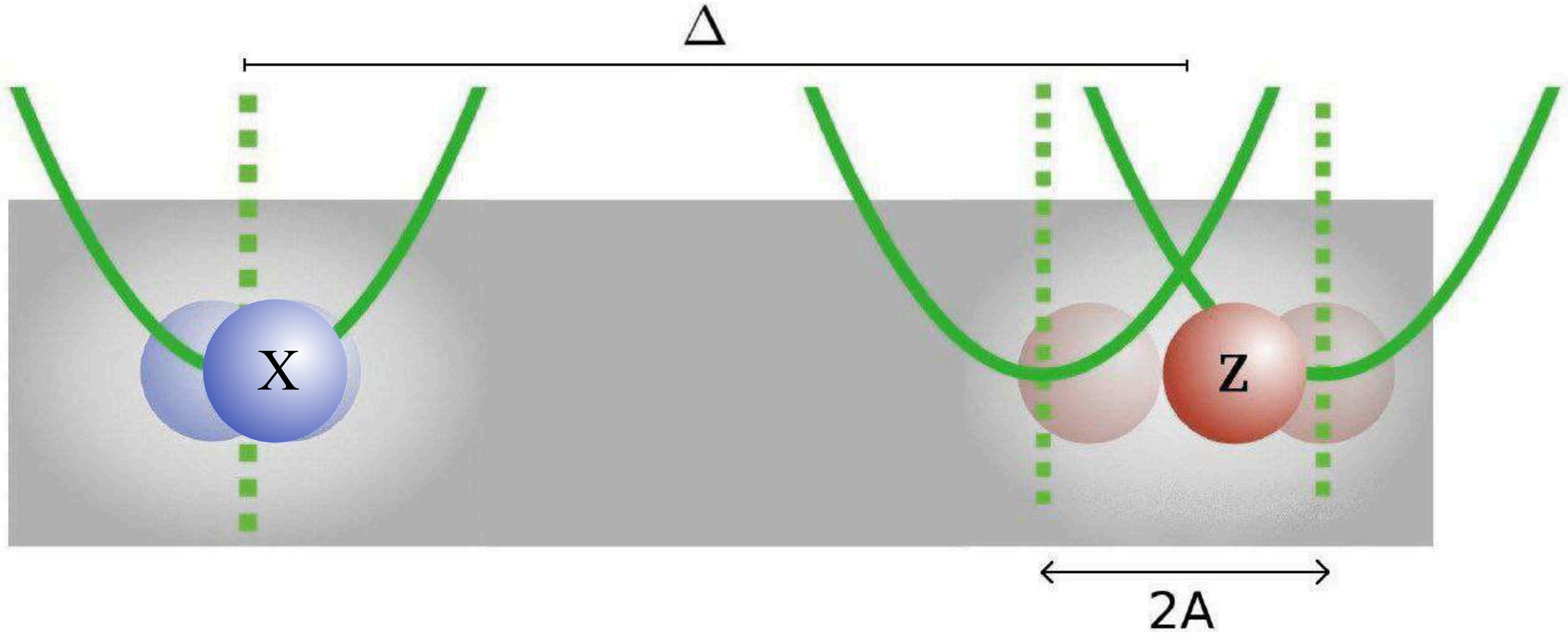
[Entropy 2017]

$$r \propto (T_c - T)/T_c \propto \xi^{-2}$$

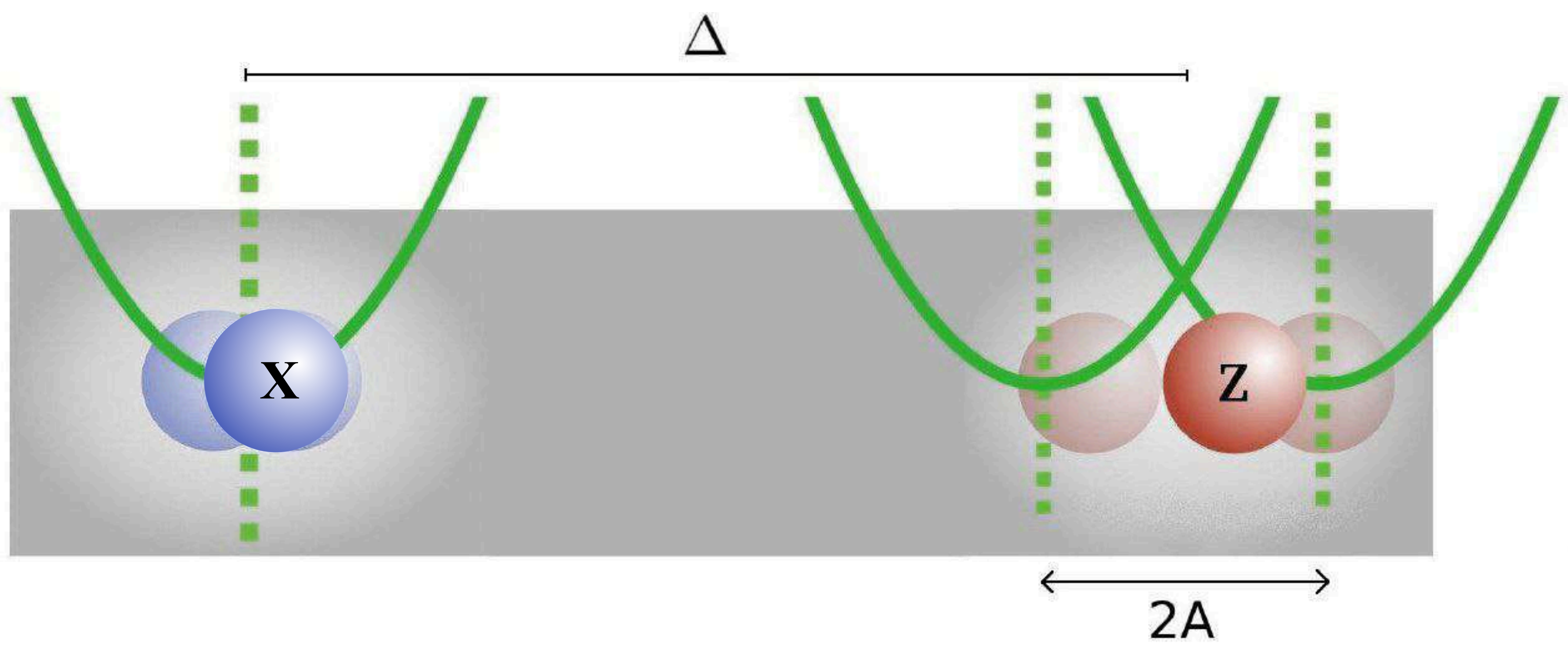


Slowly, $\Omega \sim \mathrm{hour}^{-1}$





Dynamical response

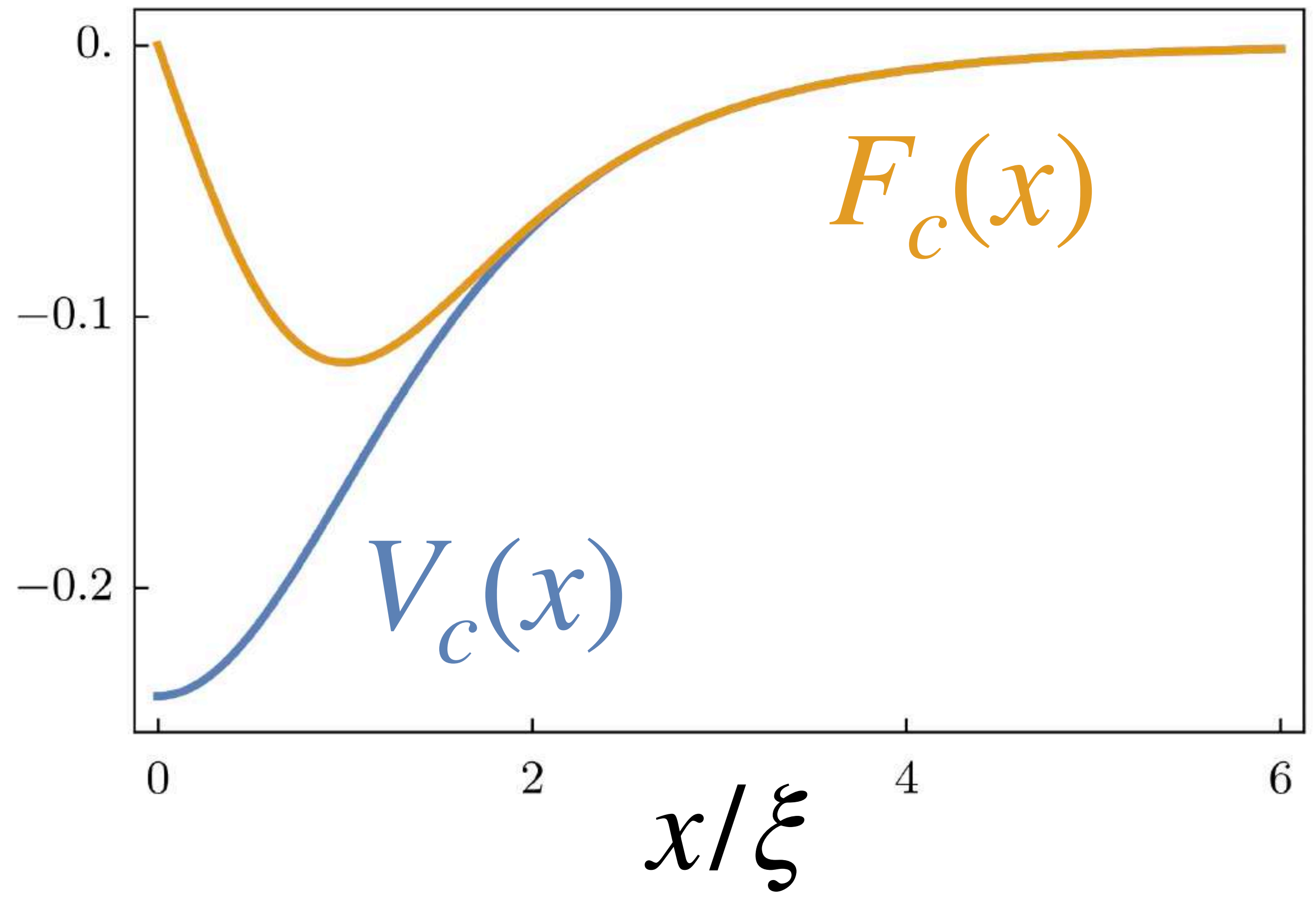


$$\mathscr{H} = \mathscr{H}_\phi + \mathscr{U}(\boldsymbol{X}) - \lambda \mathscr{H}_{\text{int}}$$

$$\mathscr{H}_{\text{int}} = \int \mathrm{d}^d \boldsymbol{x}\, \phi(\boldsymbol{x}) \left[V(\boldsymbol{x} - \boldsymbol{X}) + V(\boldsymbol{x} - \boldsymbol{Z}(t)) \right]$$

- AIM: effective dynamics of $X(t)$ upon driving $Z(t) = \Delta + A \sin(\Omega t)$
- Naive (adiabatic):

$$\int \mathscr{D}\phi \, \mathrm{e}^{-\beta \mathscr{H}} = \mathrm{e}^{-\beta[\mathscr{U}(\boldsymbol{X}) - \lambda^2 V_c(\boldsymbol{Z}(t) - \boldsymbol{X})]}$$

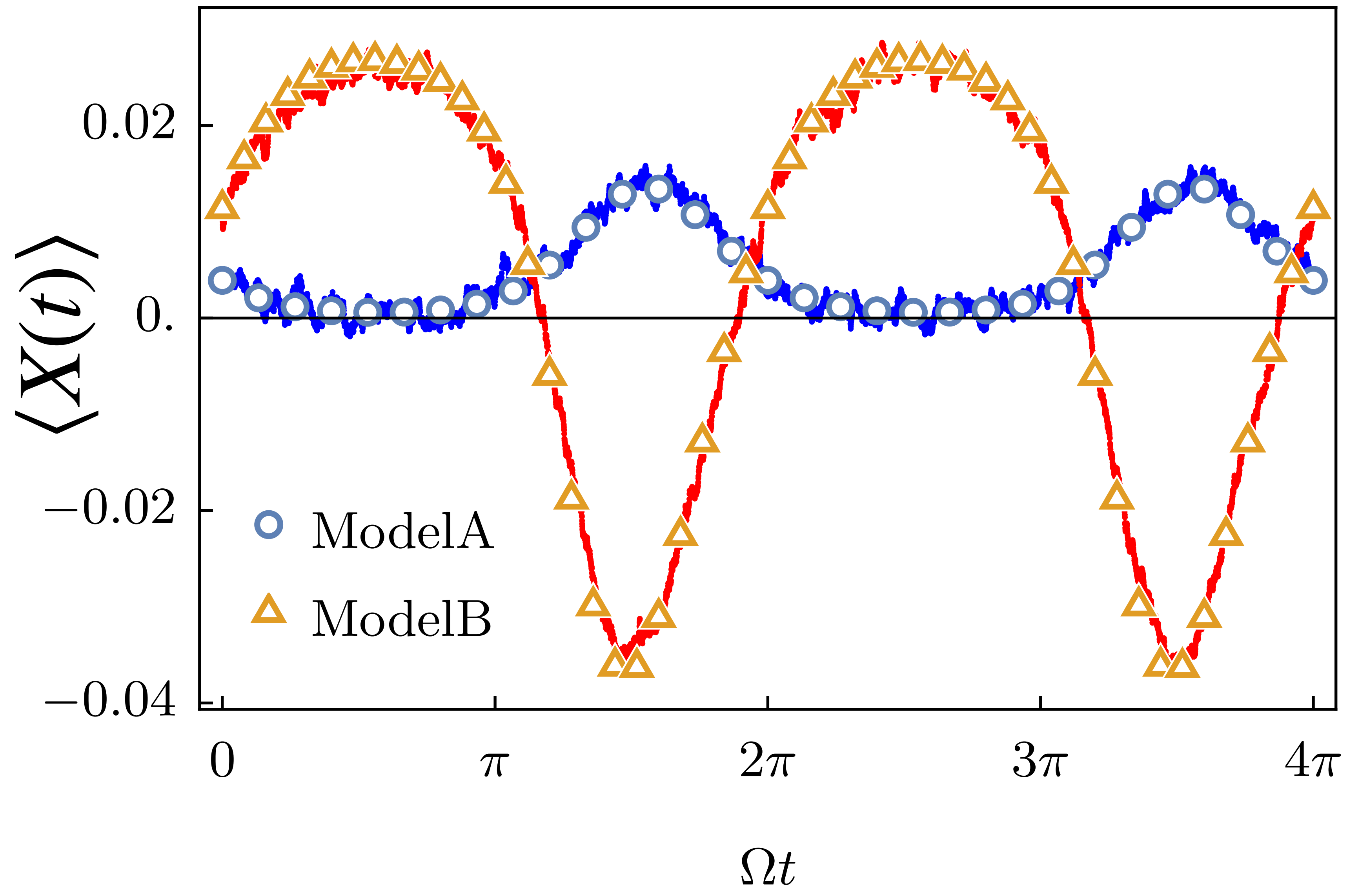


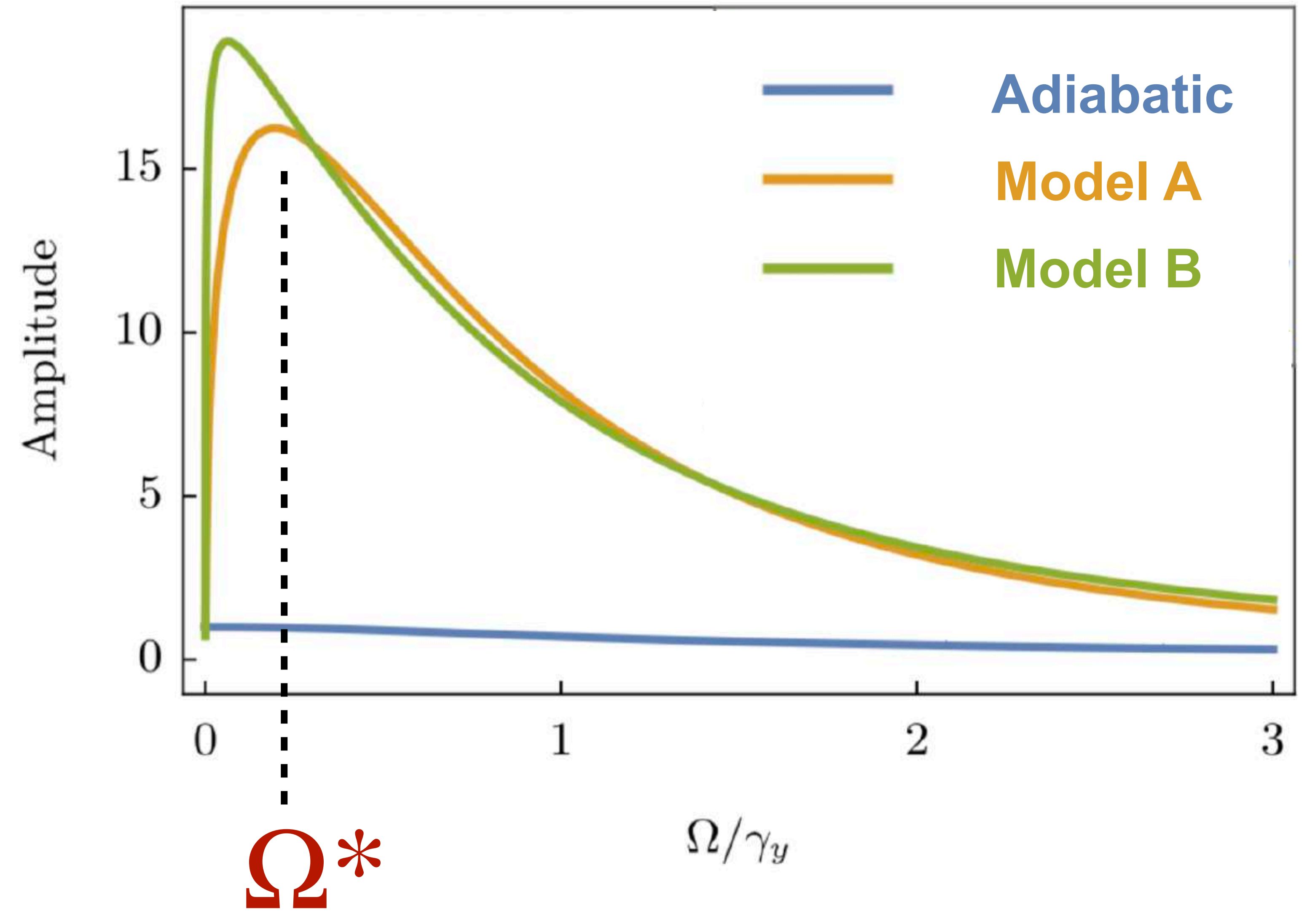
- Competing timescales $\begin{cases} \tau_\phi^{-1} \sim D q^\alpha (q^2 + r) \\ \tau_\Omega^{-1} \sim \Omega \end{cases}$

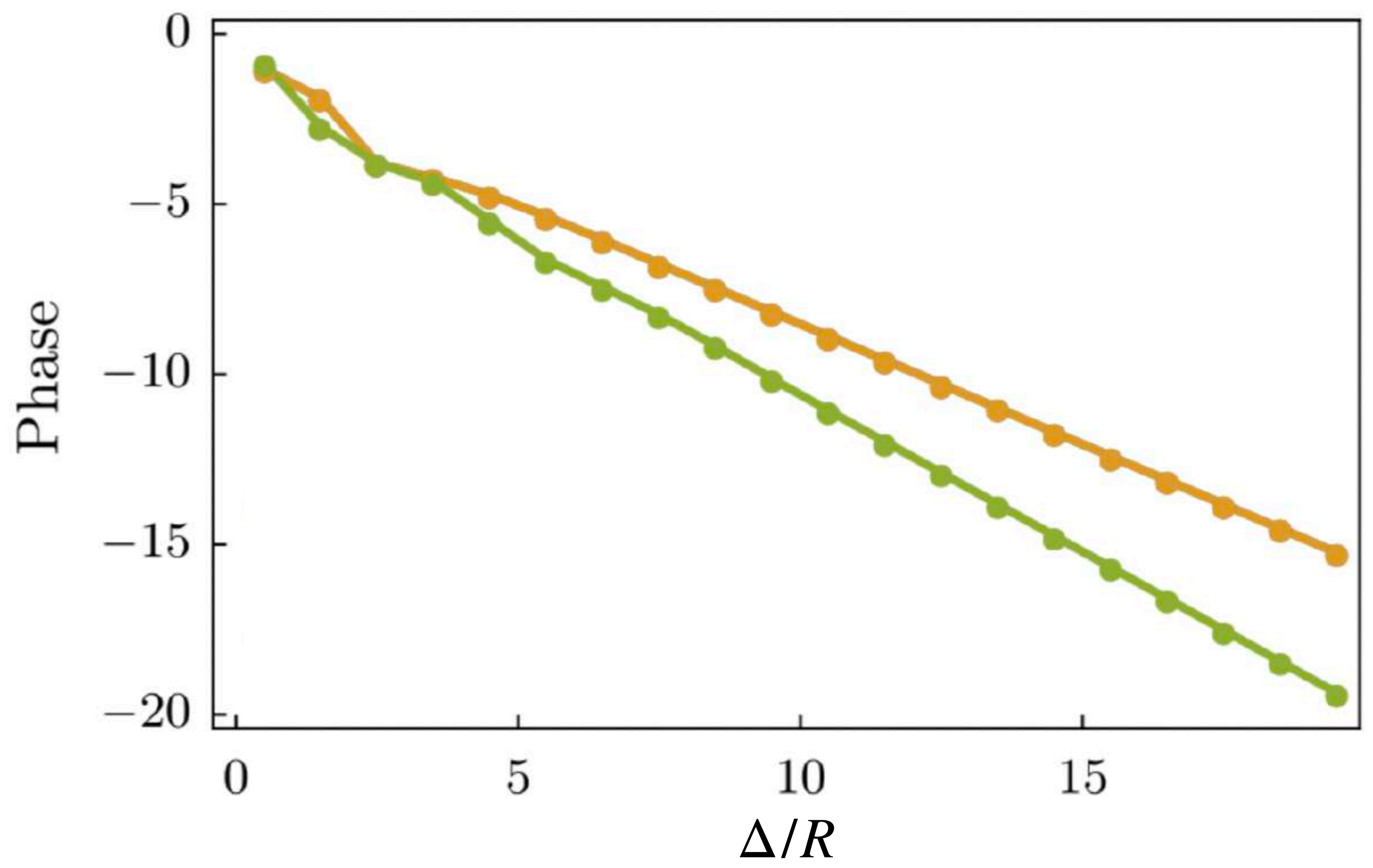
larger Ω ?

Response $\langle \exp[i\boldsymbol{q} \cdot \boldsymbol{X}(t)] \rangle$

Perturbation theory (weak coupling)







- <u>Nonequilibrium effects:</u>
  1. **Resonance** $\Omega^* \sim \tau_\phi^{-1}(q \sim \Delta^{-1}) \propto \Delta^{-z}$
  2. **Delay** (phase shift)

avg separation

new phenomena in a medium with slow response + spatial structure

[D. Venturelli, A. Gambassi, PRE **106**, 044112 (2022)]

2. Viscoelastic media

Tracer in a viscoelastic medium

- **Viscoelastic** (strain ε, stress σ): τ_s is large

$$\sigma(t) = \int_0^t \mathrm{d}t' \, G(t-t') \, \dot{\varepsilon}(t'), \qquad G(t) \xrightarrow{t \gg \tau_s} \begin{cases} G_\infty, & \text{solids} \\ 0, & \text{liquids} \end{cases}$$

shear modulus

- **GLE**

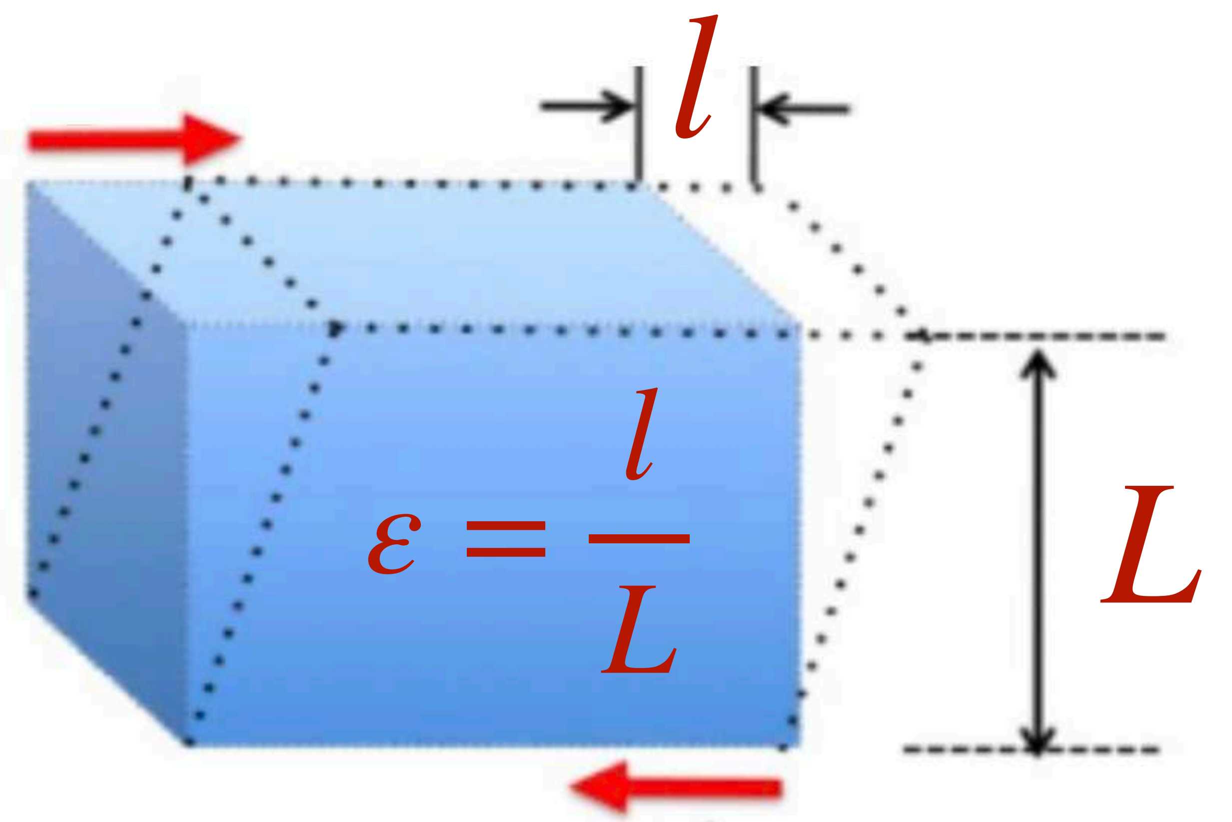
$$\gamma \dot{X}(t) + \int^t \mathrm{d}t' \, \Gamma(t-t') \, \dot{X}(t') = -\, \mathscr{U}'\left(X(t)\right) + \xi(t)$$

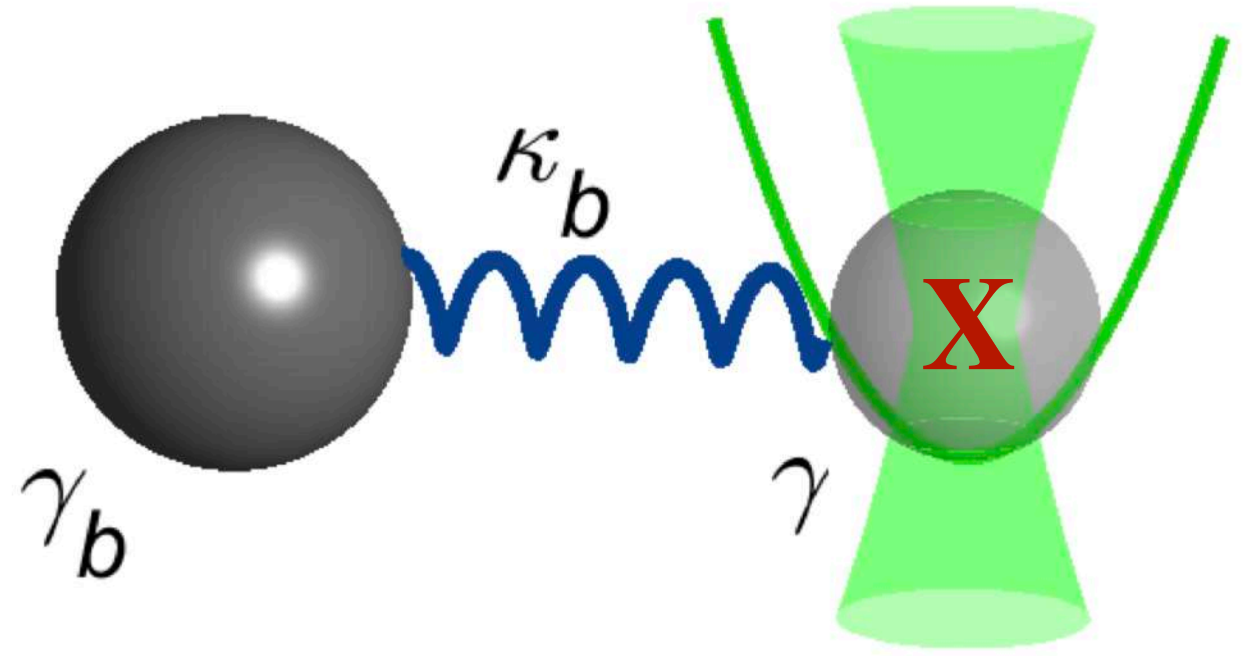
$$\langle \xi(t)\xi(t') \rangle = k_B T \left[\gamma \, \delta(t-t') + \Gamma(|t-t'|) \right]$$

[e.g. Caldeira-Leggett, Mori-Zwanzig]

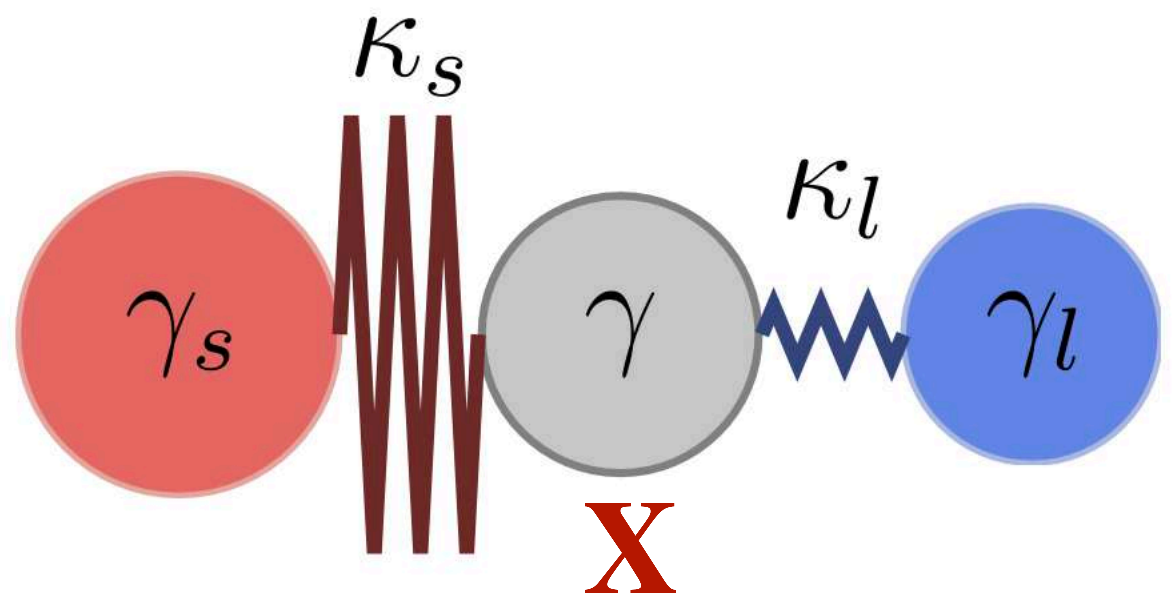
- Limitations:
  - No field-mediated interactions (no space)
  - Assumed linear in $\dot{X}$, may cause Γ to depend on $\mathscr{U}$ (***ad hoc***)

Daldrop *et al.*, PRX (2025)





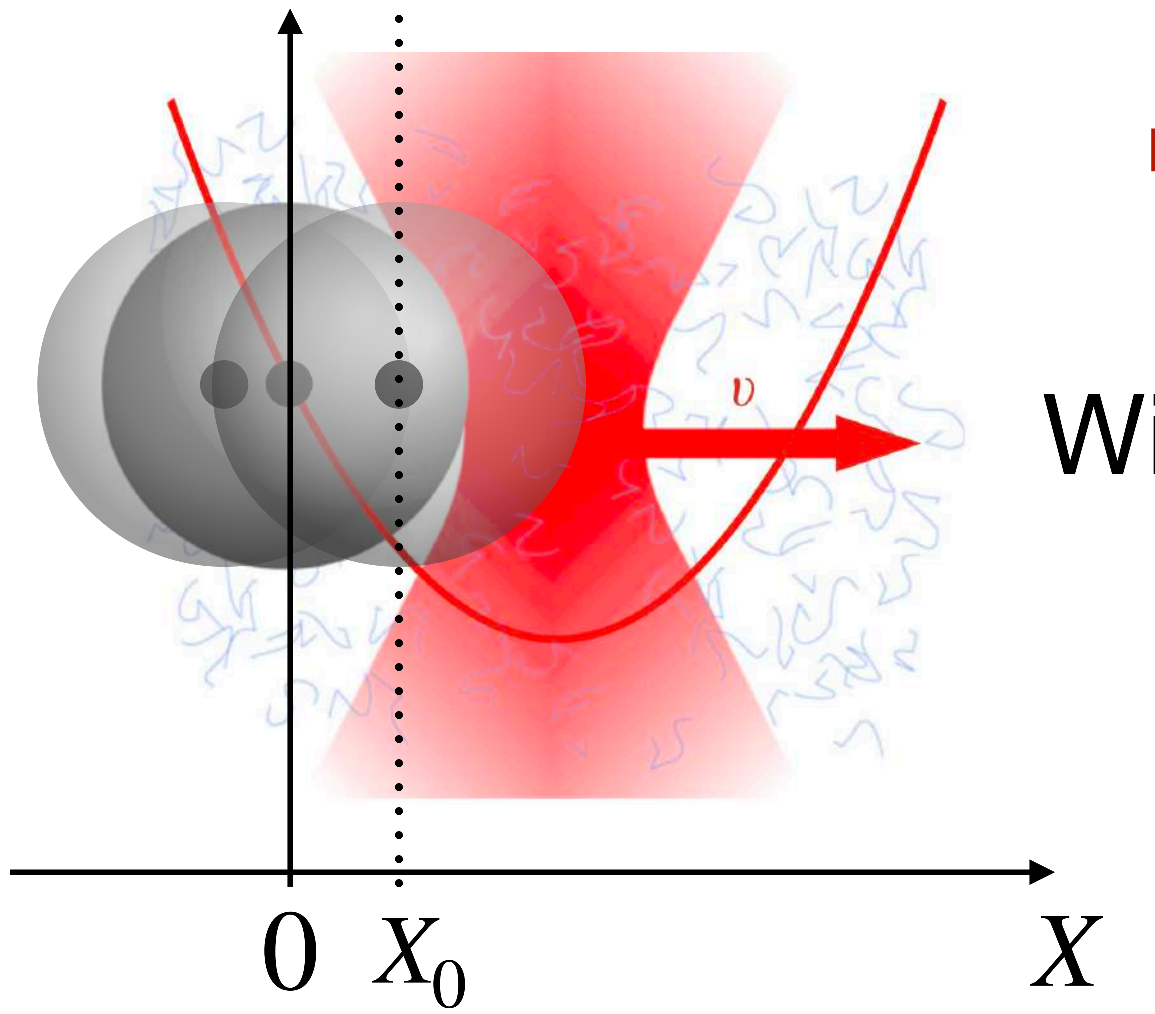
Loos *et al.*, PRX (2024)



Caspers *et al.*, J. Chem. Phys. (2023)

Memory-induced oscillations

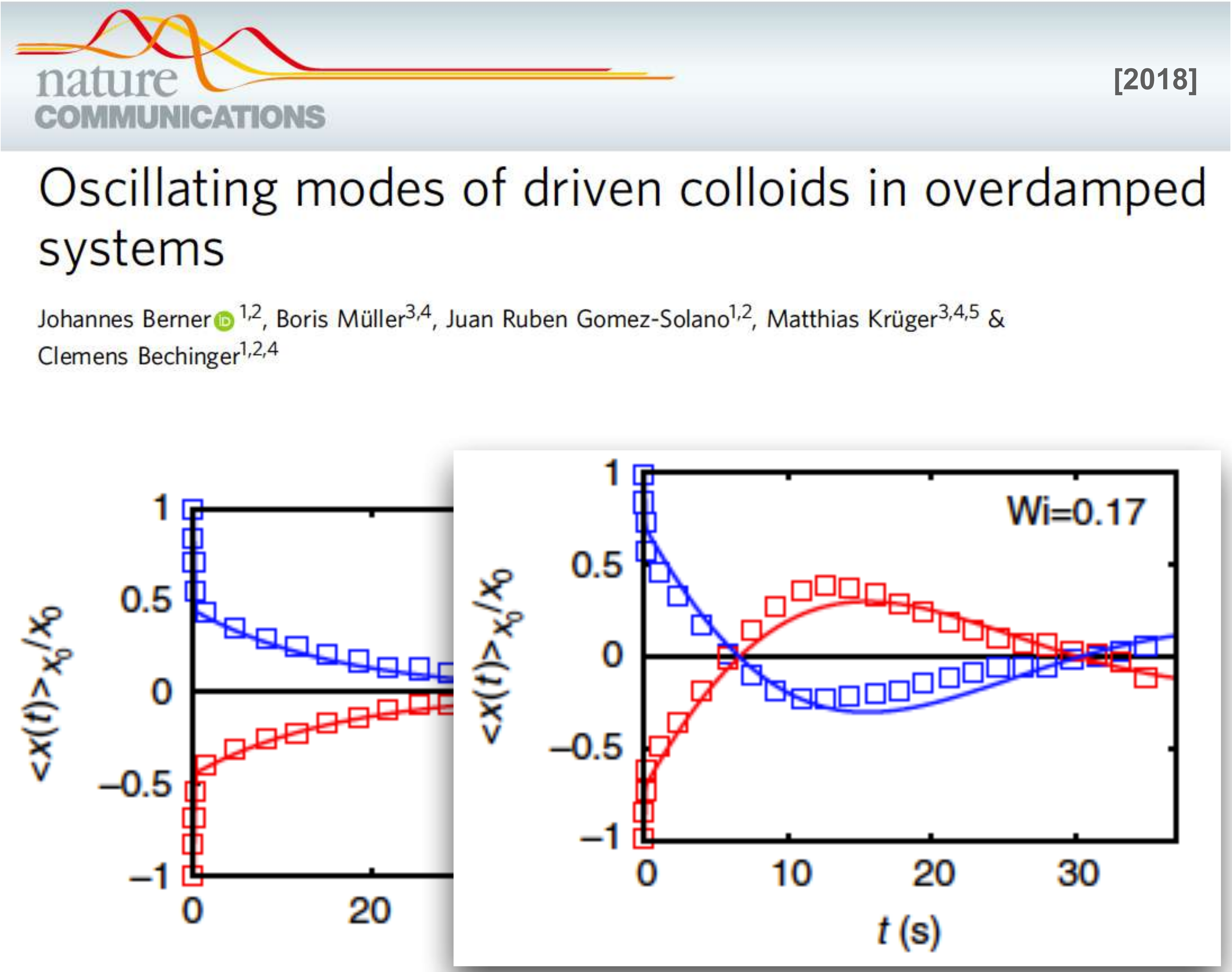
in viscoelastic media



relaxation time

$$\mathrm{Wi} = \tau_s\, v/R$$

trap speed



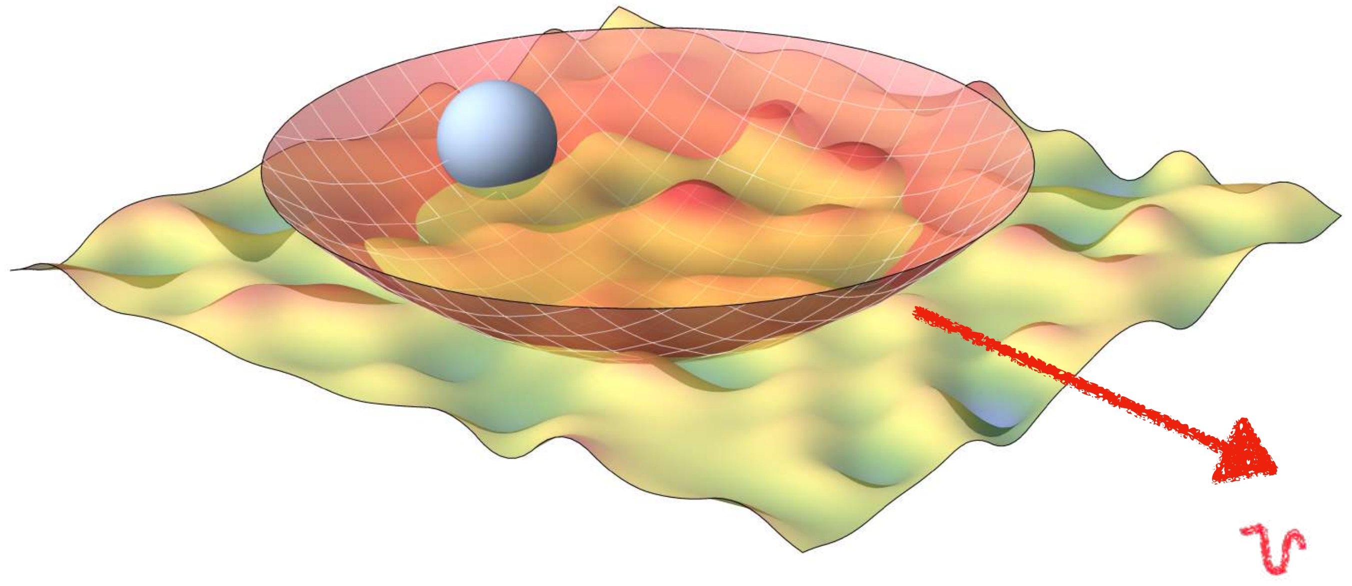
Micellar solution (overdamped!)

$$\Gamma(t) = \gamma_\infty \delta(t) + a_0\, e^{-t/\tau_s} + \sum_i a_i\, \mathrm{e}^{-t/\tau_i}$$

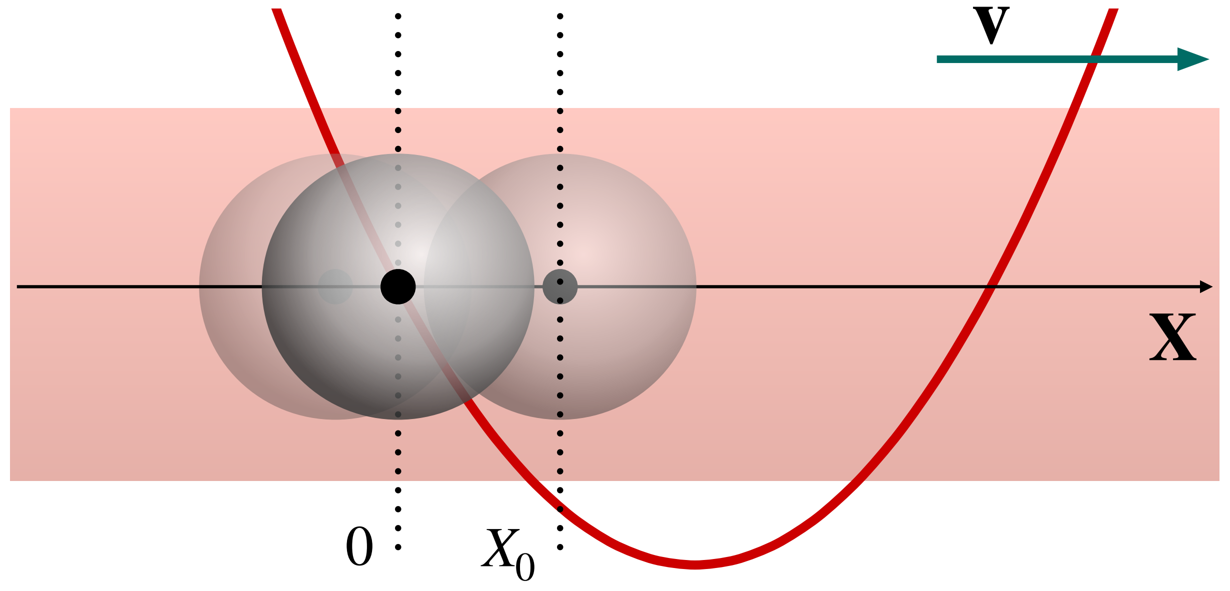
Problem: need more terms as v increases, some $a_i < 0$

Memory-induced oscillations

in correlated media



- Reintroduce **spatial** d.o.f. → bath **density** $\phi(\mathbf{x}, t)$



- Nonlinear (v-dependent) memory kernel

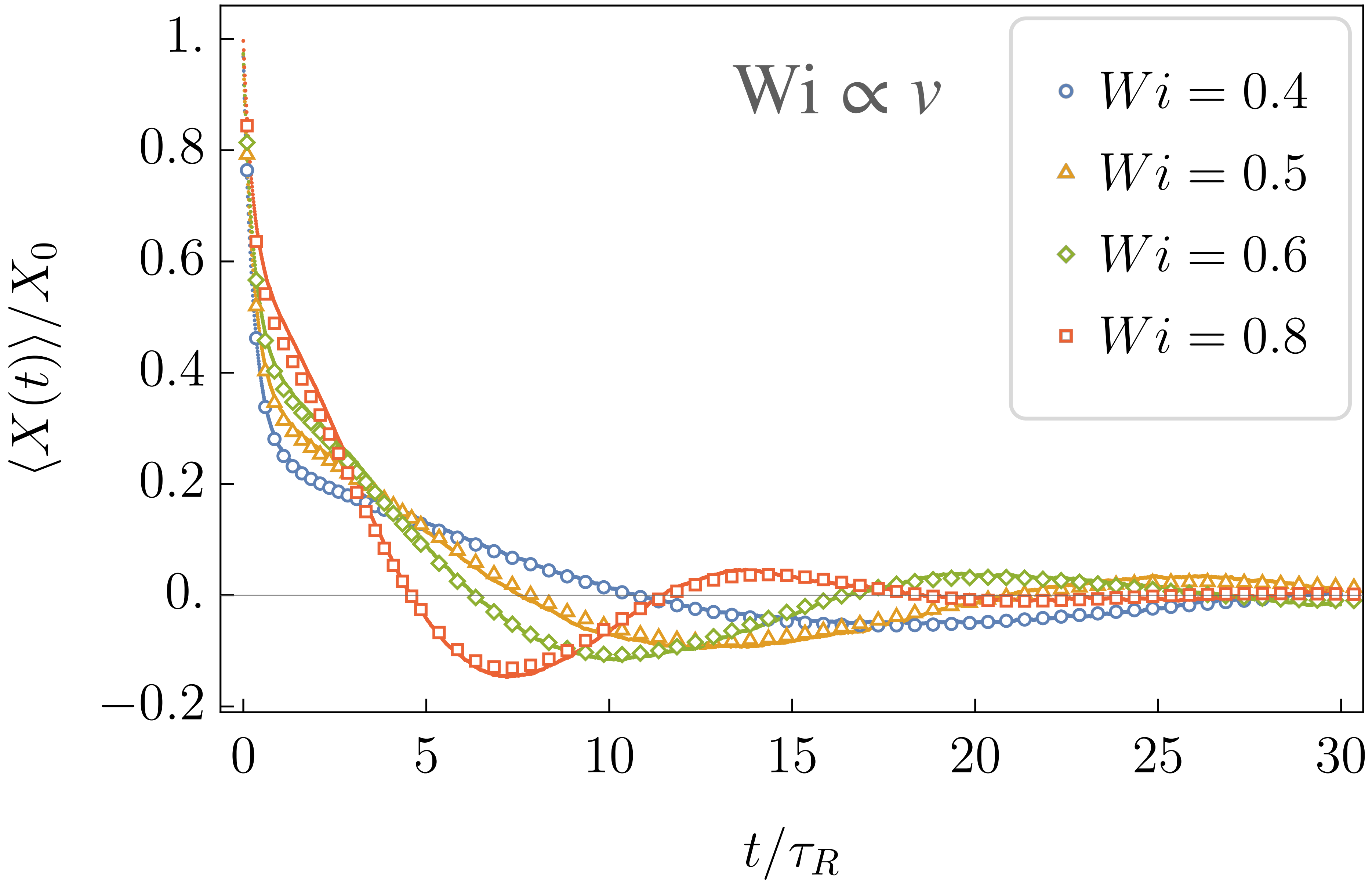
$$\partial_t \phi_q = -[Dq^\alpha(q^2 + r) - i\mathbf{q}\cdot\mathbf{v}]\phi_q + \lambda D q^\alpha V_q \, e^{-i\mathbf{q}\cdot\mathbf{X}} + \zeta_q$$

$$\phi_q(t) = \int^t d\tau \, G_q(t-\tau)[\ldots e^{-i\mathbf{q}\cdot\mathbf{X}(\tau)}\ldots]$$

$$\dot{\mathbf{X}} = -\nu\kappa\mathbf{X} - \mathbf{v} + \lambda\nu \int_{\mathbb{R}^d} \frac{d^d q}{(2\pi)^d} i\mathbf{q} V_{-q} \, e^{i\mathbf{q}\cdot\mathbf{X}(t)} \phi_q(t) + \boldsymbol{\xi}$$

- Oscillations in $\langle \mathbf{X}(t) \rangle$ upon increasing v

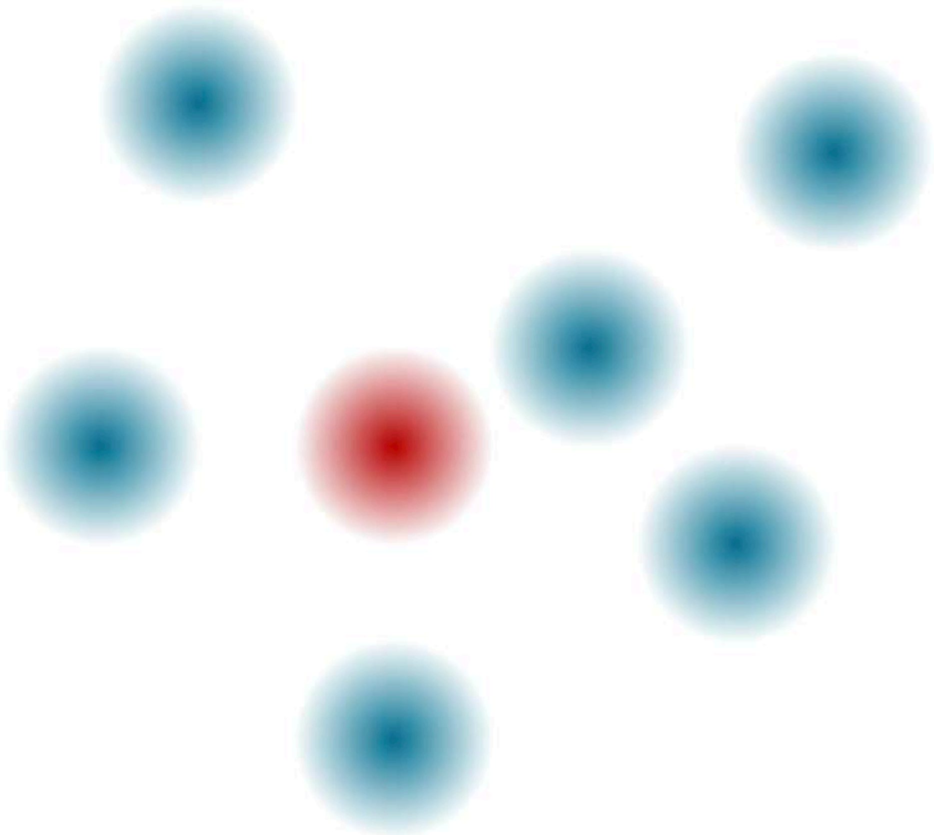
- Generic feature due to memory + spatial correlations



[D. Venturelli, A.Gambassi, 2023 New J. Phys. **25** 093025]

3. Interacting particle systems

Soft interacting particles



- N+1 overdamped Brownian particles, tracer $i = 0$

$$\dot{\mathbf{X}}_i(t) = -\mu \sum_{j\neq i} \nabla_i U\left(\mathbf{X}_i(t) - \mathbf{X}_j(t)\right) + \boldsymbol{\eta}_i(t), \quad \langle \boldsymbol{\eta}_i(t)^T \boldsymbol{\eta}_j(t')\rangle = 2\mu T \delta_{ij}\delta(t-t')I_d$$

- Dean-Kawasaki equation for $\rho(x,t) = \sum_{i=1}^{N} \delta\left(x - X_i(t)\right)$,

$$\partial_t \mathbf{X}(t) = -\mu \nabla_{\mathbf{X}} \mathcal{F}[\rho, \mathbf{X}] + \boldsymbol{\eta}_0(t),$$

$$\partial_t \rho(\mathbf{x},t) = \mu \nabla \cdot \left[\rho(\mathbf{x},t)\nabla \frac{\delta \mathcal{F}}{\delta \rho(\mathbf{x},t)}\right] + \nabla \cdot \left[\rho^{\frac{1}{2}}(\mathbf{x},t)\,\boldsymbol{\xi}(\mathbf{x},t)\right],$$

$$\mathcal{F}[\rho, \mathbf{X}] = T\int \mathrm{d}\mathbf{x}\, \rho(\mathbf{x}) \log\left(\frac{\rho(\mathbf{x})}{\rho_0}\right) + \frac{1}{2}\int \mathrm{d}\mathbf{x}\,\mathrm{d}\mathbf{y}\, \rho(\mathbf{x}) U(\mathbf{x}-\mathbf{y})\rho(\mathbf{y}) + \int \mathrm{d}\mathbf{y}\, \rho(\mathbf{y}) U(\mathbf{y} - \mathbf{X})$$

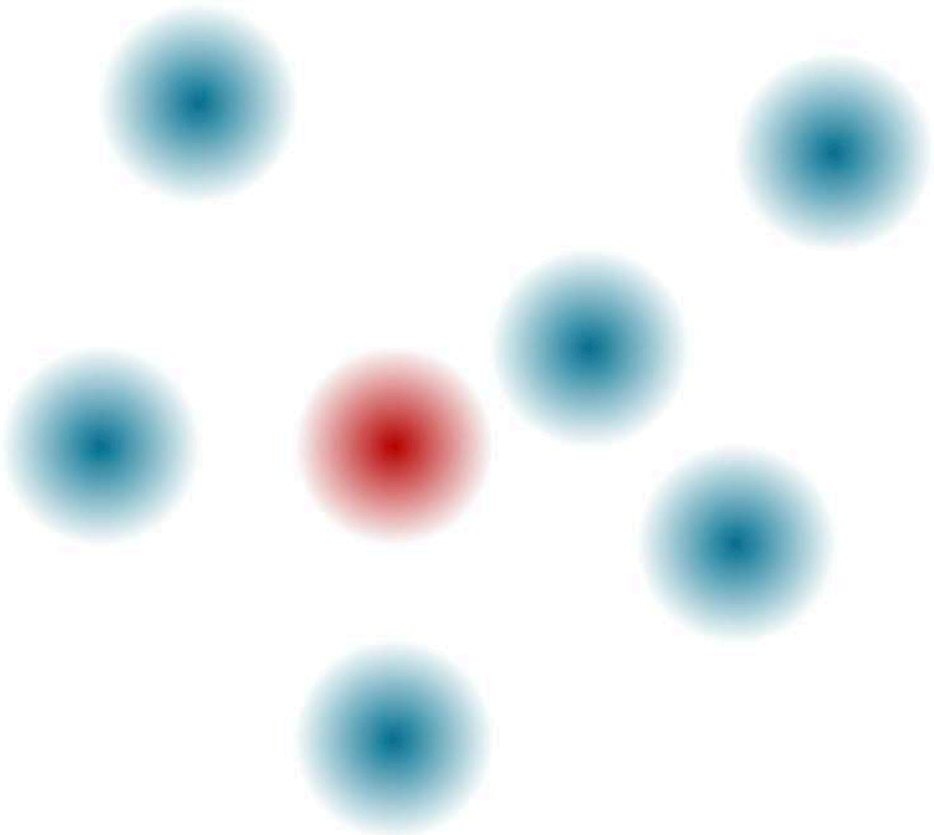
- Linearize about $\rho(\mathbf{x},t) = \rho_0 + \sqrt{\rho_0}\,\phi(\mathbf{x},t)$, assuming $\phi/\sqrt{\rho_0} \ll 1$

  → coupled eqs for $\mathbf{X}(t), \phi(\mathbf{x},t)$

FIG. 1. David Dean and Kyozi Kawasaki, who are very happy that their equation is being used for the zillionth time.

V. Démery et al., New J. Phys. **16** (2014) 053032

Tracer-bath correlation profiles



- How does $\Psi(\boldsymbol{\lambda}, t) = \ln \left\langle e^{\boldsymbol{\lambda}\cdot\mathbf{X}(t)} \right\rangle$ evolve?

$$\partial_t \Psi(\boldsymbol{\lambda}, t) = \lambda^2 \mu T + \sqrt{\rho_0}\mu\,\boldsymbol{\lambda}\cdot\int \mathrm{d}^d r\, U(\mathbf{r})\,\nabla_{\mathbf{r}}\, w(\mathbf{r}, \boldsymbol{\lambda}, t),$$

with the profile

$$w(\mathbf{r}, \boldsymbol{\lambda}, t) = \frac{\left\langle \phi(\mathbf{r}+\mathbf{X}(t), t)\, e^{\boldsymbol{\lambda}\cdot\mathbf{X}(t)} \right\rangle}{\left\langle e^{\boldsymbol{\lambda}\cdot\mathbf{X}(t)} \right\rangle} = \langle \phi(\mathbf{r}+\mathbf{X}(t), t)\rangle + \boldsymbol{\lambda}\cdot\langle \mathbf{X}(t)\,\phi(\mathbf{r}+\mathbf{X}(t), t)\rangle + \mathcal{O}(\lambda^2)$$

average density profile

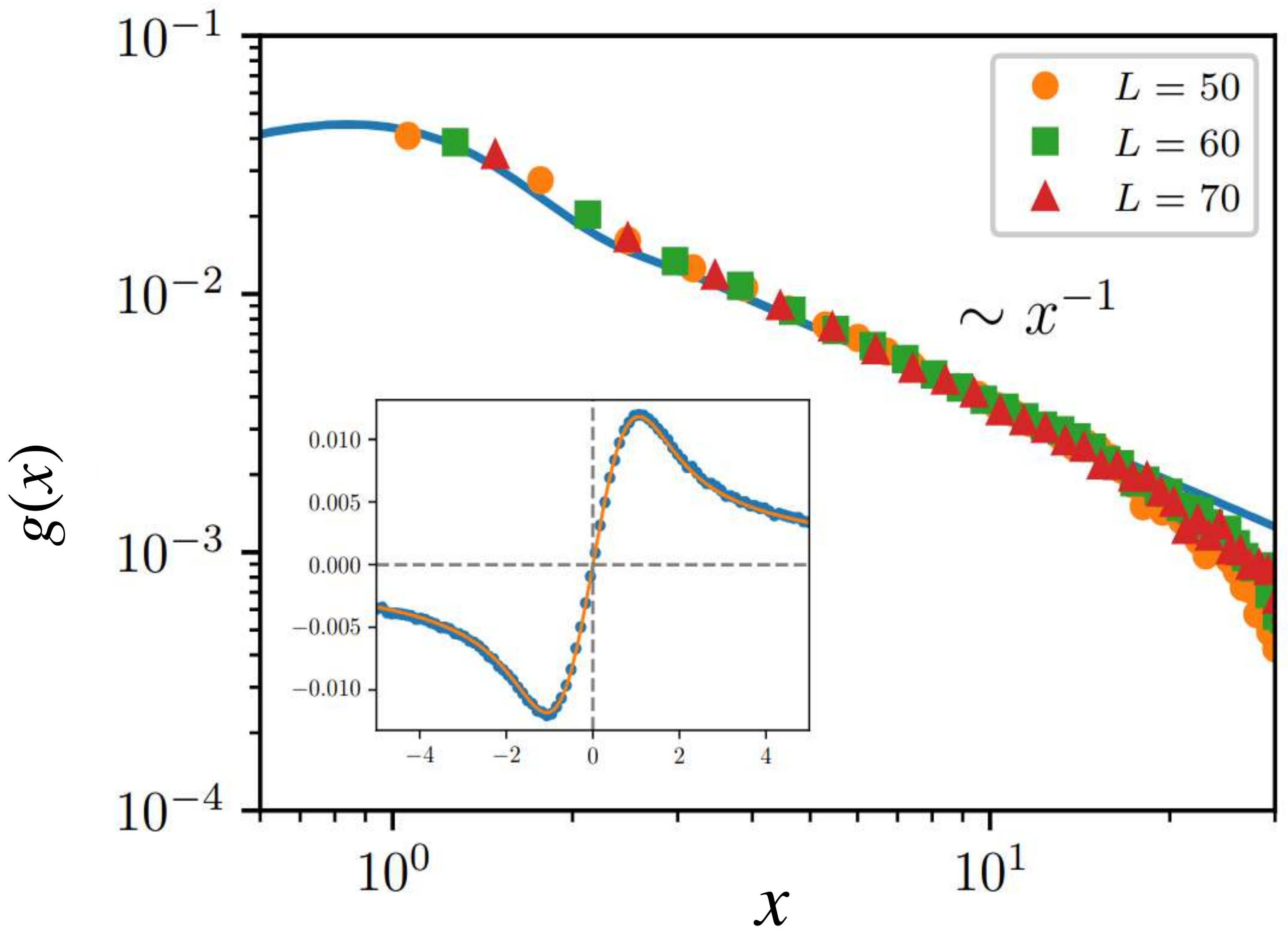
correlation profile $\mathbf{g}(\boldsymbol{r}, t)$

- Correlation profiles encode the **response** of the bath

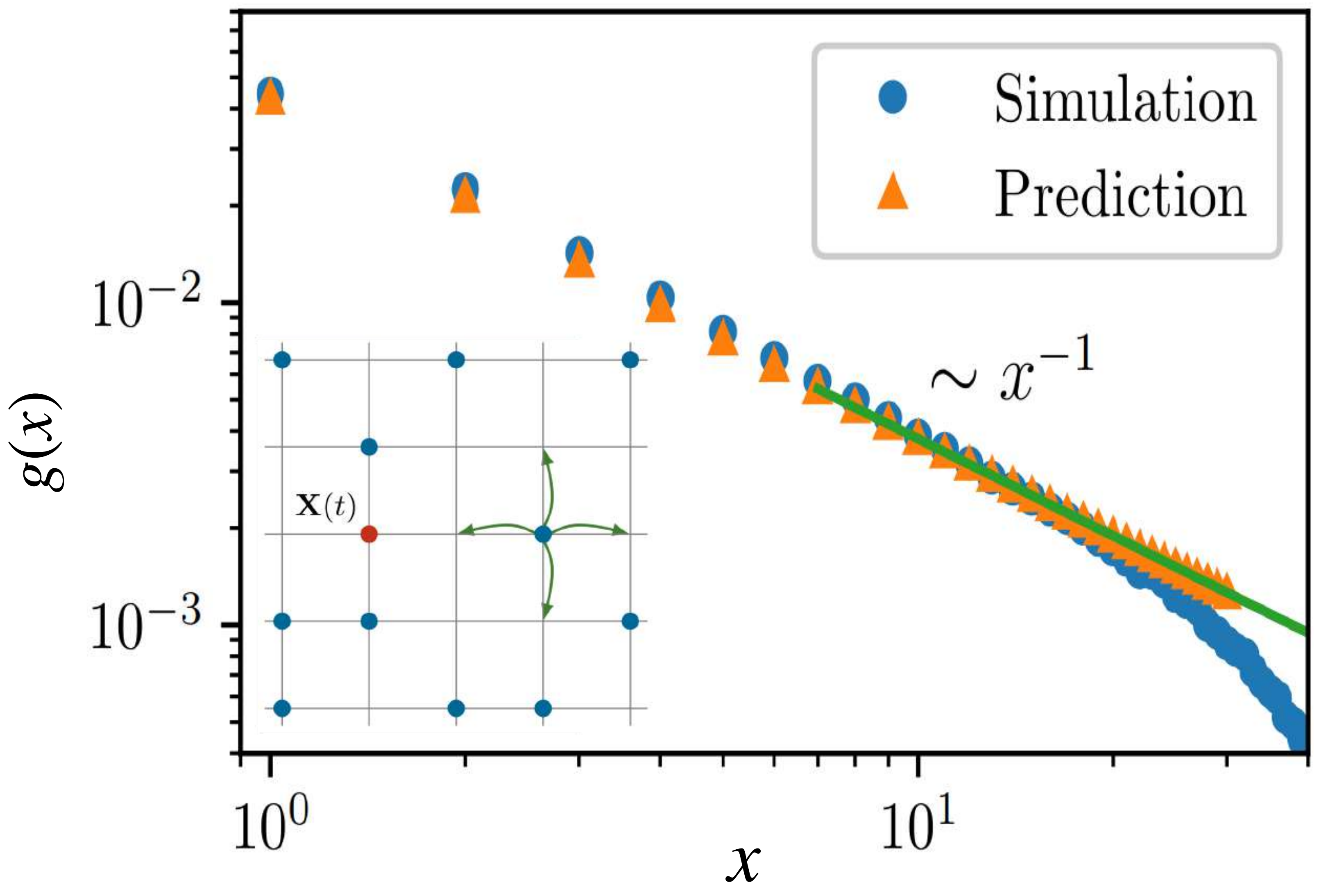
O. Bénichou, A. M. Cazabat, J. De Coninck, M. Moreau, G. Oshanin, Phys. Rev. Lett. **84**, 511 (2000)

Tracer-bath correlation profile

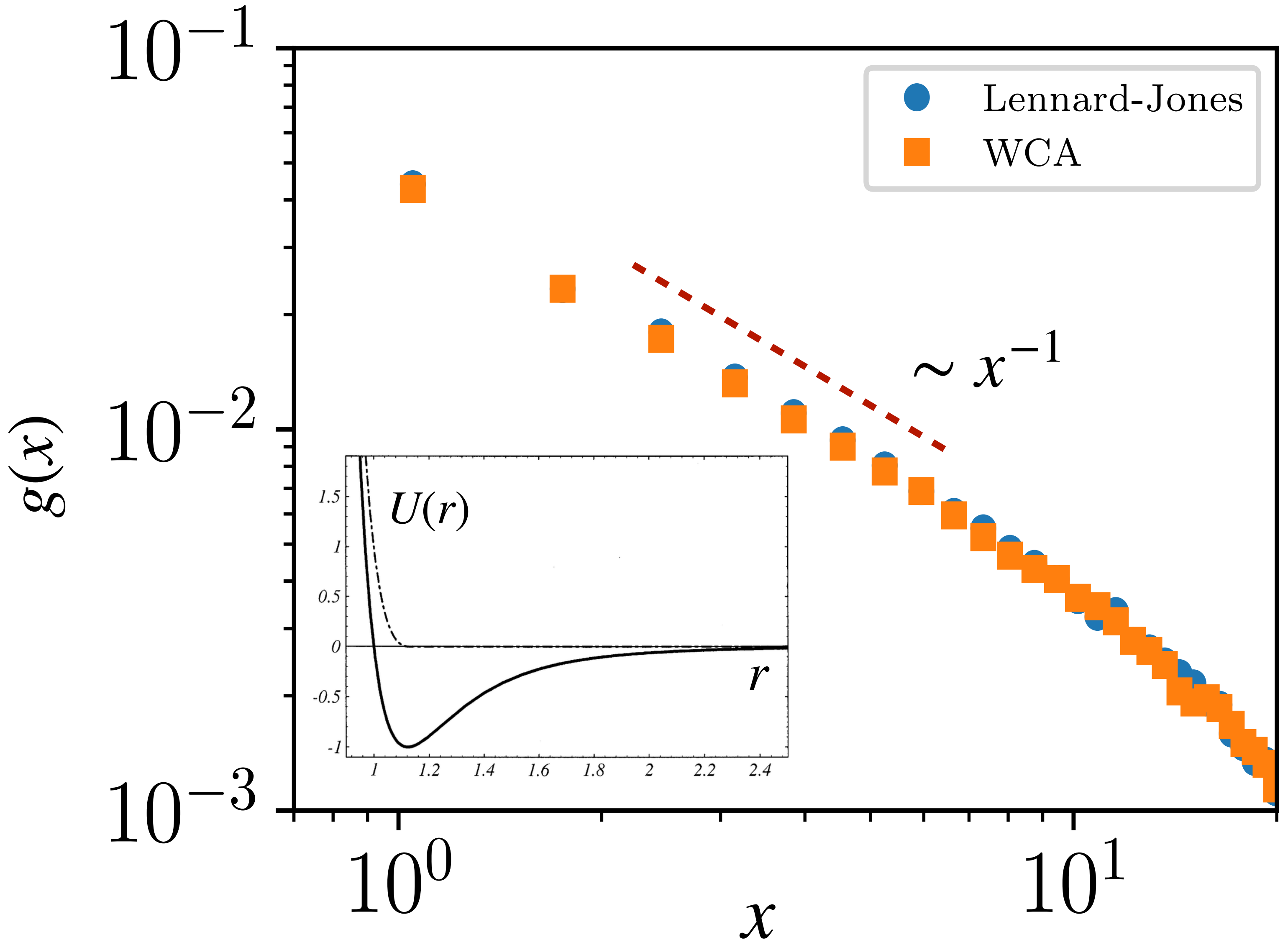
large-distance behavior @ stationary state: $\langle X\,\phi_{X+r}\rangle_c \sim r^{1-d}$



Soft-core potentials
(Dean-Kawasaki theory)



Simple exclusion process
(master equation)



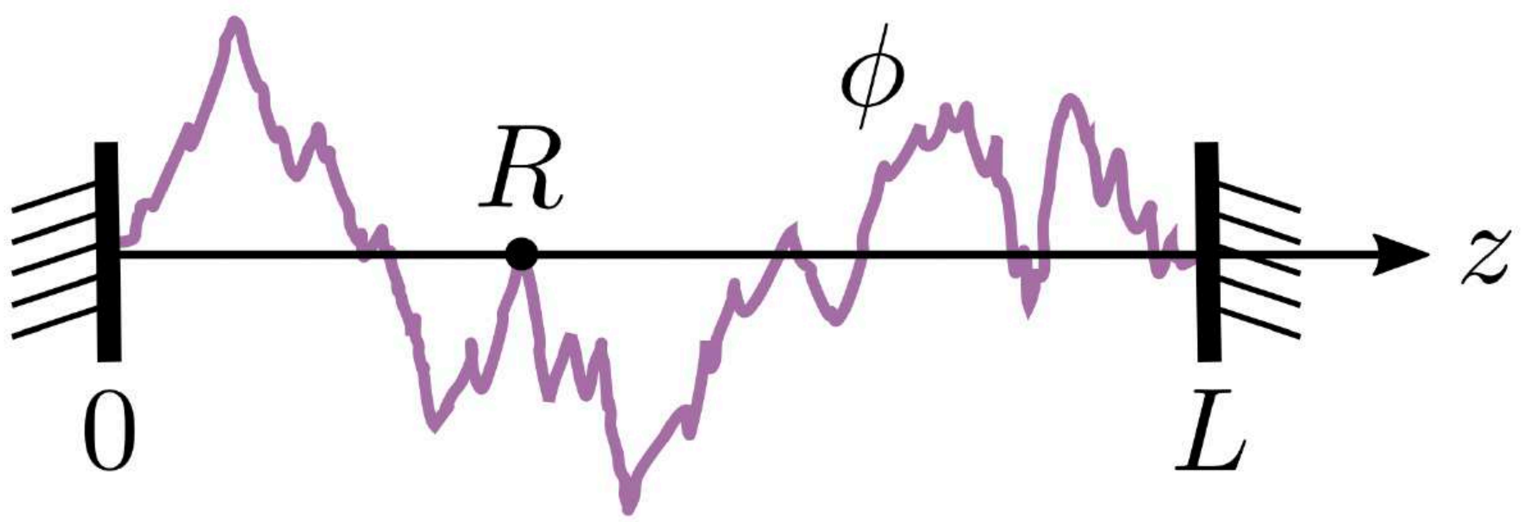
Lennard-Jones fluids
(simulation)

Universal?

D. Venturelli, P. Illien, A. Grabsch, O. Bénichou, arXiv:2411.09326 (2024)

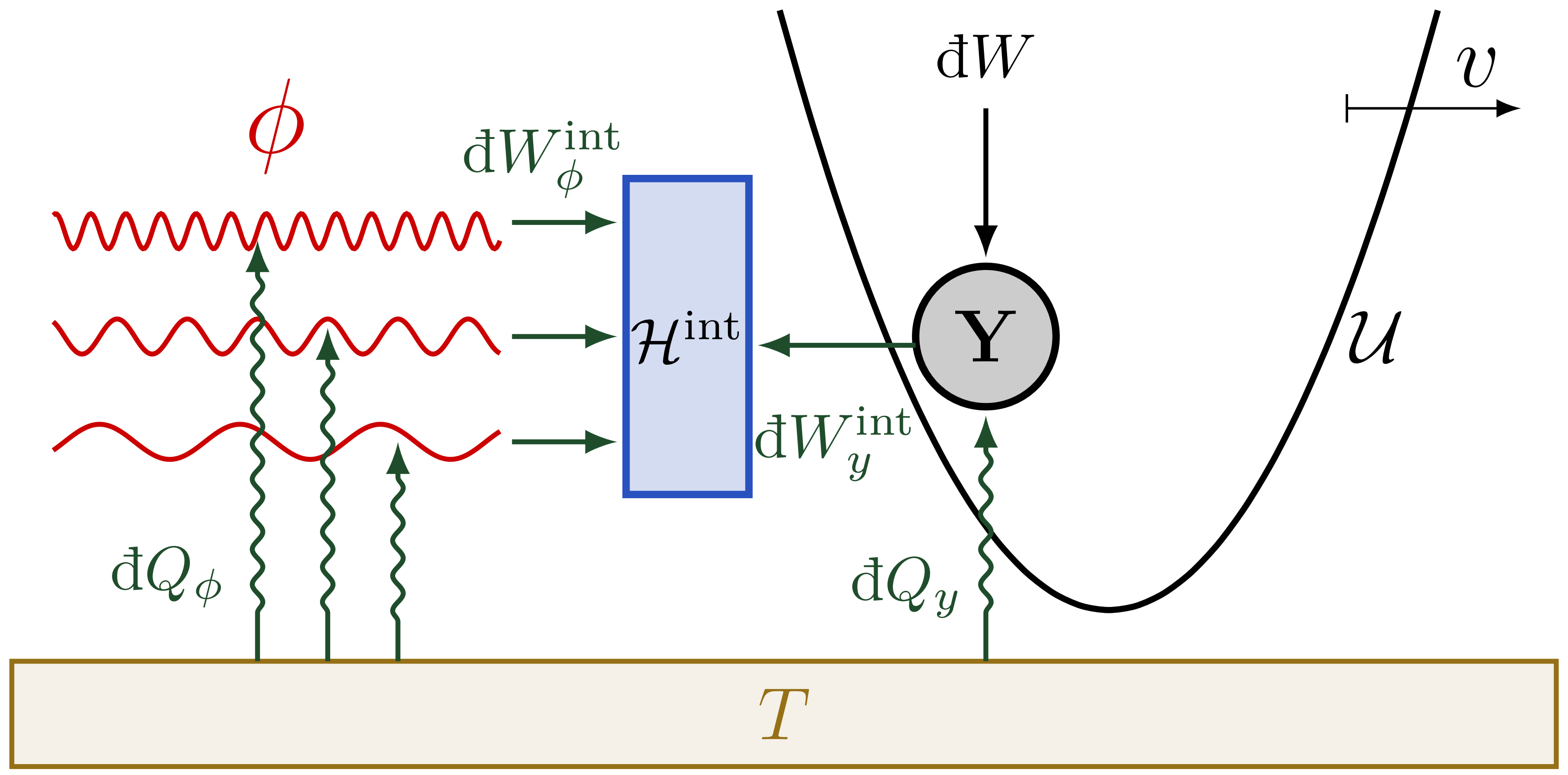
Extensions

Dynamics in fluctuating correlated media



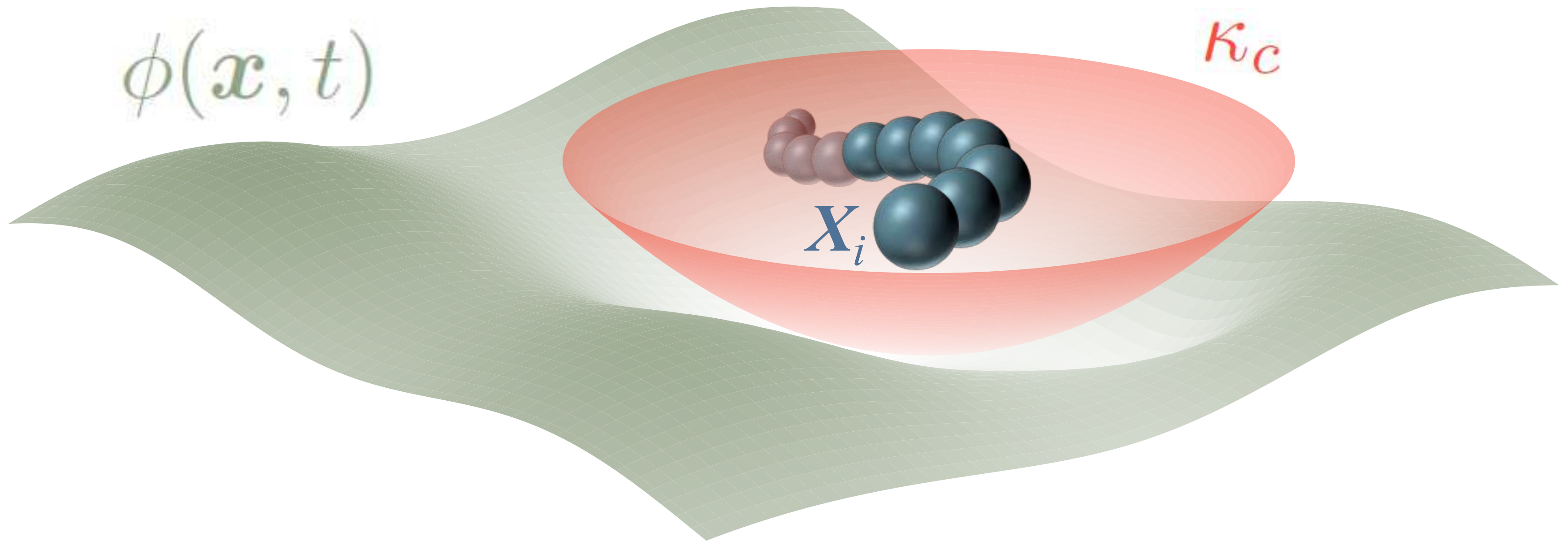
- **Spatial confinement**
  Effective tracer dynamics

  [D. Venturelli, M. Gross, J. Stat. Mech. (2022) 123210]



- **Stochastic thermodynamics**
  Spatial distribution of dissipated heat $Q(\boldsymbol{x})$

  [D. Venturelli, S. M. Loos, B. Walter, É. Roldán, A. Gambassi, 2024 *EPL* **146** 27001]

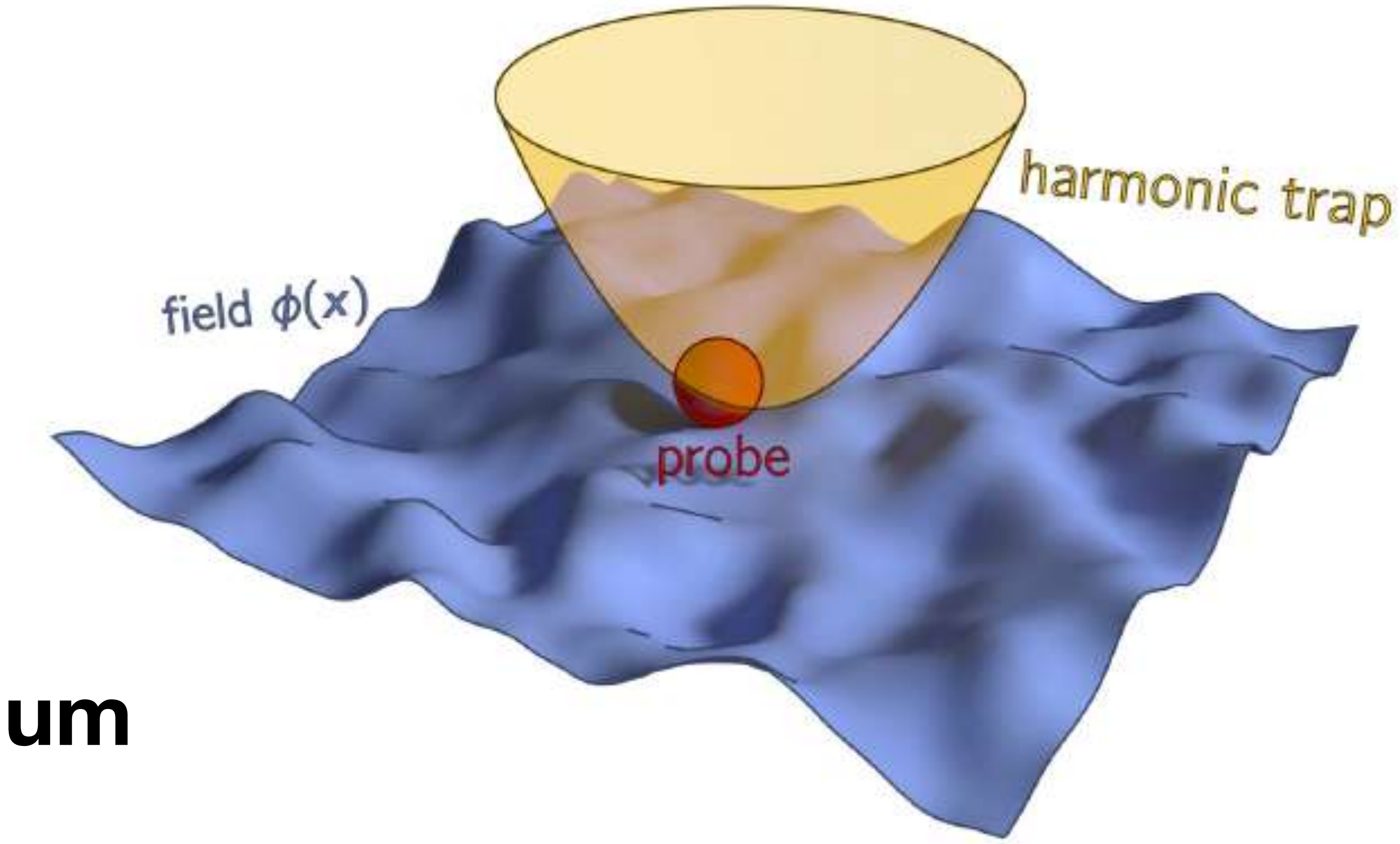


- **Polymers**
  Relaxation and structural properties

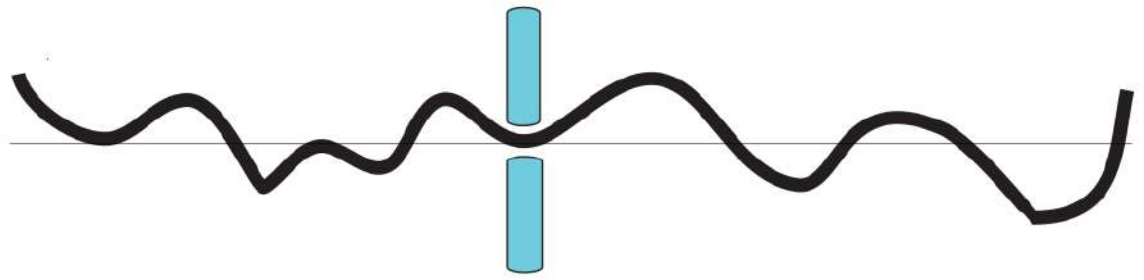
  [P. L. Muzzeddu, D. Venturelli, A. Gambassi, *arXiv:2503:03572]*

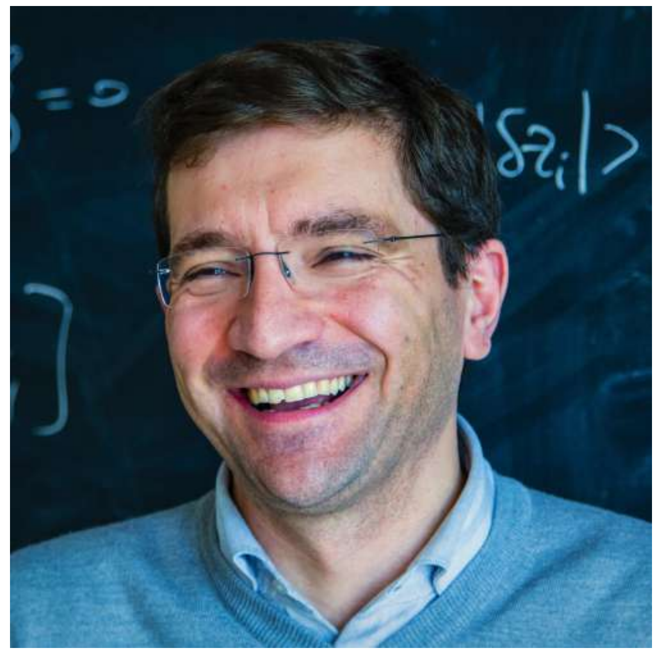
Summing up

Dynamic field-mediated forces



- ☑ **Minimal model for field-mediated interactions, out of equilibrium**
  - near-critical media, viscoelastic media, interacting particle systems
  - **fluctuating media with spatial correlations and slow relaxation**
- ☑ **Effective tracer dynamics:** algebraic relaxation, delayed response to periodic driving, oscillations
- ☑ **Tracer-bath correlations** in interacting particle systems (universal large-distance behavior)
- ☐ **Some future perspectives**:
  - Other dynamic universality classes (e.g. model H, relevant for experiments)
  - Strong-coupling regime (actual boundary conditions)

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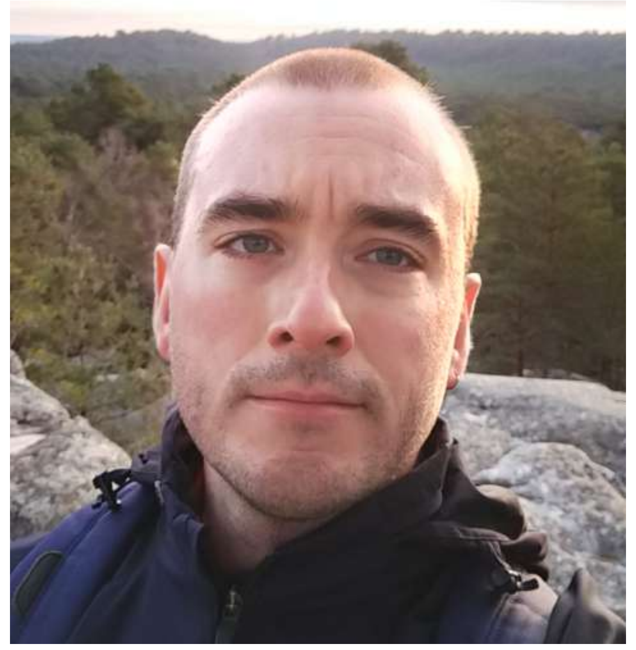
B. Walter


F. Ferraro


É. Roldán


P. L. Muzzeddu

Thank you!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T. Berlioz


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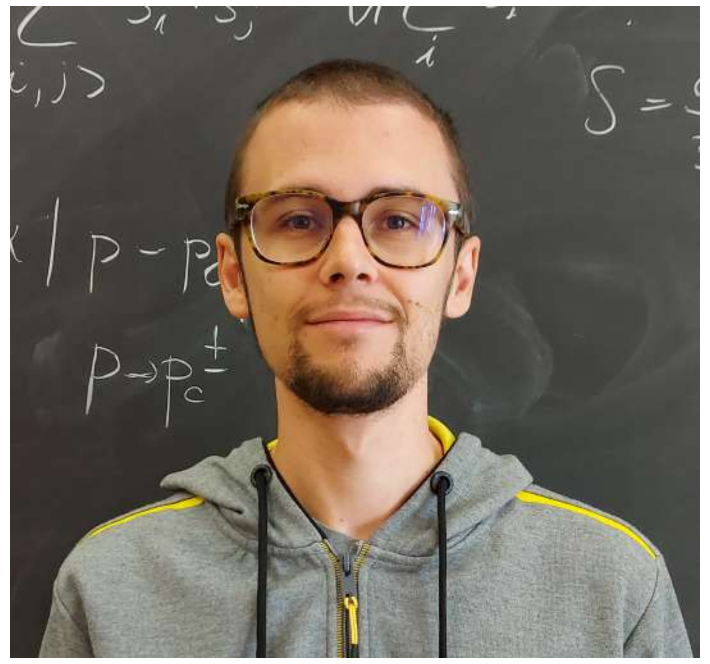
O. Bénichou


A. Jelic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