Dynamical tracer—bath correlations in interacting particle systems

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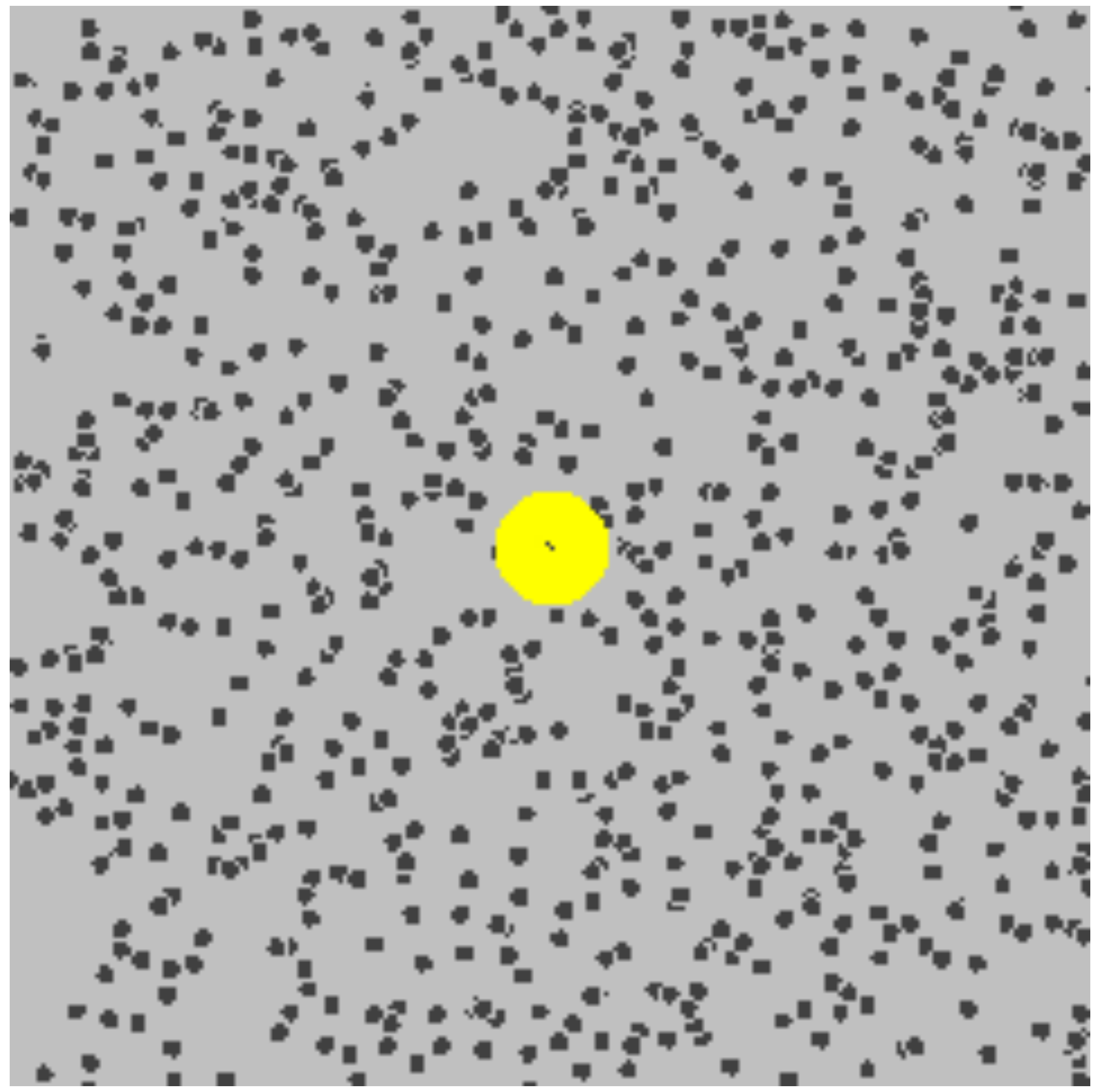
4èmes Journées du GDR IDE
Sète, 9 September 2025
Work in collaboration with T. Berlioz, P. Illien, A. Grabsch, O. Bénichou

Tracer particle

in a thermal bath

$$m\,\ddot{X}(t) = -\,\gamma\dot{X}(t) + \zeta(t)$$

$$\langle \zeta(t)\zeta(t')\rangle = 2\gamma k_B T\,\delta(t-t')$$



- **Brownian motion**: bath in **equilibrium**, **structureless**,

  no **tracer-bath** correlations, **diffusive** behaviour

  $\langle X^n(t)\,\zeta(t)\rangle = 0,$ $\left\langle X^2(t)\right\rangle \propto t$

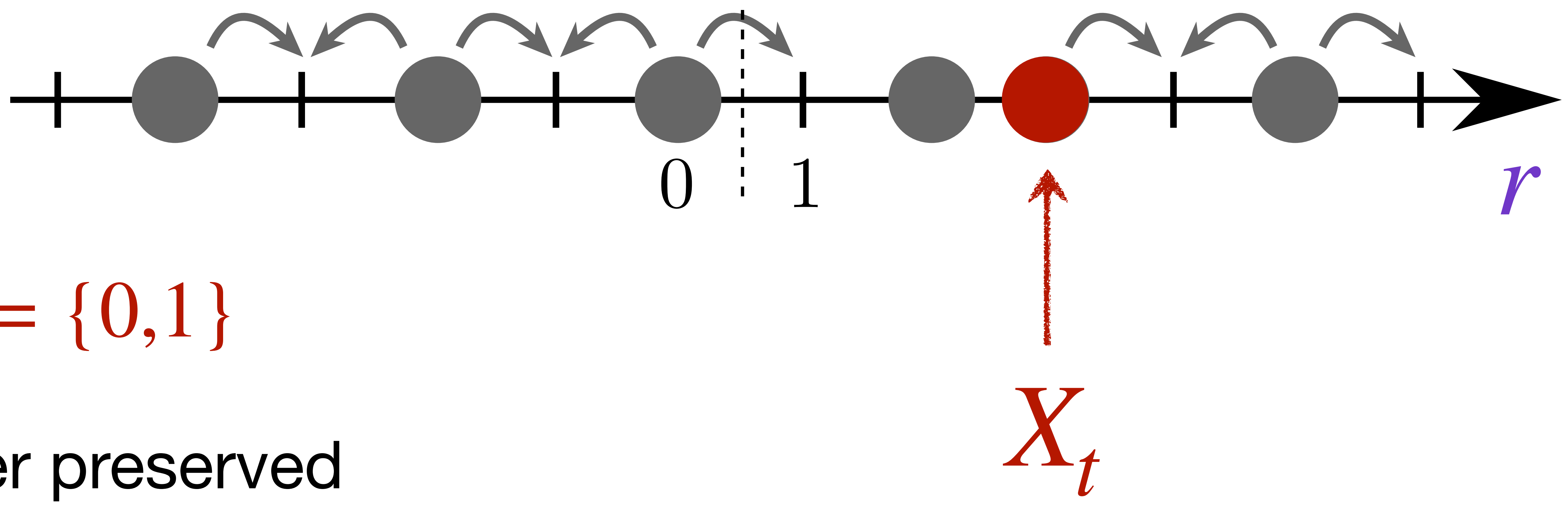
- What if particles have similar sizes?

- **Interacting particle systems**: lattice gases, interacting Brownian particles, simple liquids…

Symmetric Exclusion Process

as paradigmatic interacting particle system

- Particles on a lattice
  + random hoppings (equal rates),
  only if target site is empty



- **State of the system**: occupations $\rho_r(t) = \{0,1\}$
- In $1d$, **single-file** geometry $\rightarrow$ initial order preserved
- Subdiffusive behavior of tracer

$$\langle X_t^2 \rangle \propto \sqrt{t}$$

(zeolites, confined colloids, dipolar spheres…)

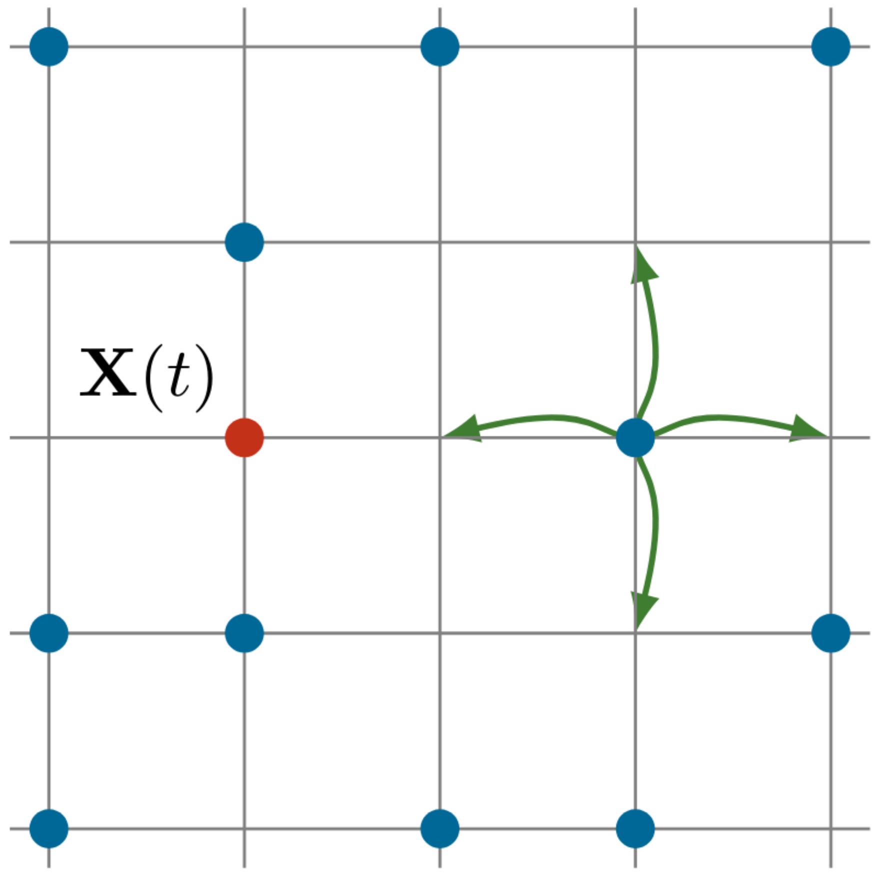
Lin, Meron, Cui, Rice, Diamant, Phys. Rev. Lett. **94** (21), 216001 (2005)

Wei, Bechinger, Leiderer, Science **287** (5453), 625-7 (2000)

Hahn, Kärger, Kukla, Phys. Rev. Lett. **76** (15), 2762-2765 (1996)

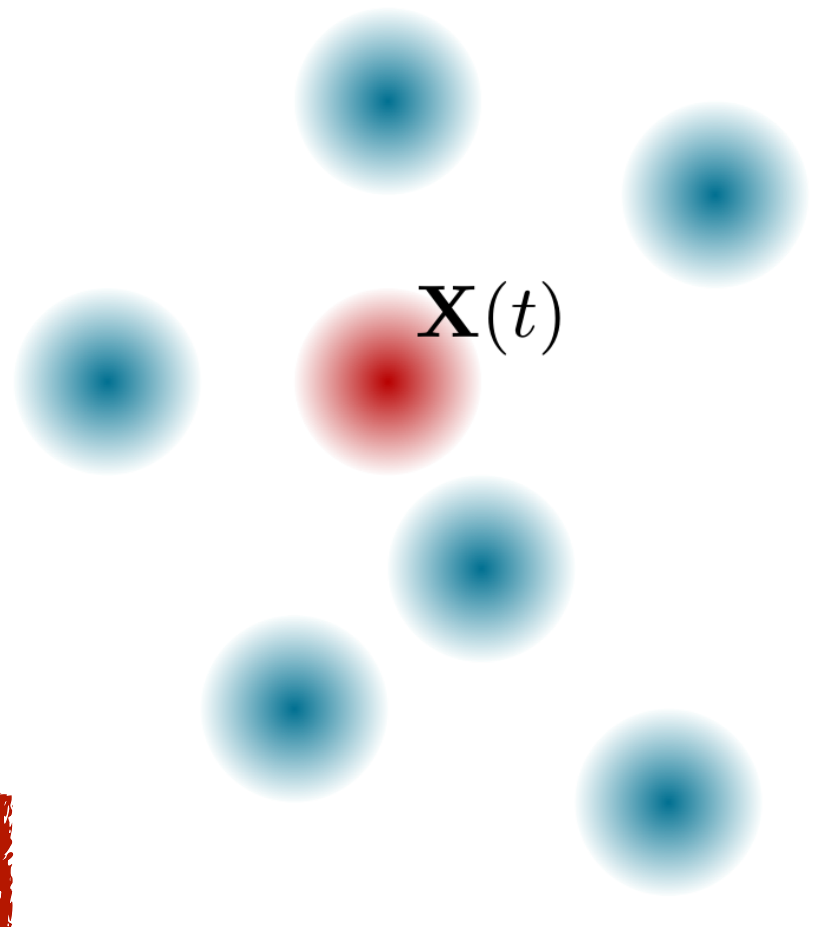
H. Spohn, *Large scale dynamics of interacting particles* (1991)

Tracer-bath correlation profiles



Hard-core lattice gas
(SEP)

Brownian particles
+ two-body potential $U(\vec{r}_i - \vec{r}_j)$

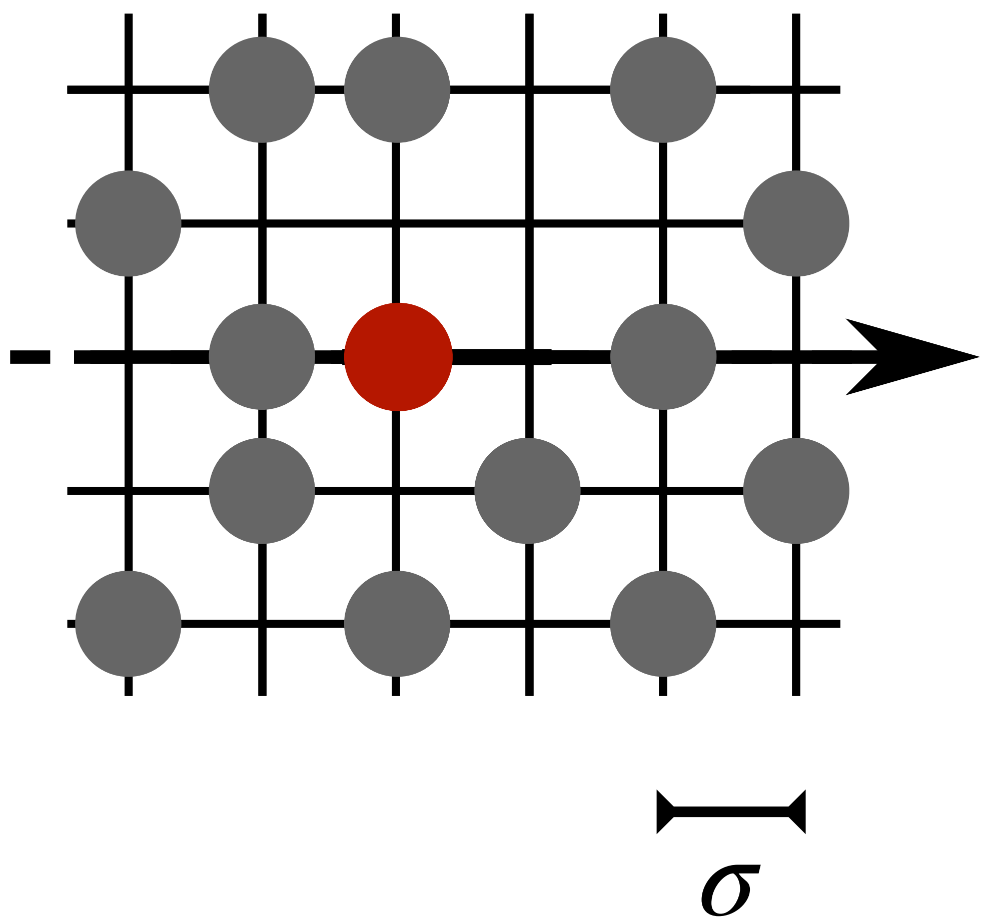


- **Key observable:** position $X(t)$ of a tagged particle
  - how does the displacement of the **tracer** affect the **bath** (= other particles)?
  - how does the **bath** affect the displacement of the **tracer**?

$$\partial_t \left\langle X^{n+1} \right\rangle_c = F\left[\dots \left\langle X^n \rho_{X+r} \right\rangle_c \dots \right]$$

density of bath particles
at distance r

Hard-core lattice gas



- Master equation $\partial_t P(X,\underline{\rho},t) = \left[\mathscr{L}_{\textbf{tracer}} + \mathscr{L}_{\textbf{bath}}\right] P(X,\underline{\rho},t)$

- Multiply by $\mathrm{e}^{\lambda\cdot X}$ and average,

$$\partial_t \Psi(\boldsymbol{\lambda},t) = \frac{1}{2d\,\tau}\sum_{\mu=-d}^{d}\left(\mathrm{e}^{\sigma\boldsymbol{\lambda}\cdot\hat{\mathbf{e}}_\mu} - 1\right)\left[1 - w_{\mathbf{e}_\mu}(\boldsymbol{\lambda},t)\right]$$

tracer cumulants

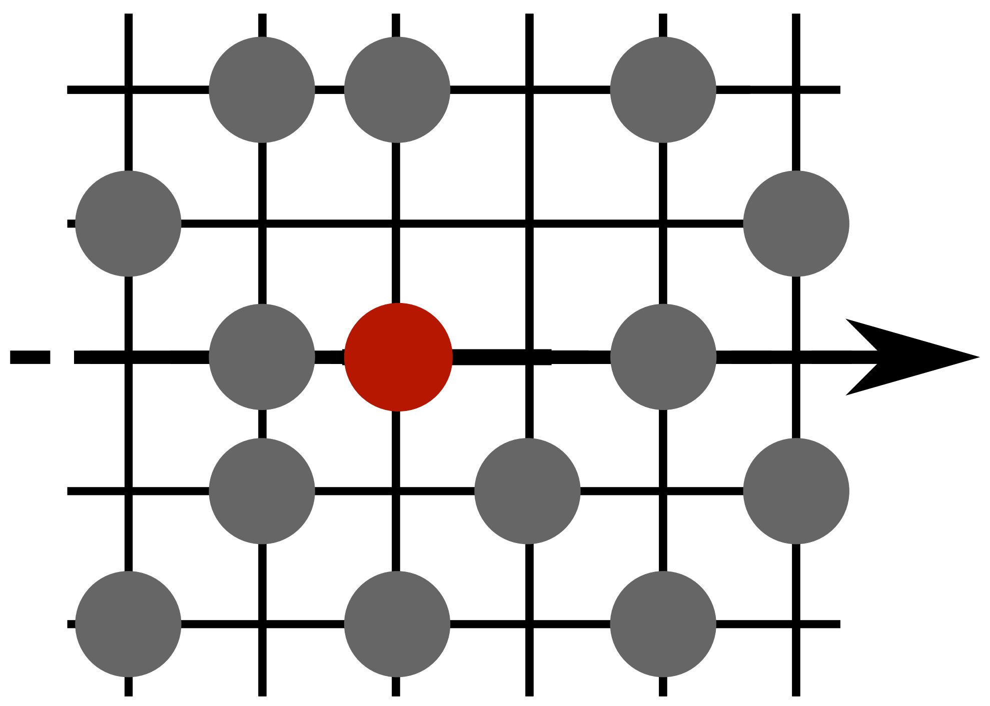
tracer-bath profiles

$$\Psi(\lambda,t) = \ln\left\langle \mathrm{e}^{\lambda\cdot X}\right\rangle = \sum_{n=1}^{\infty}\frac{\lambda^n}{n!}\langle X^n\rangle_c,$$

$$w_r(\lambda,t) = \frac{\left\langle \rho_{X+r}\,\mathrm{e}^{\lambda\cdot X}\right\rangle}{\left\langle \mathrm{e}^{\lambda\cdot X}\right\rangle} = \sum_{n=0}^{\infty}\frac{\lambda^n}{n!}\left\langle \rho_{X+r}X^n\right\rangle_c$$

Role of correlations

with surrounding bath



$$\partial_t \Psi(\boldsymbol{\lambda}, t) = \frac{1}{2d\,\tau} \sum_{\mu=-d}^{d} \left(\mathrm{e}^{\sigma \boldsymbol{\lambda} \cdot \hat{\mathbf{e}}_\mu} - 1 \right) \left[1 - w_{\mathbf{e}_\mu}(\boldsymbol{\lambda}, t) \right]$$

- Knowing $w_{\mathbf{e}_\mu}(\boldsymbol{\lambda}, t)$ (= **response** of the bath) on neighbouring sites is enough to deduce $\Psi(\boldsymbol{\lambda}, t)$

- $\partial_t w_{\mathbf{e}_\mu}(\boldsymbol{\lambda}, t) = \dots \left[w_{\mathbf{r}}(\boldsymbol{\lambda}, t) \right] \dots$
  generically depends on $w_{\mathbf{r}}(\boldsymbol{\lambda}, t)$ even from far away $\boldsymbol{r}$

Context: $w_{\mathbf{r}}(\boldsymbol{\lambda}, t)$ fully understood in 1d SEP $\longrightarrow \left\langle \mathrm{e}^{\lambda \cdot X} \right\rangle$ as a **byproduct**

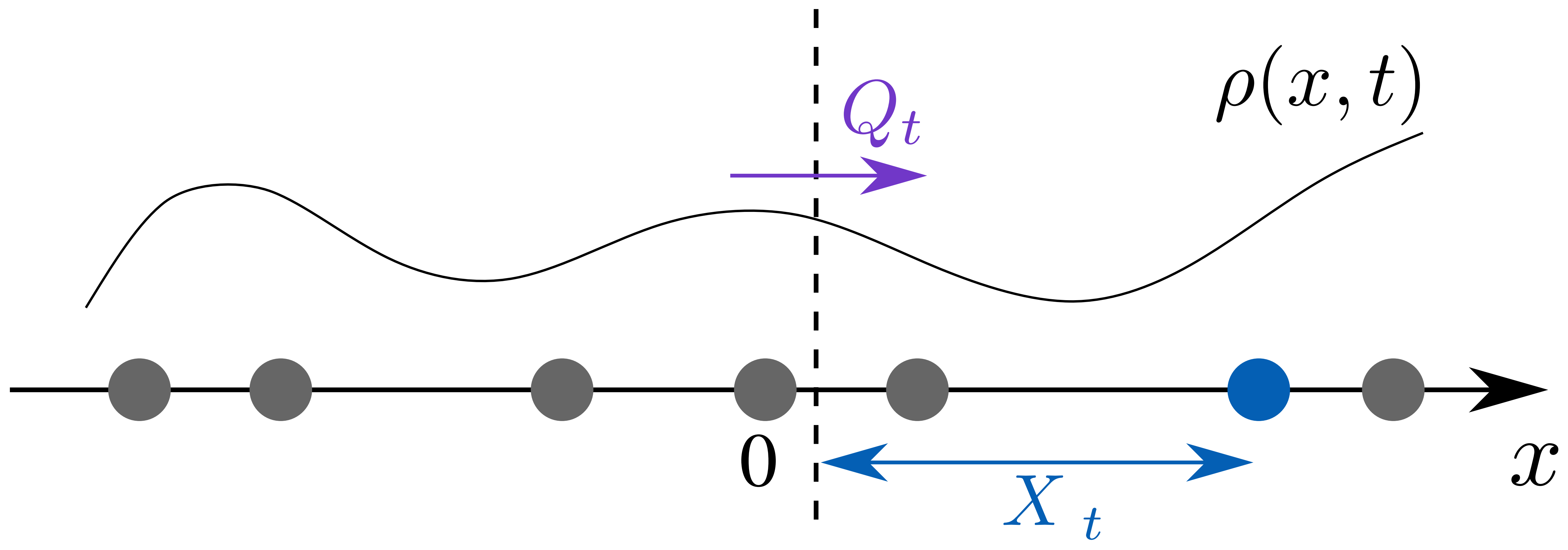
Aim: characterize these profiles in $d > 1$

Grabsch, Poncet, Rizkallah, Illien, Bénichou, Sci. Adv. 8, eabm5043 (2022)

Integrated current Q_t

- Q_t = net # of particles crossing (0—1)
  $\rightarrow$ in $d = 1$, $\langle Q_t^2 \rangle \propto \sqrt{t}$
- A positive fluctuation of Q_t is correlated with an increase of $\rho_r(t)$ on its r.h.s.
  $\rightarrow$ **correlations dictate the subdiffusive behavior of Q_t**
- $\langle \rho_r(t)\, Q_t^n \rangle$ encodes the **response** of the bath

bath structure **local observable**



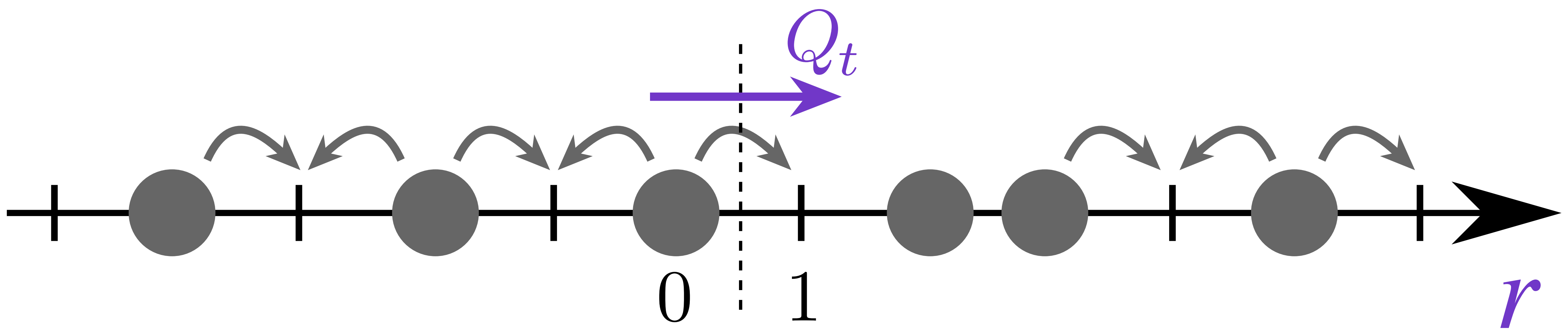
- Fully understood in 1d SEP, open problem in $d > 1$

Grabsch, Poncet, Rizkallah, Illien, Bénichou, Sci. Adv. 8, eabm5043 (2022)

1. Current fluctuations in SEP

- As **first step**, focus on

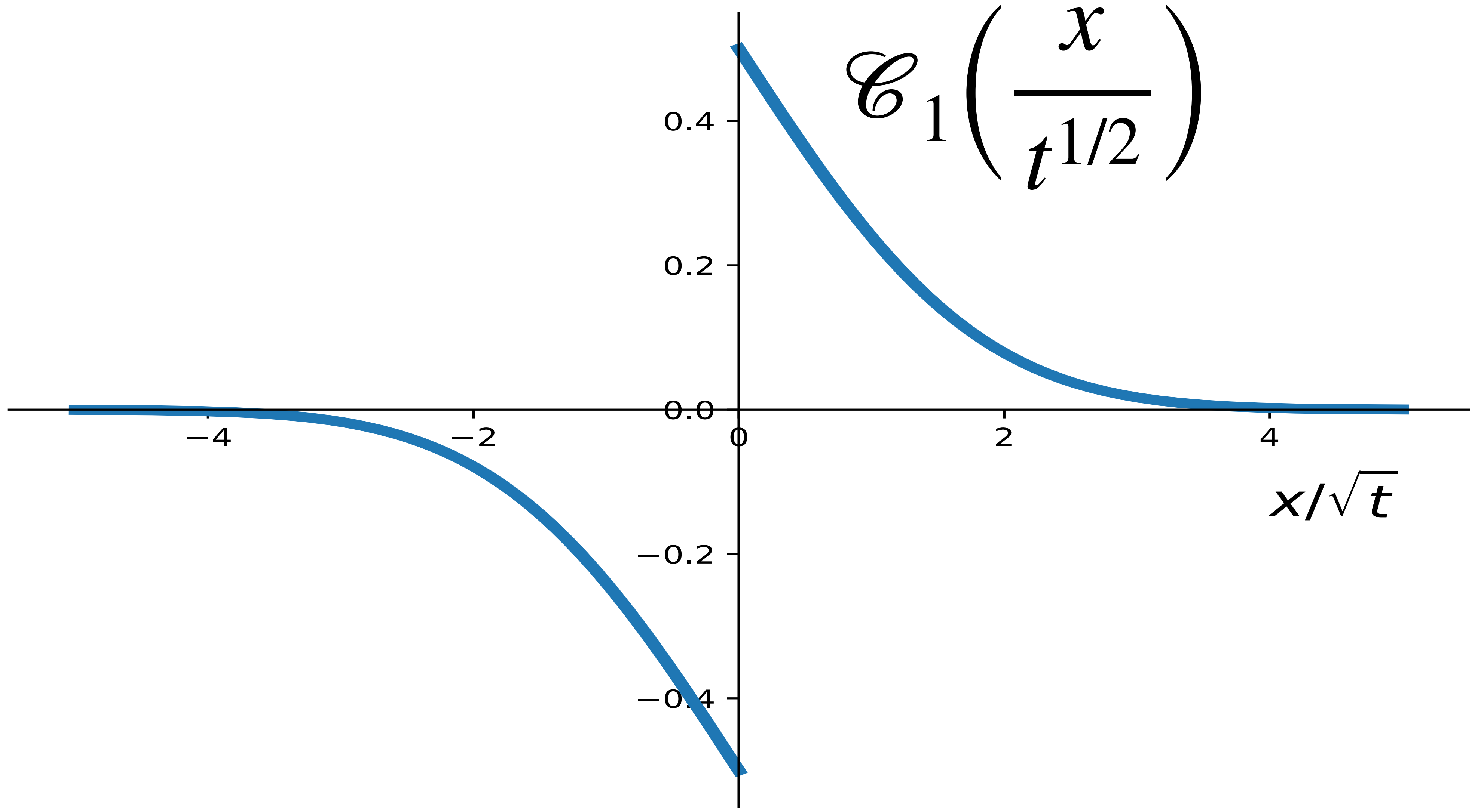
$$c_r(t) \equiv \langle Q_t \, \rho_r(t) \rangle$$



- **Fact 1:** infinite lattice $d = 1$ (no reservoirs, no PBC)

$$c_r(t) \xrightarrow[t \to \infty]{} \bar{\rho}(1-\bar{\rho})\, \mathscr{C}_1\left(\frac{r}{t^{1/2}}\right),$$

$$\langle Q_t^2 \rangle \propto \bar{\rho}(1-\bar{\rho})\, t^{1/2}$$



- **Fact 2:** finite systems (any spatial dimension d)

$$c_{\vec{r}}(t) \xrightarrow[t \to \infty]{} c_{\vec{r}}\,, \qquad \langle Q_t^2 \rangle \propto t$$

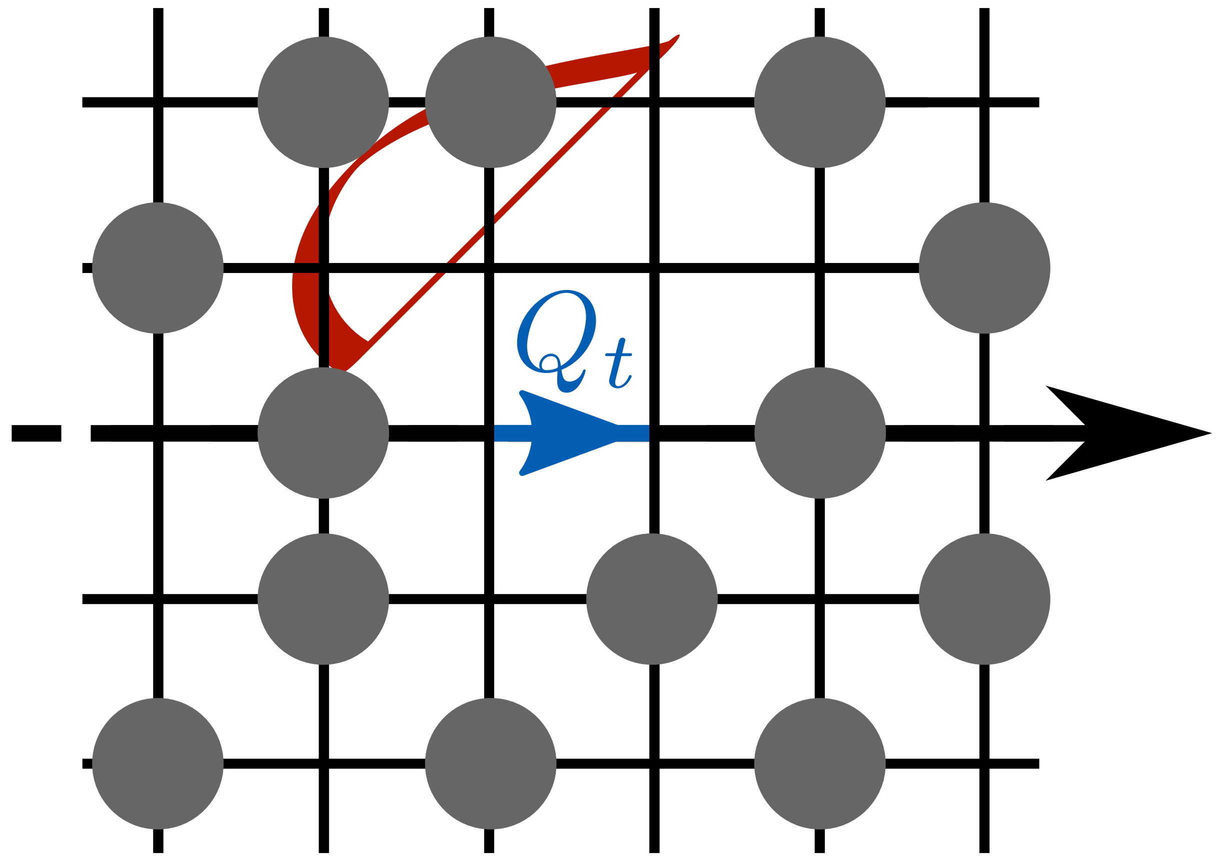
Recall $\partial_t \langle Q_t^{n+1} \rangle = \ldots \; F\left[\langle Q_t^n \, \rho_{\vec{r}}(t) \rangle\right] \; \ldots$

$$\partial_t \langle Q_t^2 \rangle = \ldots \; F\left[c_{\vec{r}}(t)\right] \; \ldots$$

infinite lattices in $d > 1$?

From $d = 1$ to higher d

passing through the comb



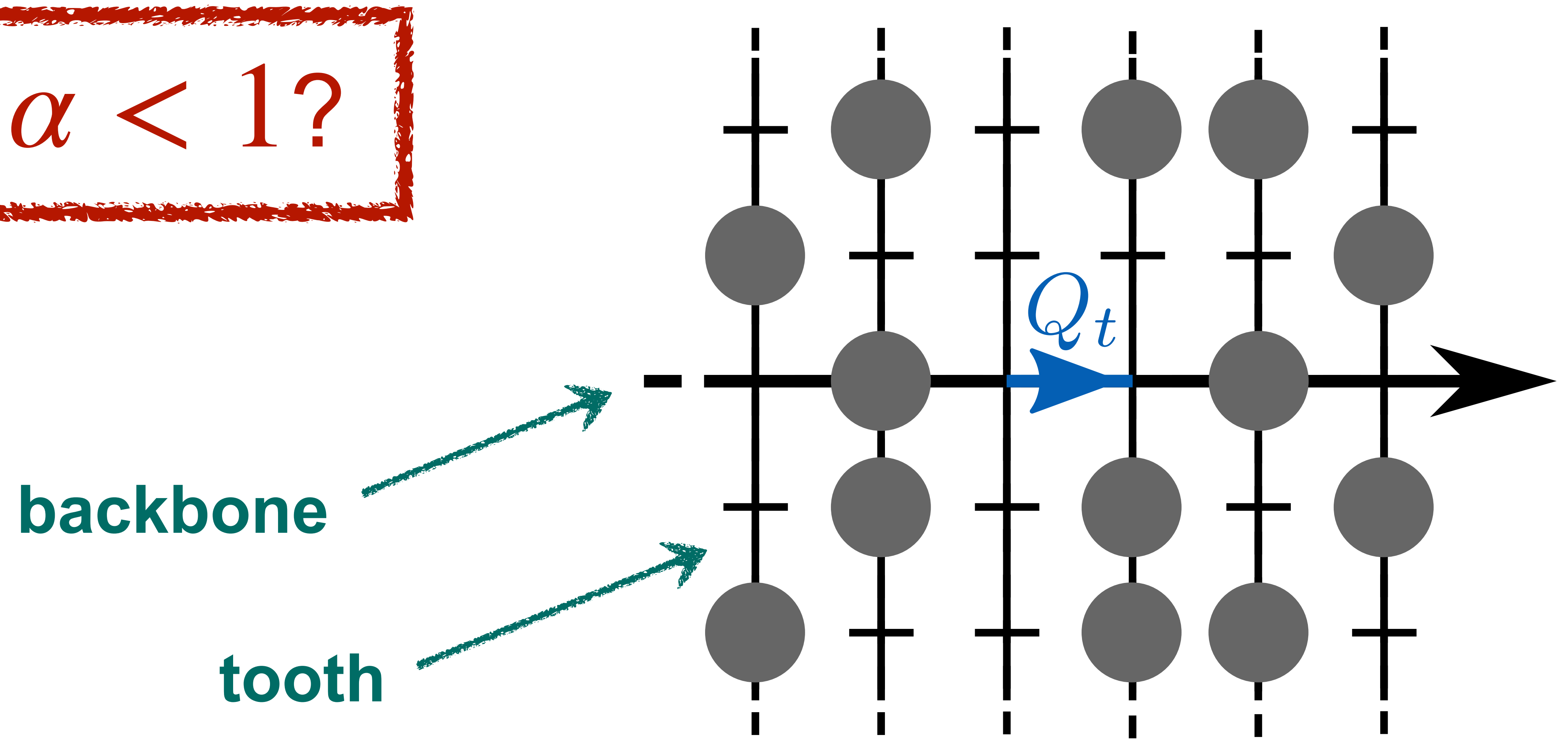
1. Order-preserving (single-file)

2. Tree-like (no loops!)

$\boxed{\text{Do we need both, to have } \langle Q_t^2 \rangle \propto t^\alpha \text{ with } \alpha < 1\text{?}}$

- **Higher d** : ~~(1)~~ ~~(2)~~
- **Comb lattice** : ~~(1)~~ (2)

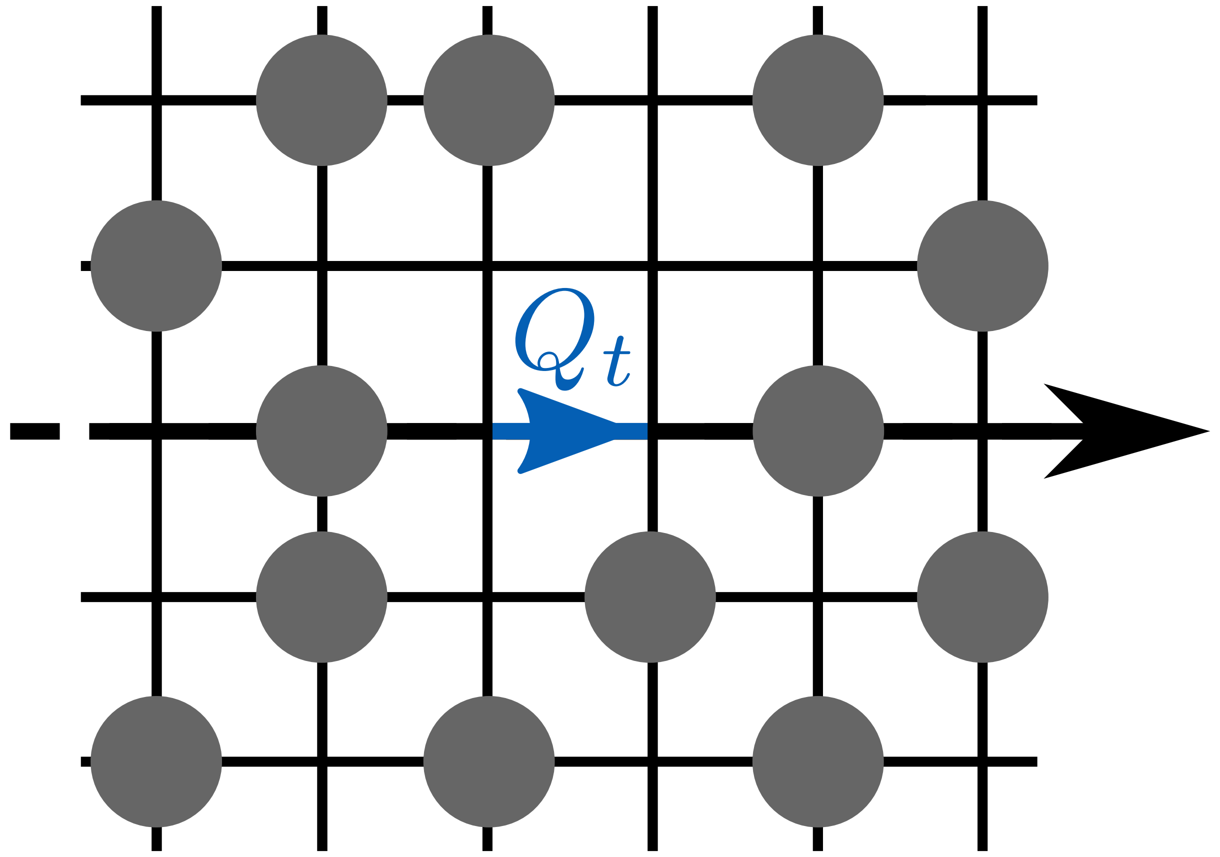
Transport in porous media,
subdiffusion (even single particle!)



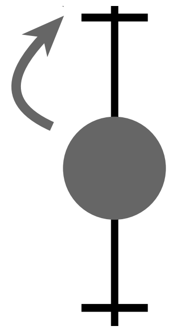
Bénichou, Illien, Oshanin, Sarracino, Voituriez, Phys. Rev. Lett. **115**, 220601 (2015)

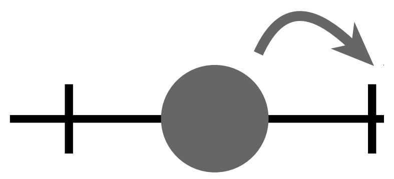
Ben-Avraham, Havlin, *Diffusion and reactions in fractals and disordered systems* (2000)

Microscopic calculation

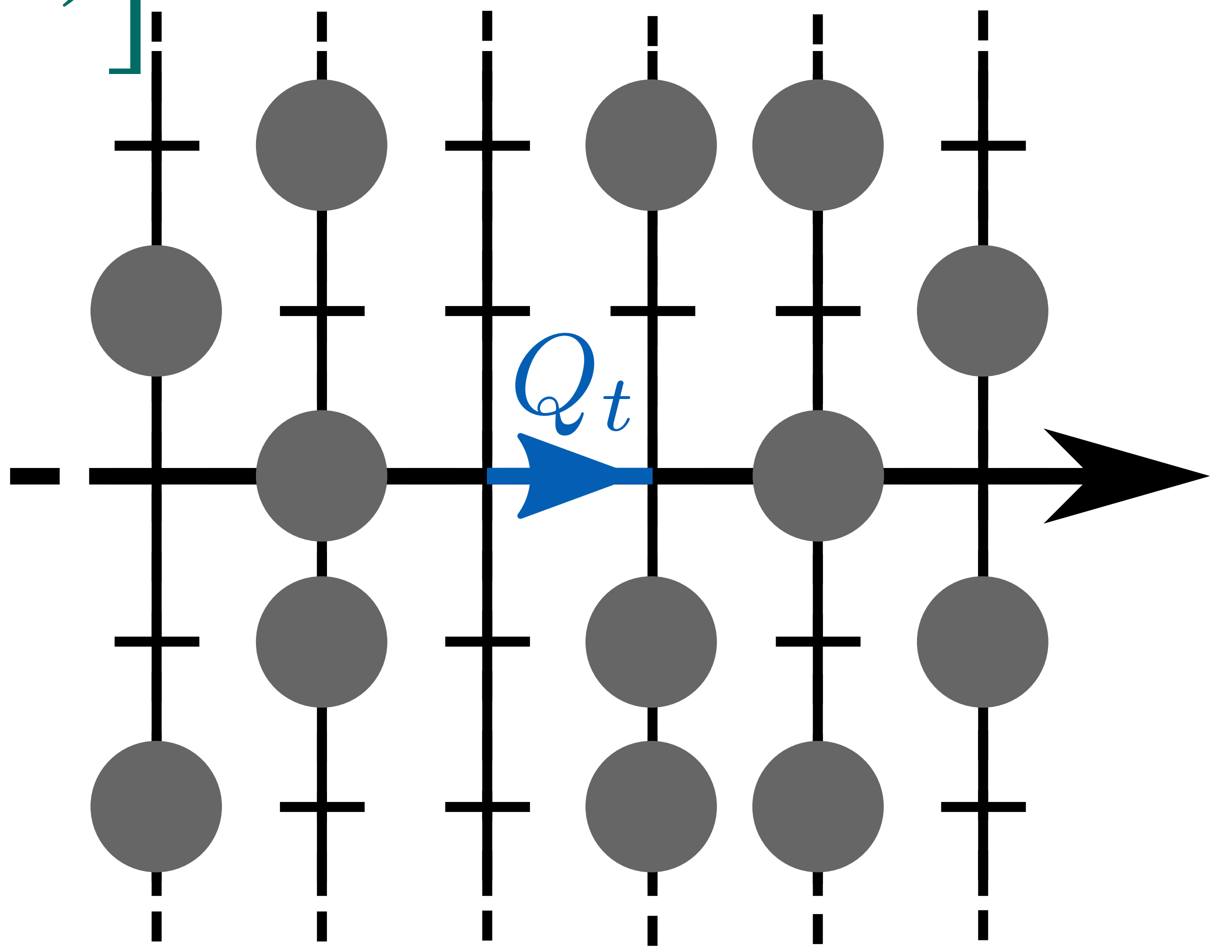


- Master equation





$$\partial_t P_t(\underline{\rho}) = \sum_{x,y} \left[P(\underline{\rho}^{x,y+}, t) - P(\underline{\rho}, t) \right] + \sum_{x,y} \left[P(\underline{\rho}^{x+,y}, t) - P(\underline{\rho}, t) \right]$$

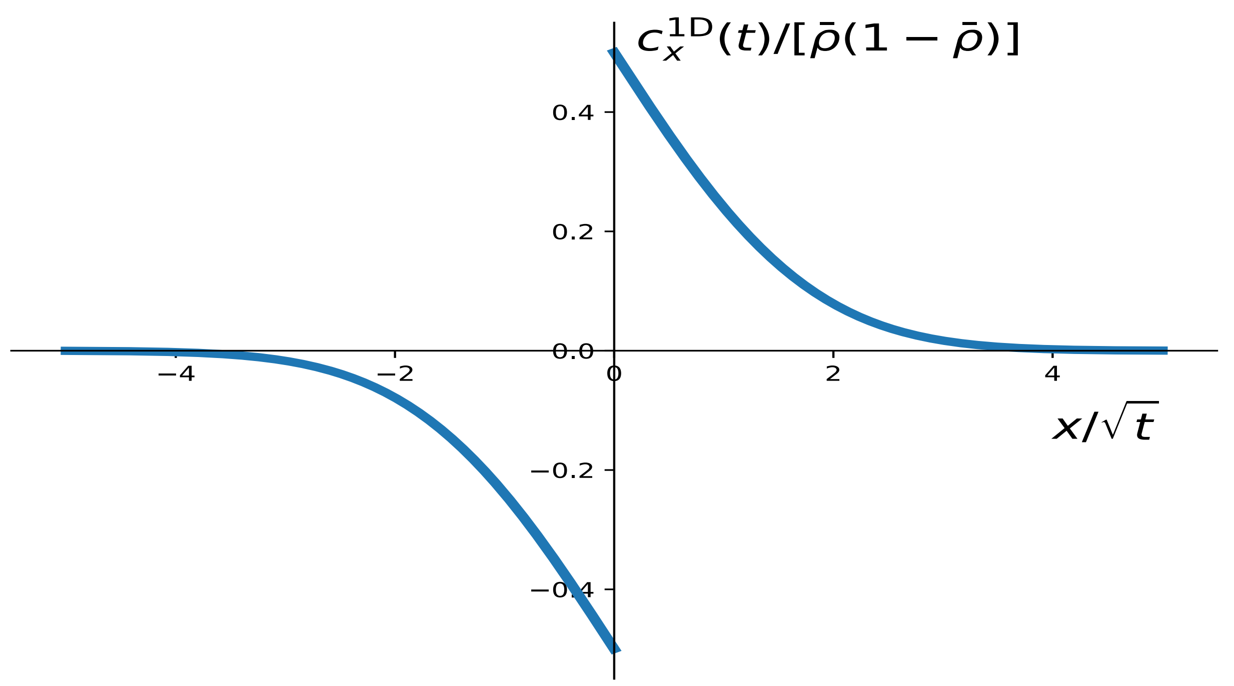


- Look for moments, e.g. $\partial_t \left\langle Q_t^2 \right\rangle = \dots \left[c_{\vec{r}}(t) \right] \dots$
- Find exact closed equations $\partial_t c_{\vec{r}}(t)$ for $c_{\vec{r}}(t) = \left\langle Q_t \, \rho_{\vec{r}}(t) \right\rangle$
- Solve them in Fourier-Laplace (self-consistently)

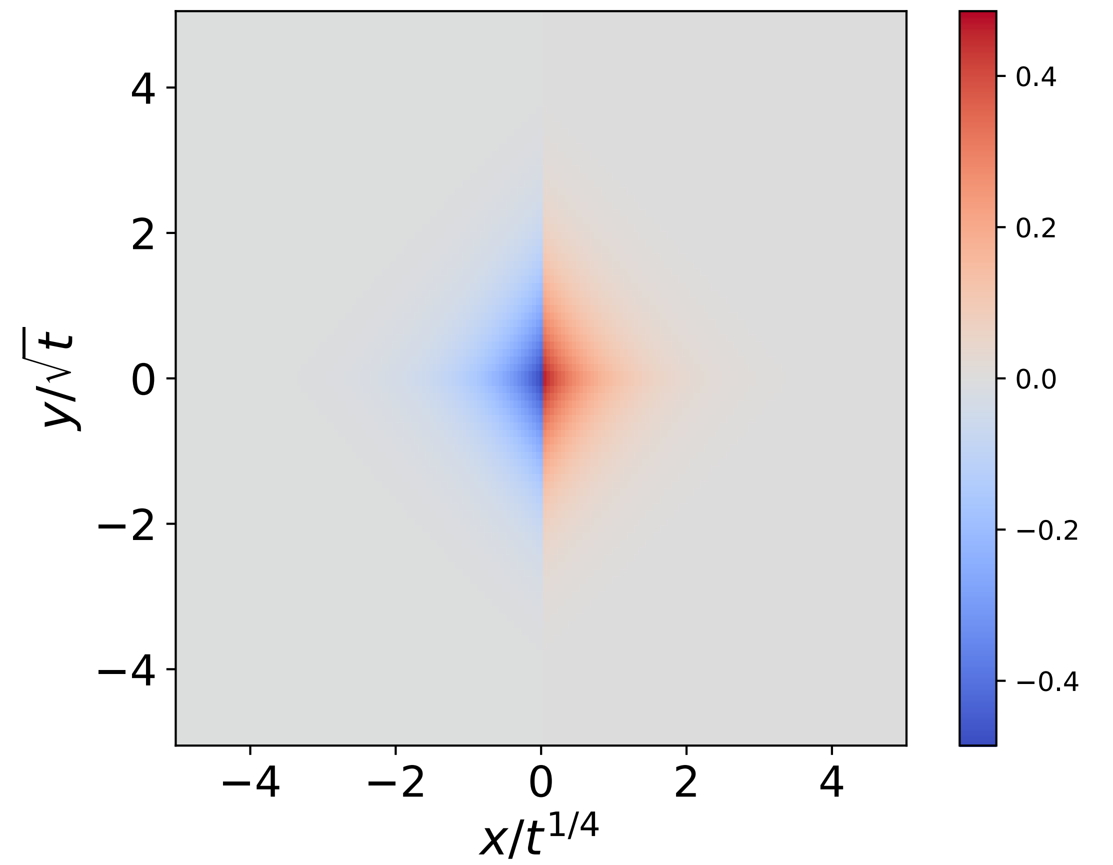
T. Berlioz, D. Venturelli, A. Grabsch, O. Bénichou, J. Stat. Mech. (2024) 113208

Results

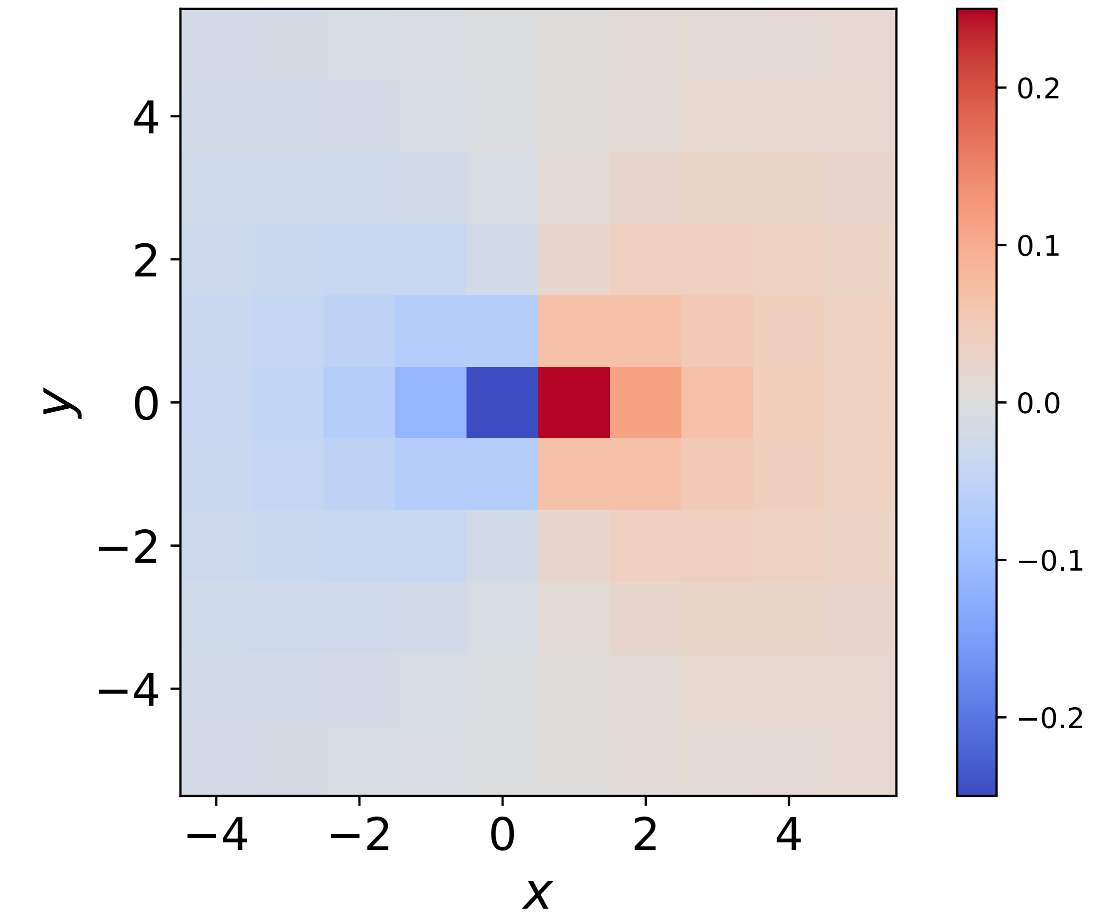
- $d = 1$ $$c_r(t) \xrightarrow[t\to\infty]{} \bar{\rho}(1-\bar{\rho})\,\mathscr{C}_1\!\left(\frac{r}{t^{1/2}}\right)$$ $$\left\langle Q_t^2 \right\rangle = n_1\,\bar{\rho}(1-\bar{\rho})\ t^{1/2}$$



- Comb $$c_{\vec{r}}(t) \xrightarrow[t\to\infty]{} \bar{\rho}(1-\bar{\rho})\,\mathscr{C}_c\!\left(\frac{x}{t^{1/4}},\frac{y}{t^{1/2}}\right)$$ $$\left\langle Q_t^2 \right\rangle = n_c\,\bar{\rho}(1-\bar{\rho})\ t^{3/4}$$



- $d = 2$ $$c_{\vec{r}}(t) \xrightarrow[t\to\infty]{} \bar{\rho}(1-\bar{\rho})\,\mathscr{C}_2(\vec{r})$$ $$\left\langle Q_t^2 \right\rangle = n_2\,\bar{\rho}(1-\bar{\rho})\ t$$

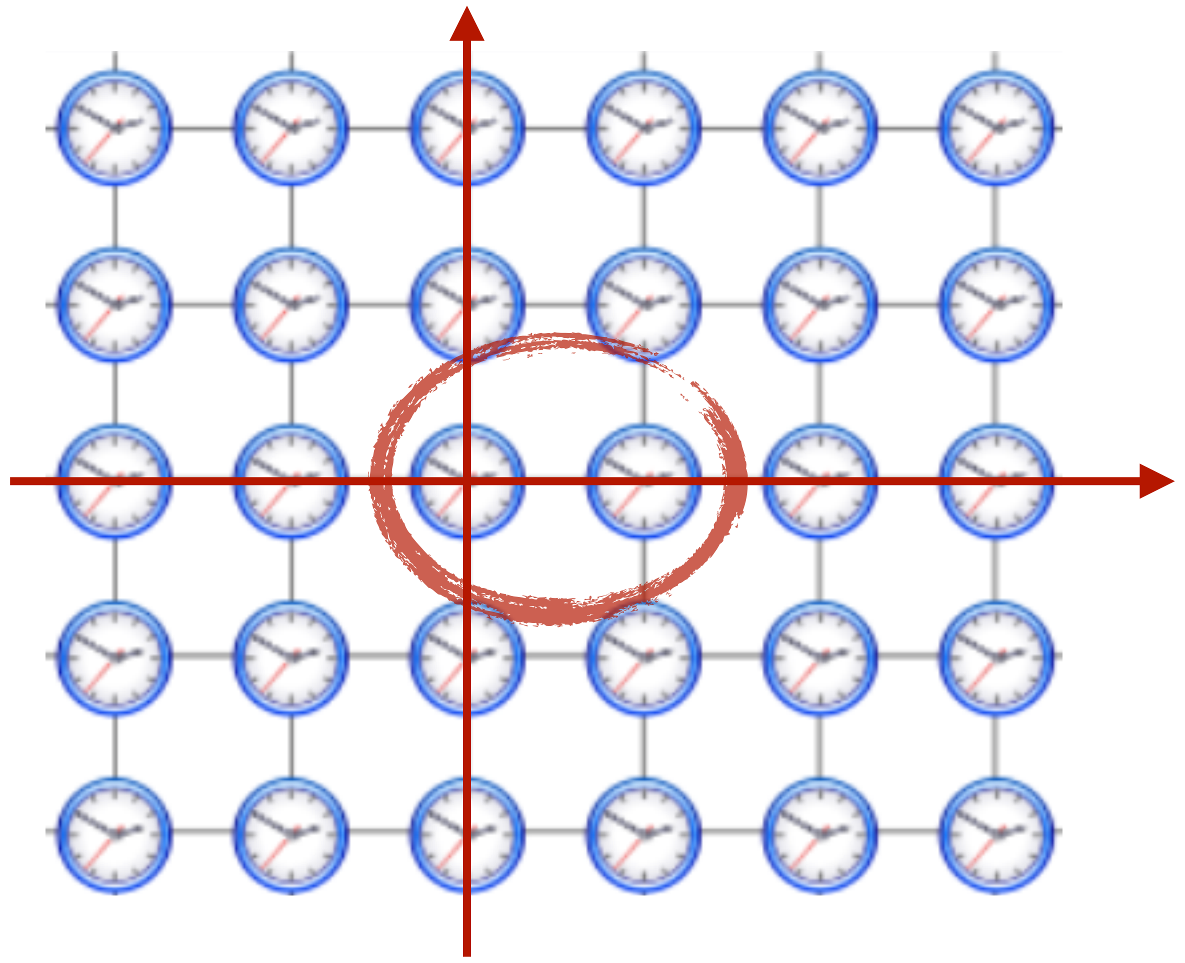


$\boxed{\left\langle Q_t^2 \right\rangle \propto t \text{ requires loops!}}$

Microscopic path-integral representation

$$\rho_{\vec{r}}(t+dt) - \rho_{\vec{r}}(t) = dt \sum_{\nu} \left(\vec{j}_{\vec{r}-\vec{\nu}}(t) - \vec{j}_{\vec{r}}(t) \right) \cdot \vec{\nu} \,,$$

$$\vec{j}_{\vec{r}}(t)\, dt = \sum_{\vec{\nu}} \left[\rho_{\vec{r}} (1-\rho_{\vec{r}+\vec{\nu}})\, \xi_{\vec{r},\vec{\nu}}(t) - \rho_{\vec{r}+\vec{\nu}} (1-\rho_{\vec{r}})\, \xi_{\vec{r}+\vec{\nu},-\vec{\nu}}(t) \right] \vec{\nu} \,,$$



equivalent to the M.E. if Poissonian noise is $\xi_{\vec{r},\vec{\mu}}(t) = \begin{cases} 1 & \textbf{prob. } \gamma\, dt \,, \\ 0 & \textbf{prob. } 1-\gamma\, dt \,. \end{cases}$

Usual MSR machinery gives

$$\left\langle e^{\lambda Q_T} \right\rangle = \int \mathscr{D}\theta_{\vec{r}}\, \mathscr{D}\overrightarrow{\varphi}_{\vec{r}}\, e^{-S[\rho_{\vec{r}}, \vec{j}_{\vec{r}}, \theta_{\vec{r}}, \overrightarrow{\varphi}_{\vec{r}}] + \lambda Q_T} \longrightarrow \quad Q_T = \int_0^T dt \left(\vec{j}_{\vec{r}=\vec{0}}(t) \right)_1$$

A. Lefèvre, G. Biroli, J. Stat. Mech. (2007) P07024

T. Berlioz, D. Venturelli, A. Grabsch, O. Bénichou, J. Stat. Mech. (2024) 113208

Microscopic path-integral representation

- Saddle-point eqs are difference equations for $\rho_{\vec{r}}, \vec{j}_{\vec{r}}, \theta_{\vec{r}}, \overrightarrow{\varphi}_{\vec{r}}$, to be solved order by order, e.g.

$$\vec{j}_{\vec{r}} = \vec{j}^{(0)}_{\vec{r}} + \lambda \vec{j}^{(1)}_{\vec{r}} + \dots \qquad \left\langle e^{\lambda Q_T} \right\rangle = \int \mathscr{D}\theta_{\vec{r}}\, \mathscr{D}\overrightarrow{\varphi}_{\vec{r}}\, e^{-S[\rho_{\vec{r}}, \vec{j}_{\vec{r}}, \theta_{\vec{r}}, \overrightarrow{\varphi}_{\vec{r}}] + \lambda Q_T}$$

- Turn out to relax to a **stationary** limit,

$$\mathcal{S} = \int_0^T dt\, \mathscr{L}[\{\rho, \vec{j}, \theta, \overrightarrow{\varphi}\}] \simeq T\, \mathscr{L}[\{\rho^*, \vec{j}^*, \theta^*, \overrightarrow{\varphi}^*\}]$$

- Can be used to recover

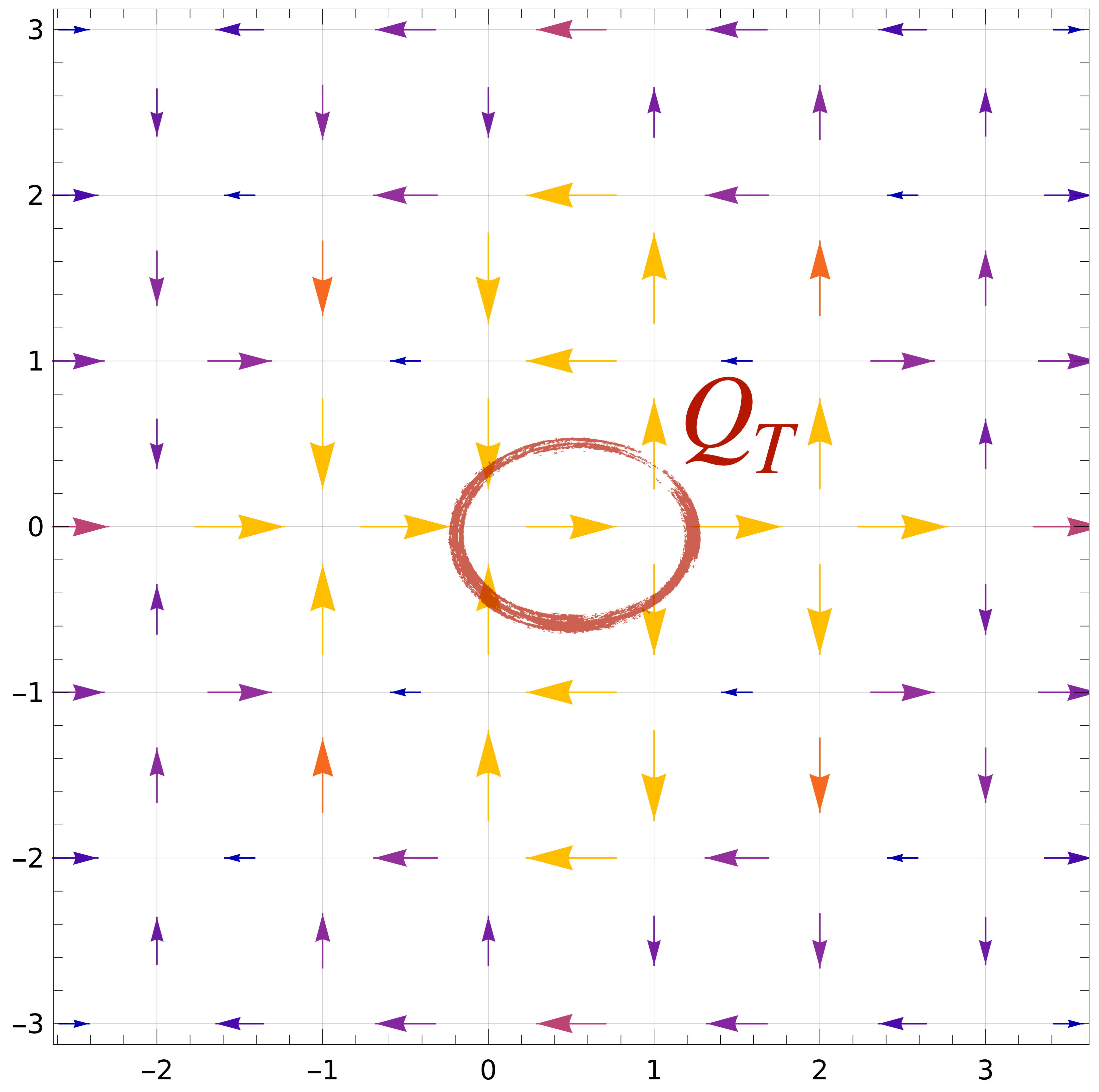
$$\left\langle e^{\lambda Q_T} \right\rangle \simeq \exp\{-T[\mathscr{L}^* - \lambda (\vec{j}^*_{\vec{r}=\vec{0}})_1]\} \longrightarrow \left\langle Q_t^2 \right\rangle = 2\gamma \left(1 - \frac{1}{d}\right) \bar{\rho}(1-\bar{\rho})\, t$$

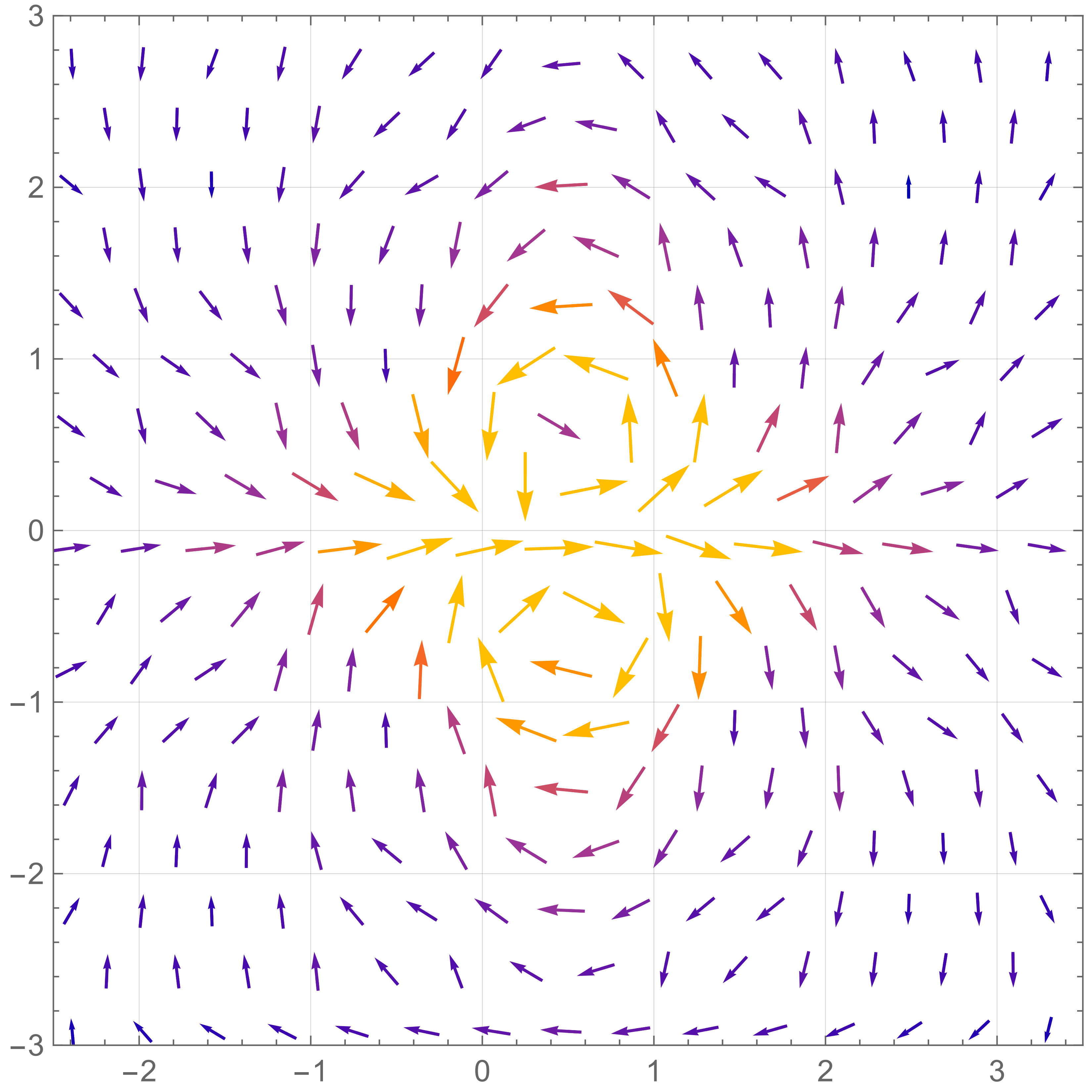
correlation profile $\longrightarrow$ $\left\langle \rho_{\vec{r}}(t)\, e^{\lambda Q_T} \right\rangle \Big/ \left\langle e^{\lambda Q_T} \right\rangle \simeq \rho^*_{\vec{r}}(t)$ $\longleftarrow$ optimal profile

Role of loops

$$\frac{\left\langle \vec{j}_{\vec{r}}(t)\, e^{\lambda Q_T} \right\rangle}{\left\langle e^{\lambda Q_T} \right\rangle} \simeq \vec{j}^{\,*}_{\vec{r}}(t) \longrightarrow \left\langle \vec{j}_{\vec{r}}(t)\, Q_T \right\rangle$$

$\vec{j}^{\,*}_{\vec{r}}$



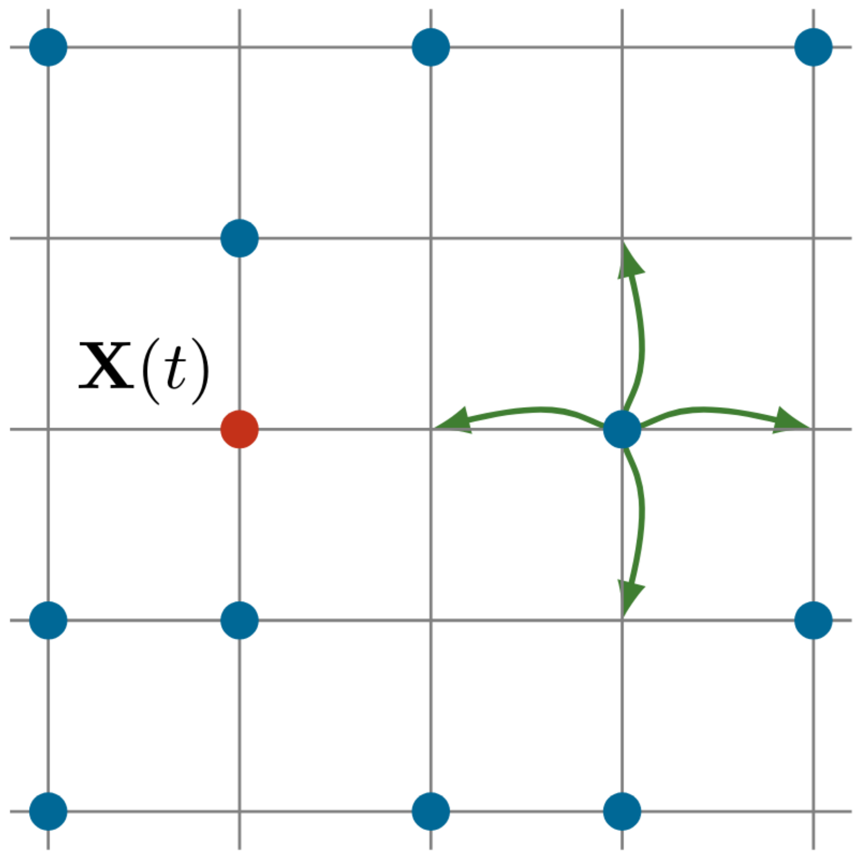


looped structure of the lattice allows for **vortex** configurations, and thus for $\langle Q_t^2 \rangle \propto t$

2. Tracer-bath correlations

Hard-core lattice gas

in d dimensions: $\langle X_t \, \rho_{X+r}(t) \rangle$



- Using the ME, write an equation for $g_r(t) = \langle X_t \, \rho_{X+r}(t) \rangle \to$ **not closed!**
- $\partial_t g_r(t) = (\ldots)$ can be closed upon ***decoupling*** (exact for large/small $\bar{\rho}$)

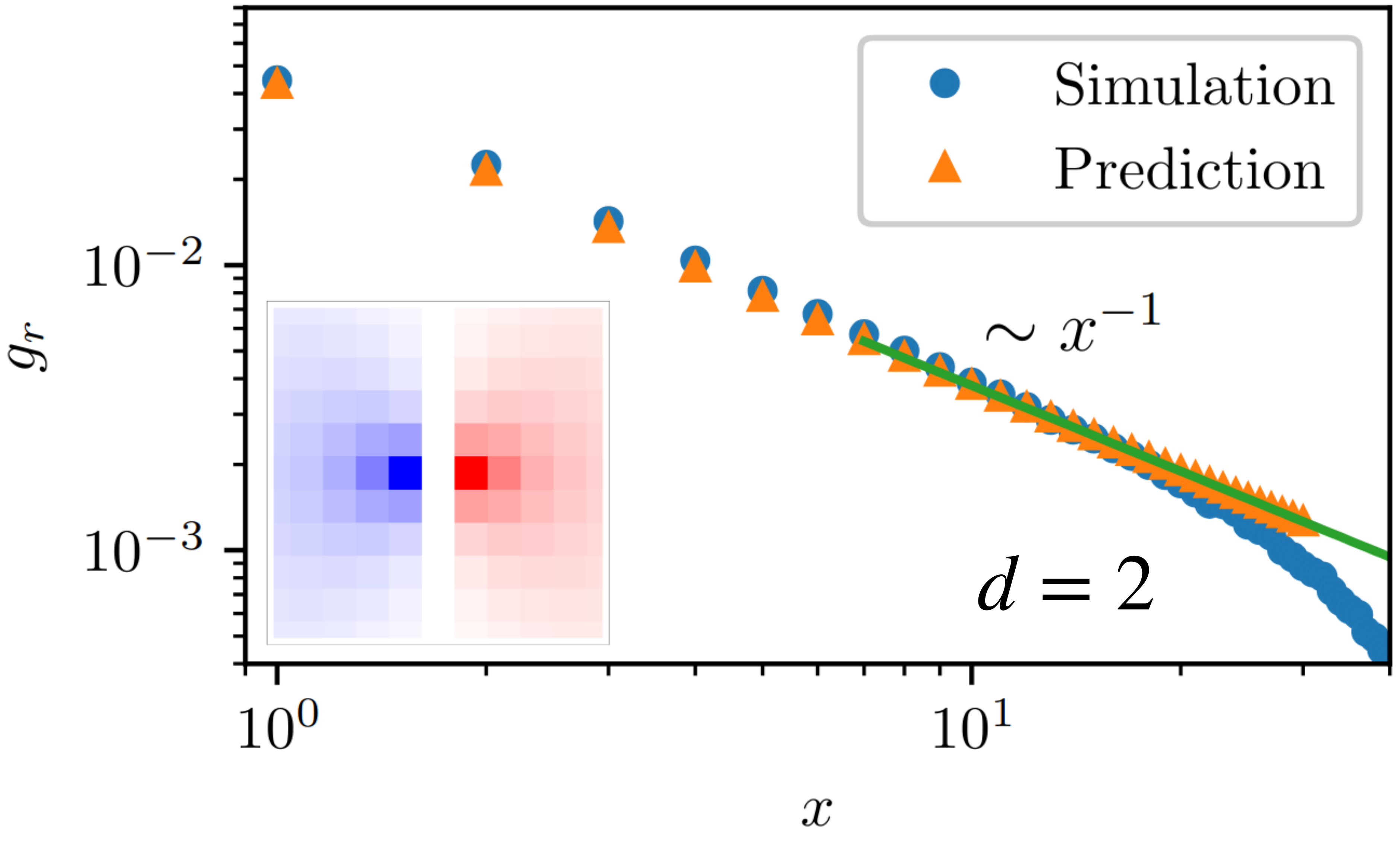
$$\langle \rho_{X+r} \, \rho_{X+r'} \rangle \simeq \langle \rho_{X+r} \rangle \langle \rho_{X+r'} \rangle$$

$$\langle X_t \, \rho_{X+r} \, \rho_{X+r'} \rangle \simeq \langle X_t \, \rho_{X+r} \rangle \langle \rho_{X+r'} \rangle + \langle X_t \, \rho_{X+r'} \rangle \langle \rho_{X+r} \rangle$$

- Solve for $g_r(t) \to g_r$ at long t. For large $x = \mathbf{r} \cdot \hat{\mathbf{e}}_1$,

$$g_x \sim x^{1-d}$$

Q: is this specific to SEP?



Bénichou, Illien, Oshanin, Sarracino, Voituriez, Phys. Rev. Lett. **115**, 220601 (2015)

D. Venturelli, P. Illien, A. Grabsch, O. Bénichou, arXiv:2411.09326 (to appear in Phys. Rev. Lett.)

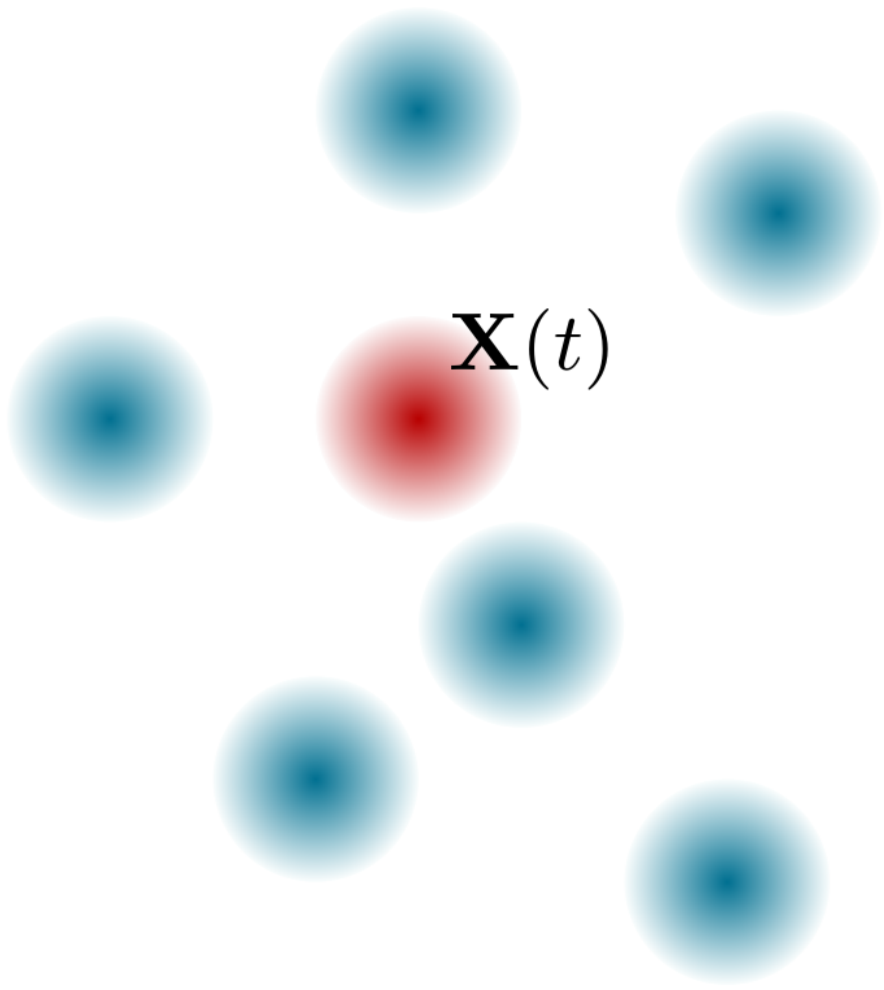
Interacting Brownian particles

$$\dot{\mathbf{X}}_i(t) = -\mu \sum_{j \neq i} \nabla_i U\left(\mathbf{X}_i(t) - \mathbf{X}_j(t)\right) + \boldsymbol{\eta}_i(t), \qquad \langle \boldsymbol{\eta}_i(t)^T \boldsymbol{\eta}_j(t') \rangle = 2\mu T \delta_{ij} \delta(t - t') I_d$$

- **Tracer** $i = 0$ (omitted)
- Dean-Kawasaki equation for $\rho(\mathbf{r}, t) = \sum_{i=1}^{N} \delta\left(\mathbf{r} - \mathbf{X}_i(t)\right)$,

$$\partial_t \mathbf{X}(t) = -\mu \nabla_{\mathbf{X}} \mathscr{F}[\rho, \mathbf{X}] + \boldsymbol{\eta}_0(t),$$

$$\partial_t \rho(\mathbf{r}, t) = \mu \nabla \cdot \left[\rho(\mathbf{r}, t) \nabla \frac{\delta \mathscr{F}}{\delta \rho(\mathbf{r}, t)}\right] + \nabla \cdot \left[\rho^{\frac{1}{2}}(\mathbf{r}, t)\, \boldsymbol{\xi}(\mathbf{r}, t)\right],$$



$$\mathscr{F}[\rho, \mathbf{X}] = T \int \mathrm{d}\mathbf{x}\, \rho(\mathbf{x}) \log\left(\frac{\rho(\mathbf{x})}{\rho_0}\right) + \frac{1}{2} \int \mathrm{d}\mathbf{x}\, \mathrm{d}\mathbf{y}\, \rho(\mathbf{x}) U(\mathbf{x} - \mathbf{y}) \rho(\mathbf{y}) + \int \mathrm{d}\mathbf{y}\, \rho(\mathbf{y}) U(\mathbf{y} - \mathbf{X})$$

V. Démery et al., New J. Phys. **16** (2014) 053032

Tracer statistics
from tracer-bath profiles

hard-core lattice gas

$$\boxed{\partial_t \Psi(\boldsymbol{\lambda}, t) = \frac{1}{2d\,\tau} \sum_{\mu=-d}^{d} \left(e^{\sigma \boldsymbol{\lambda} \cdot \hat{\mathbf{e}}_\mu} - 1 \right) \left[1 - w_{\mathbf{e}_\mu}(\boldsymbol{\lambda}, t) \right]}$$

- How does $\Psi(\lambda, t) = \ln \left\langle e^{\lambda \cdot \mathbf{X}(t)} \right\rangle$ evolve?

$$\partial_t \Psi(\boldsymbol{\lambda}, t) = \lambda^2 \mu T + \mu \, \boldsymbol{\lambda} \cdot \int d^d r \, U(\mathbf{r}) \, \nabla_{\mathbf{r}} \, w(\mathbf{r}, \boldsymbol{\lambda}, t),$$

with the profile

average bath density profile

$$w(\mathbf{r}, \lambda, t) = \frac{\left\langle \rho(\mathbf{r} + \mathbf{X}(t), t) \, e^{\lambda \cdot \mathbf{X}(t)} \right\rangle}{\left\langle e^{\lambda \cdot \mathbf{X}(t)} \right\rangle} = \langle \rho(\mathbf{r} + \mathbf{X}(t), t) \rangle + \boldsymbol{\lambda} \cdot \langle \mathbf{X}(t) \, \rho(\mathbf{r} + \mathbf{X}(t), t) \rangle + \mathcal{O}(\lambda^2)$$

tracer-bath correlation profile

D. Venturelli, P. Illien, A. Grabsch, O. Bénichou, arXiv:2411.09326
(to appear in Phys. Rev. Lett.)

Beware

of zero modes

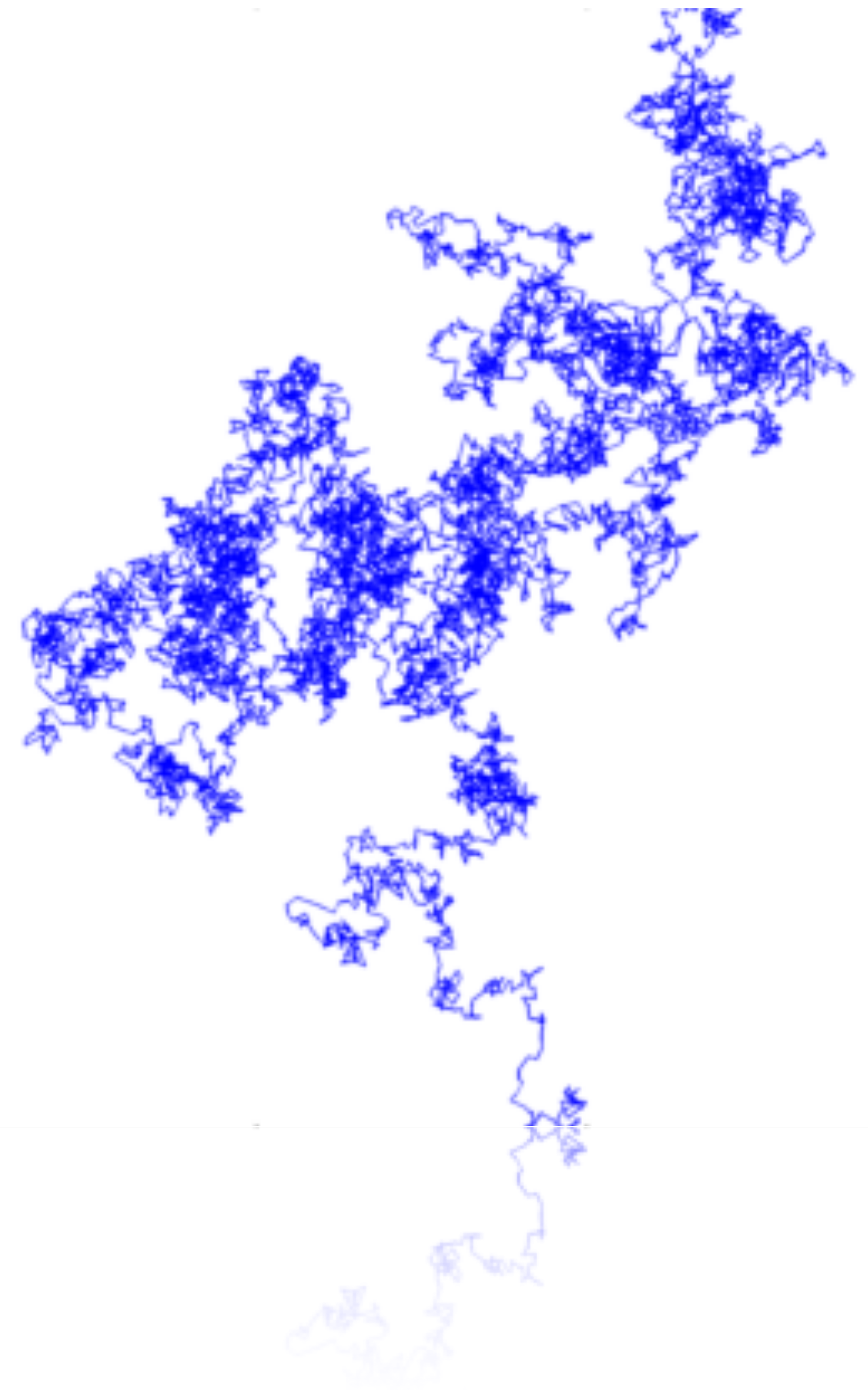
- For simple liquids, stationary quantities are computed as

$$\langle \mathbf{X}\,\rho(\mathbf{x}+\mathbf{X})\rangle \propto \int \mathscr{D}\rho \int \mathrm{d}\mathbf{X}\,\mathrm{e}^{-\frac{1}{T}\mathscr{F}[\rho,\mathbf{X}]}\,\left[\mathbf{X}\,\rho(\mathbf{x}+\mathbf{X})\right] = 0$$

upon defining $\rho'(\mathbf{x}) = \rho(\mathbf{x}+\mathbf{X})$.

- Thermodynamic quantities only depend on $|\mathbf{X}_i - \mathbf{X}_j|$, whereas $\left[\mathbf{X}\,\rho(\mathbf{x}+\mathbf{X})\right]$ depends also on **COM**

- Trivial zero! → Need to use EOM to predict stationary profiles.



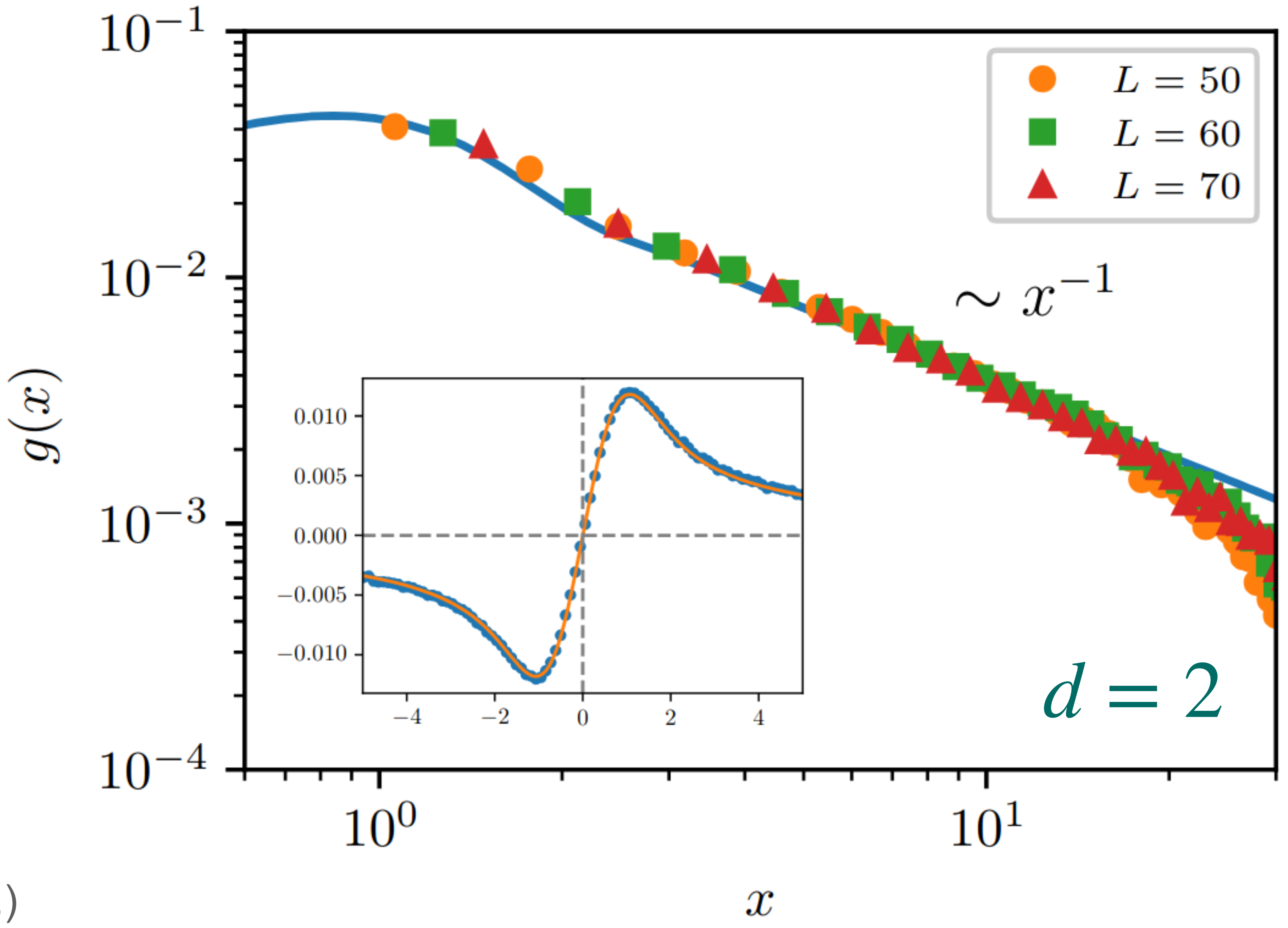
Stationary tracer-bath profile $\langle X \rho_{X+r} \rangle$

- $\rho(\mathbf{r}, t) = \rho_0 + \sqrt{\rho_0}\, \phi(\mathbf{r}, t) \rightarrow$ linearize the DK eqs assuming $\phi/\sqrt{\rho_0} \ll 1$
- From coupled eqs for $\partial_t \mathbf{X}(t)$ and $\partial_t \phi(\mathbf{r}, t)$, write one for

$$g(\mathbf{r}, t) = \hat{\mathbf{e}}_1 \cdot \langle \mathbf{X}(t)\, \rho(\mathbf{r} + \mathbf{X}(t), t) \rangle$$

- Solve for $g(\mathbf{r})$ perturbatively, in the stationary limit
- At large distance $x = \mathbf{r} \cdot \hat{\mathbf{e}}_1$,

$$g(x) \sim x^{1-d}$$



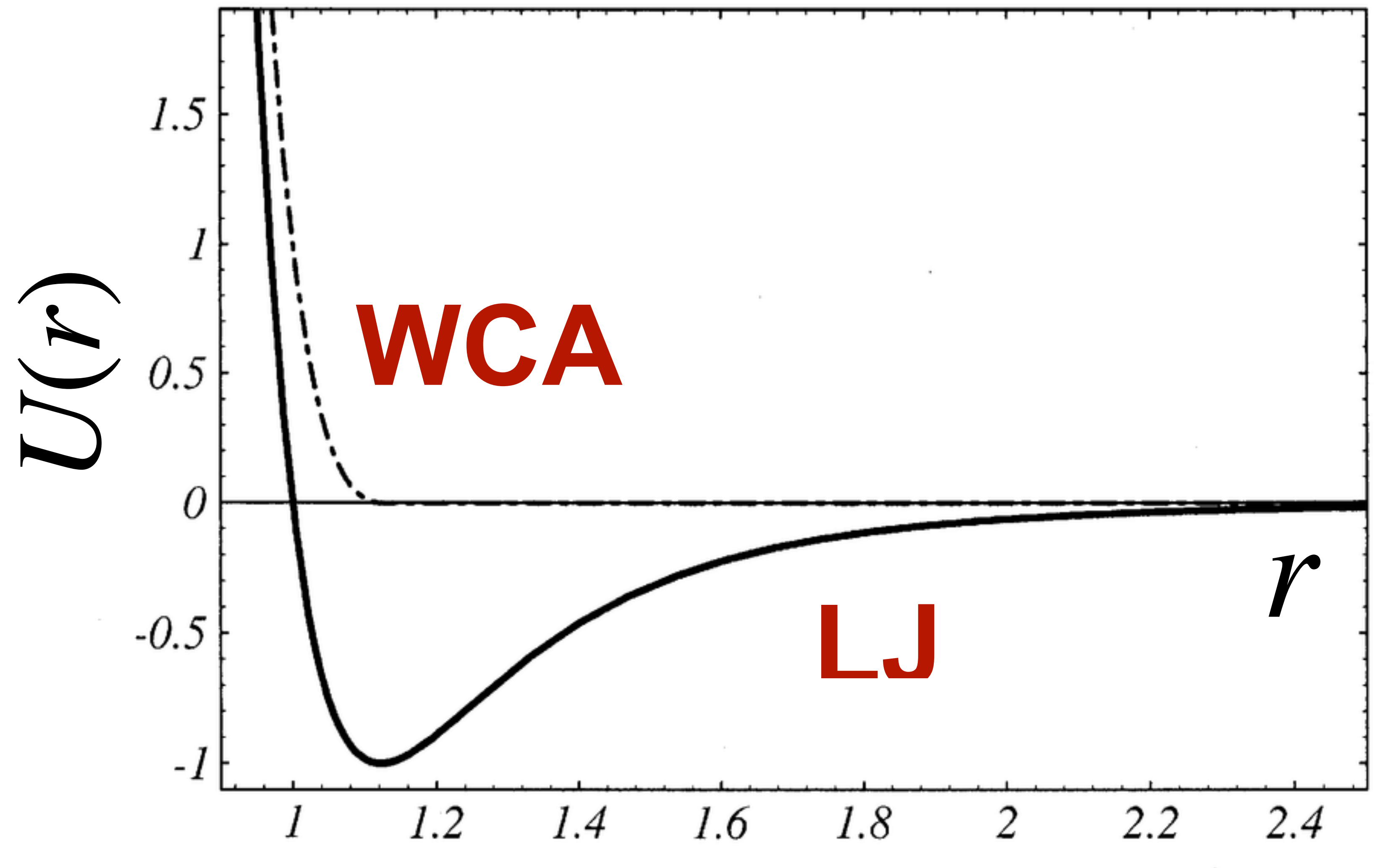
D. Venturelli, P. Illien, A. Grabsch, O. Bénichou, arXiv:2411.09326 (to appear in Phys. Rev. Lett.)

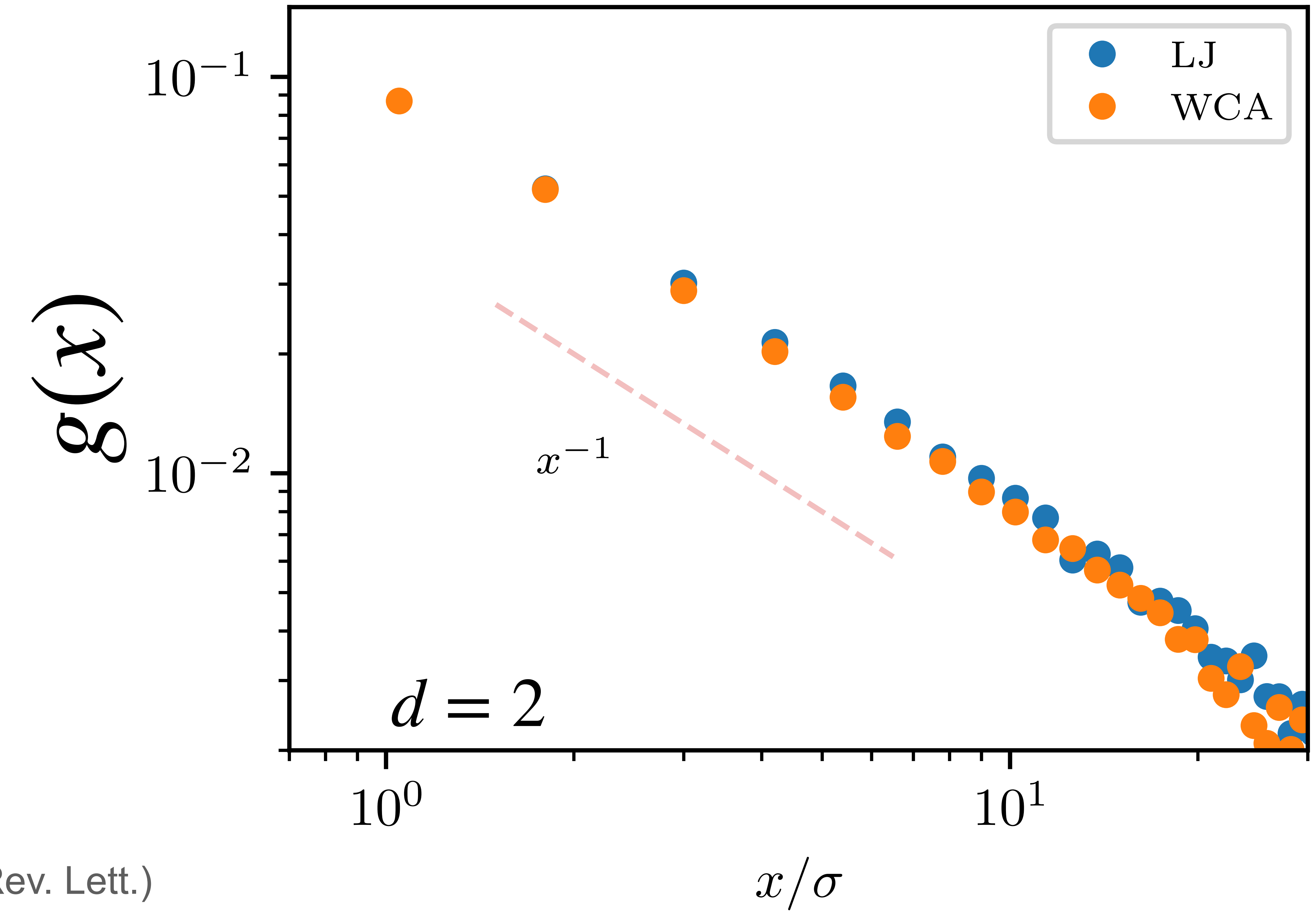
Lennard-Jones fluids

stationary tracer-bath profile

- Strong **repulsion** beyond linearised D-K theory
- **Simulate** LJ and WCA suspensions

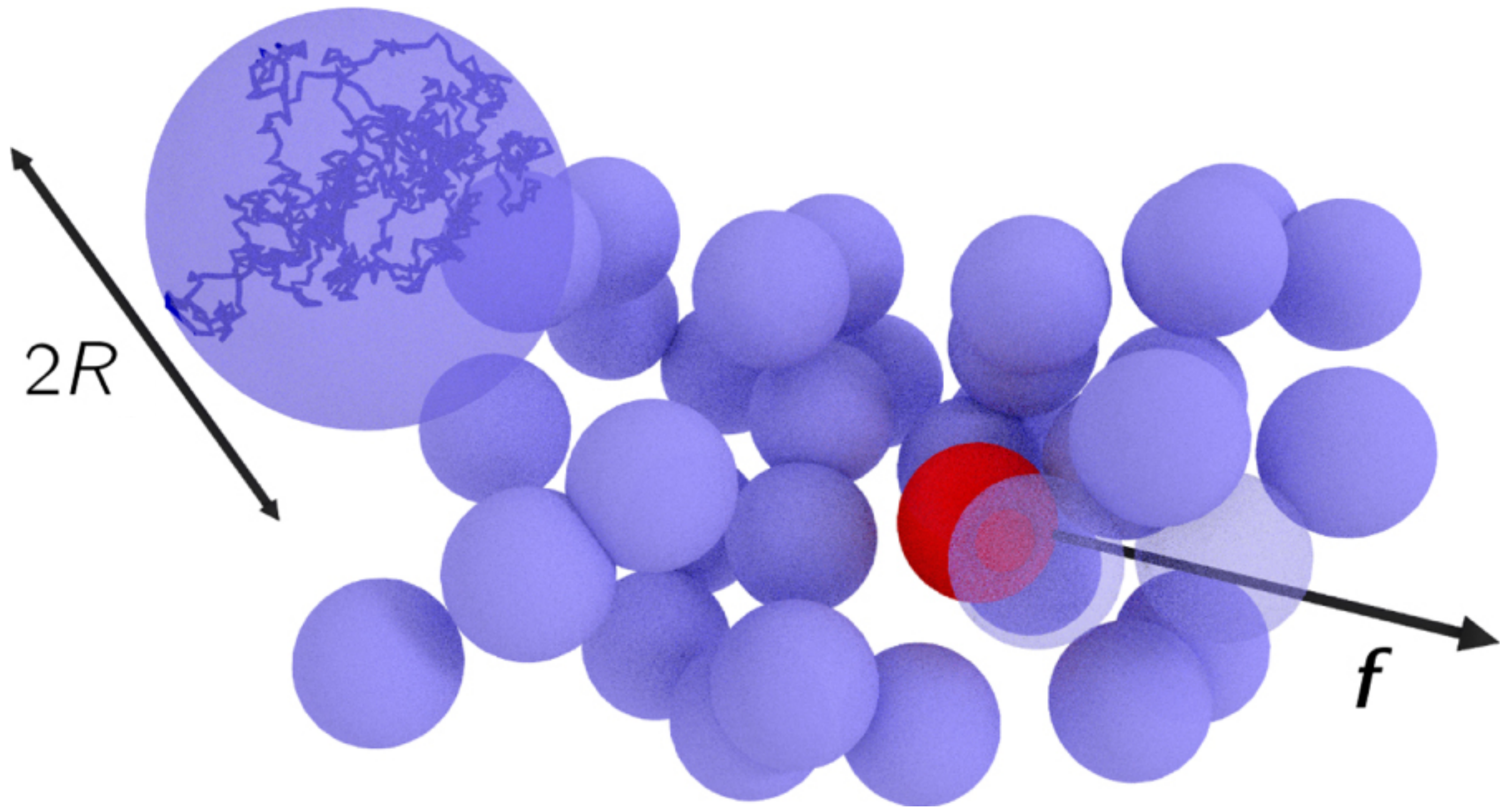
Short-distance details of interactions irrelevant for large-distance behaviour of the bath response?





D. Venturelli, P. Illien, A. Grabsch, O. Bénichou, arXiv:2411.09326 (to appear in Phys. Rev. Lett.)

Average density profile
under steady driving



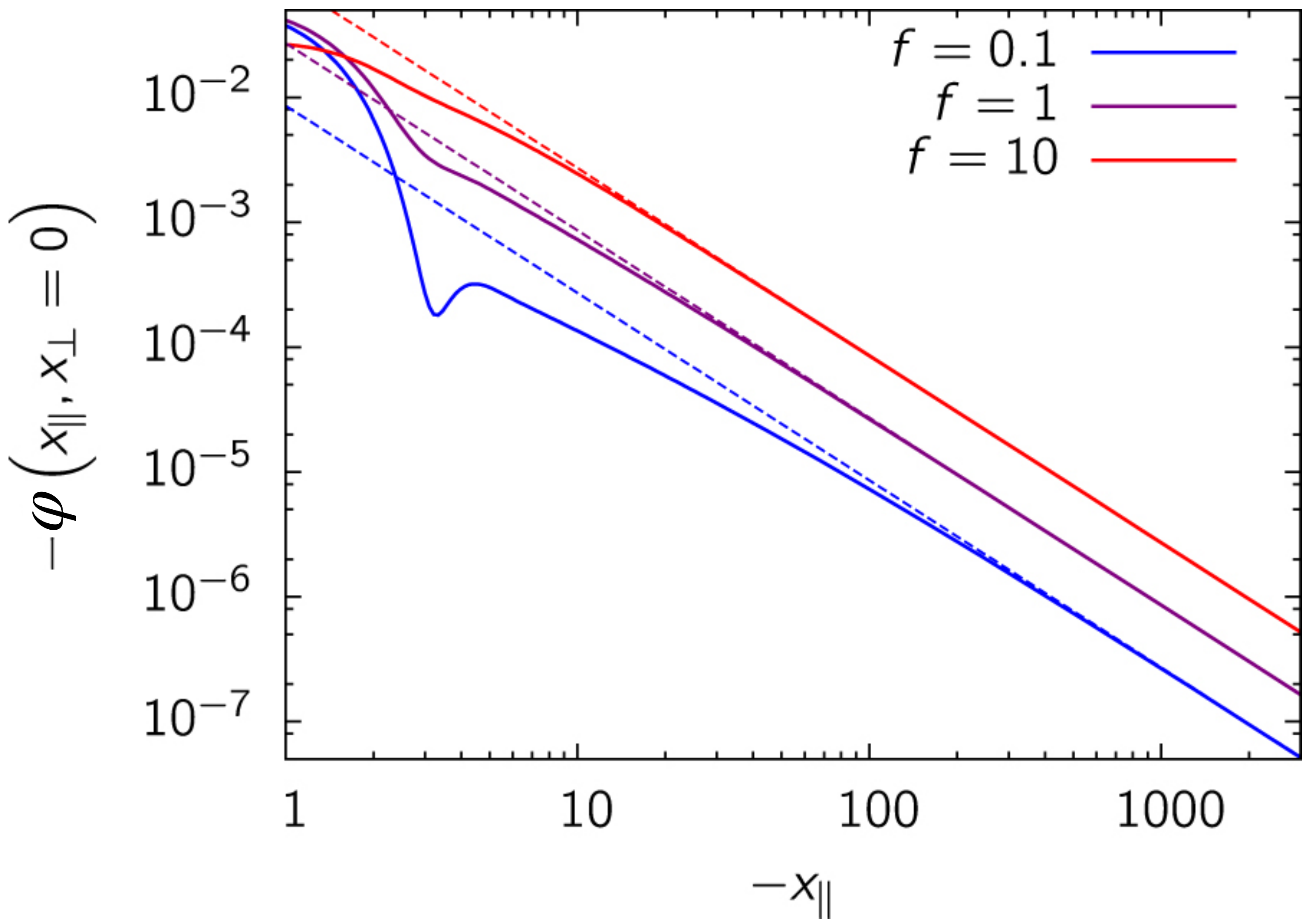
- Stationary density profile in the frame of the tracer,

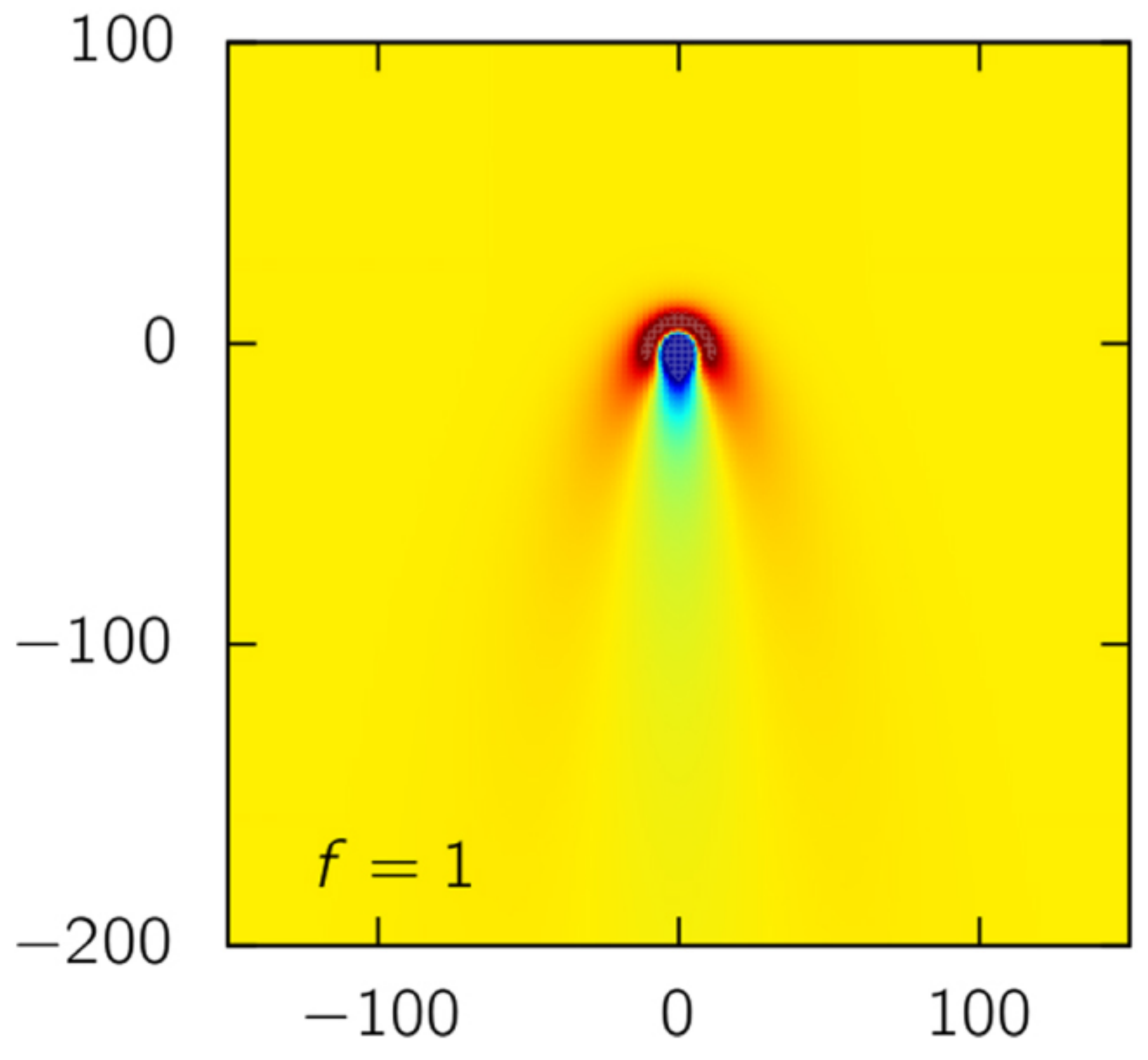
$$\varphi(\mathbf{x}, t) = \langle \rho(\mathbf{x} + \mathbf{X}(t), t) \rangle$$

- For $x_{||} \to -\infty$,

$$\varphi(x_{||}, \mathbf{x}_\perp = \mathbf{0}) \sim -|x_{||}|^{-\frac{1+d}{2}}$$

both for discrete and continuum



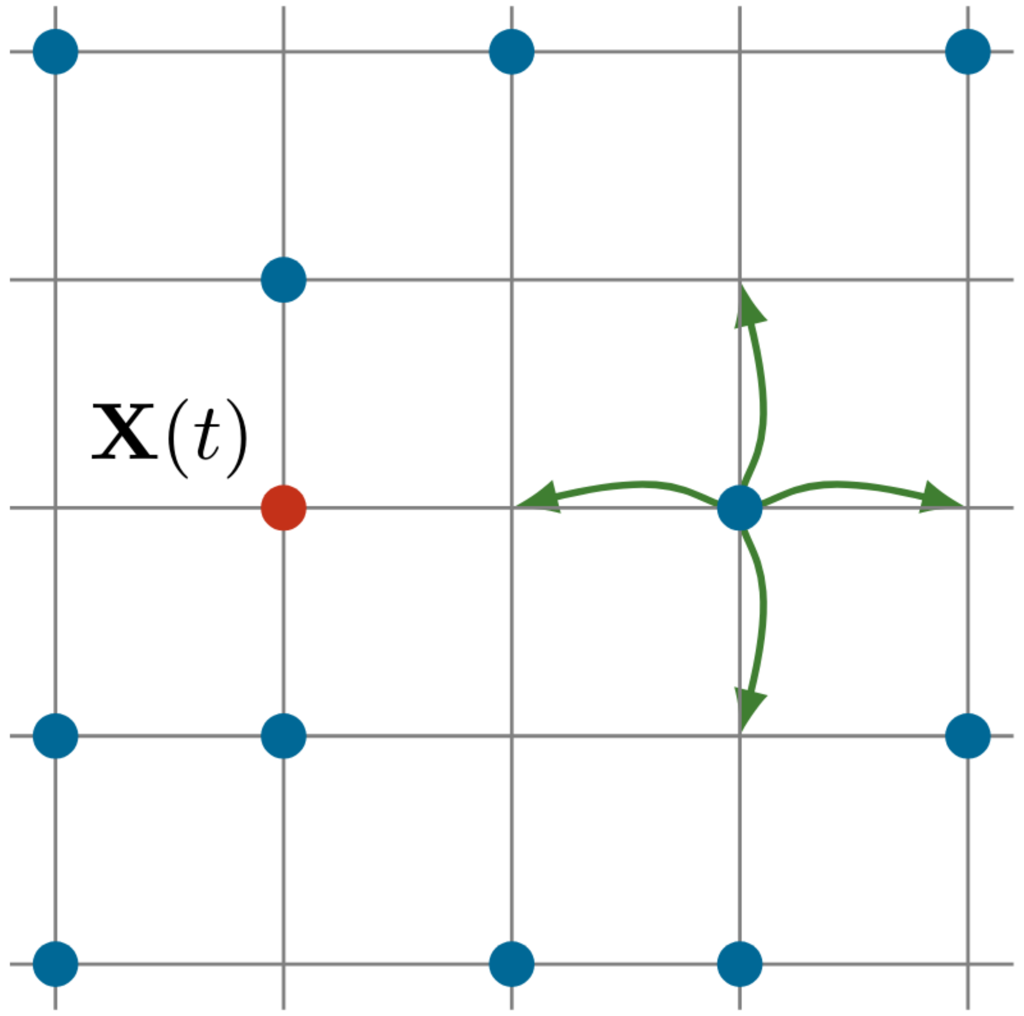


V. Démery et al., New J. Phys. **16** (2014) 053032

O. Bénichou et al., Phys. Rev. Lett. **84**, 511 (2000)

A common language

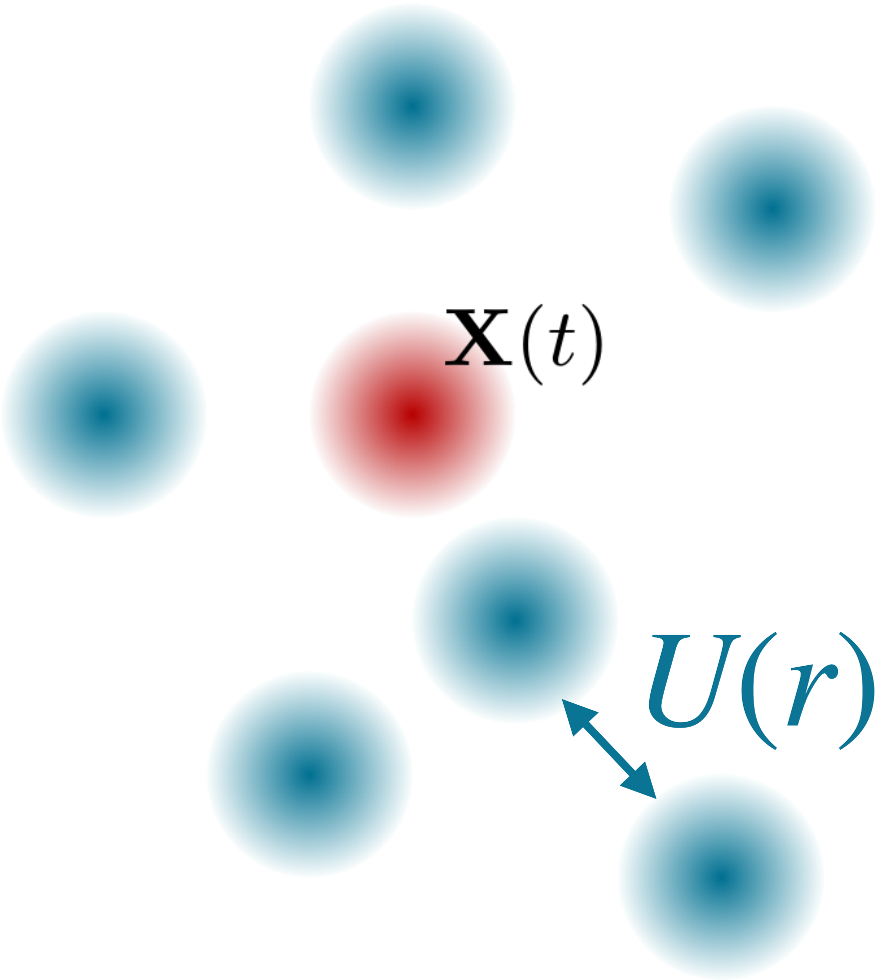
for diffusive systems?



large-distance behaviour
of tracer-bath correlations

$$\longleftrightarrow$$

$$\langle X \rho_{X+r} \rangle_c$$



$\downarrow$ $\boxed{\text{master equation}}$

$\overset{??}{\longleftrightarrow}$

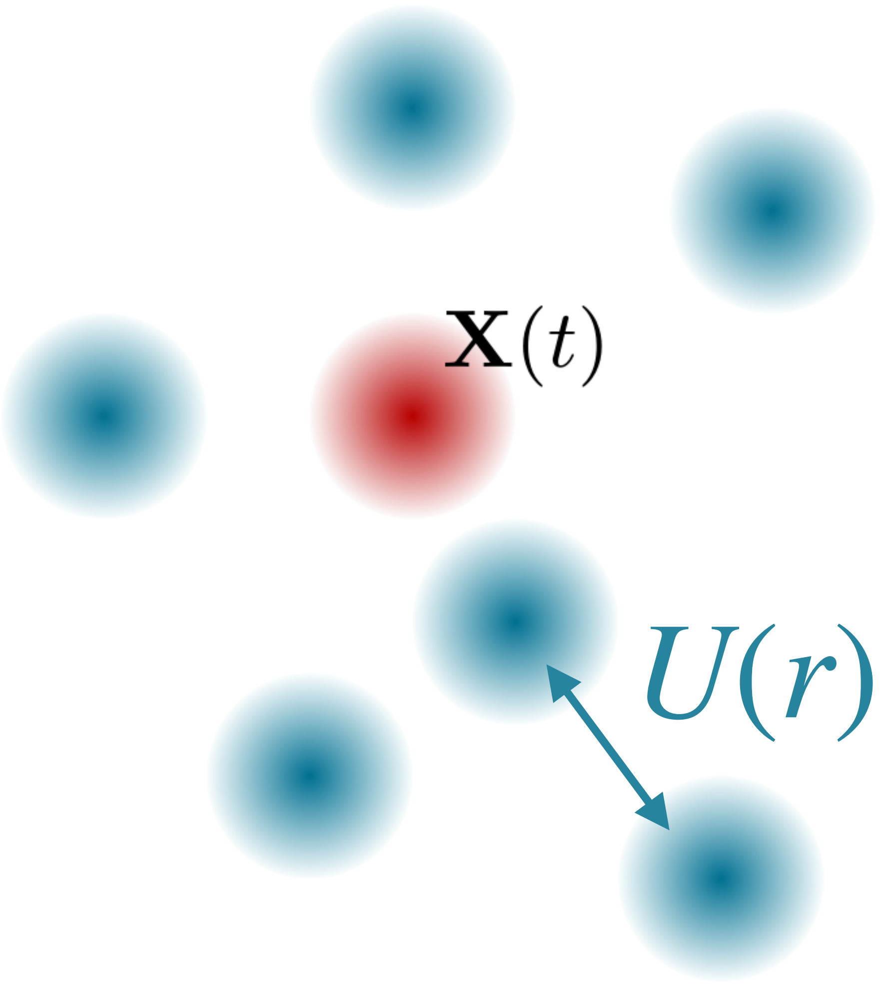
$\downarrow$ $\boxed{\text{Dean-Kawasaki theory}}$

Macroscopic fluctuation theory

for interacting Brownian particles

$$\partial_t \rho(x,t) = \partial_x [D(\rho)\partial_x \rho + \sqrt{\sigma(\rho)}\,\eta(x,t)]$$

diffusion — mobility — white Gaussian noise



- Large-scale properties of **diffusive** systems
- **Example**: $D(\rho) = 1$ and $\sigma(\rho) = 2\rho(1-\rho)$ for SEP

(1) Who are $D(\rho)$ and $\sigma(\rho)$ for generic $U(r)$?

$$\dot{\mathbf{X}}_i = -\mu \sum_{j\neq i} \nabla_i U\left(\mathbf{X}_i - \mathbf{X}_j\right) + \sqrt{2\mu k_B T}\boldsymbol{\eta}_i,$$

$$\sigma(\rho) = 2\mu k_B T\rho, \quad D(\rho) = \mu\, \partial_\rho P(\rho)$$

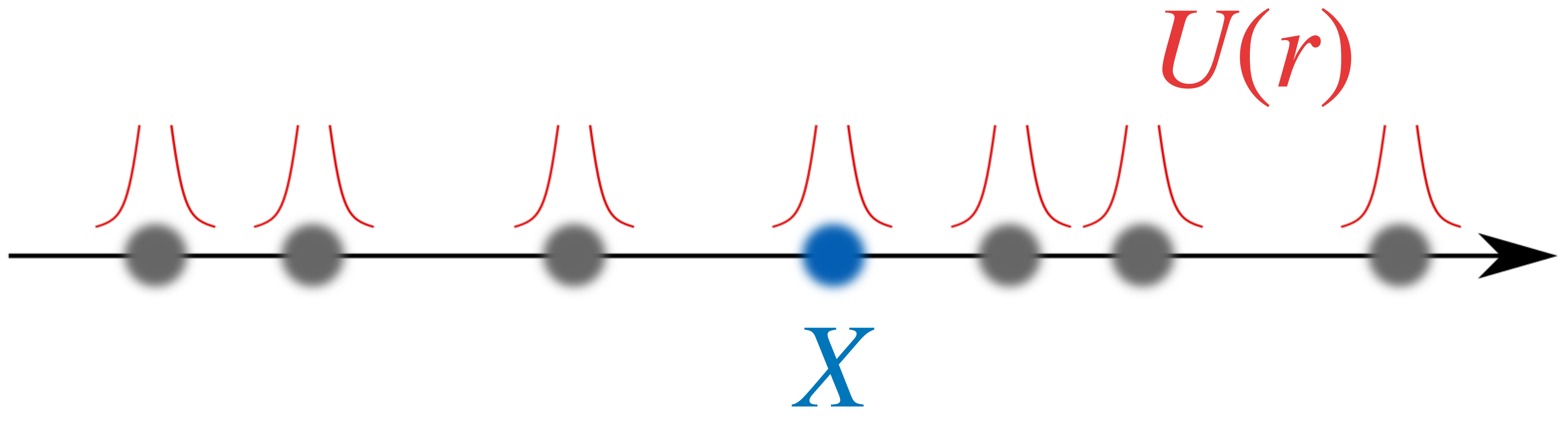
equilibrium pressure, conceptually solved problem (e.g. virial expansion)

A. Grabsch, D. Venturelli, O. Bénichou, arXiv:2504.08560 (to appear in Phys. Rev. Lett.)

Macroscopic fluctuation theory

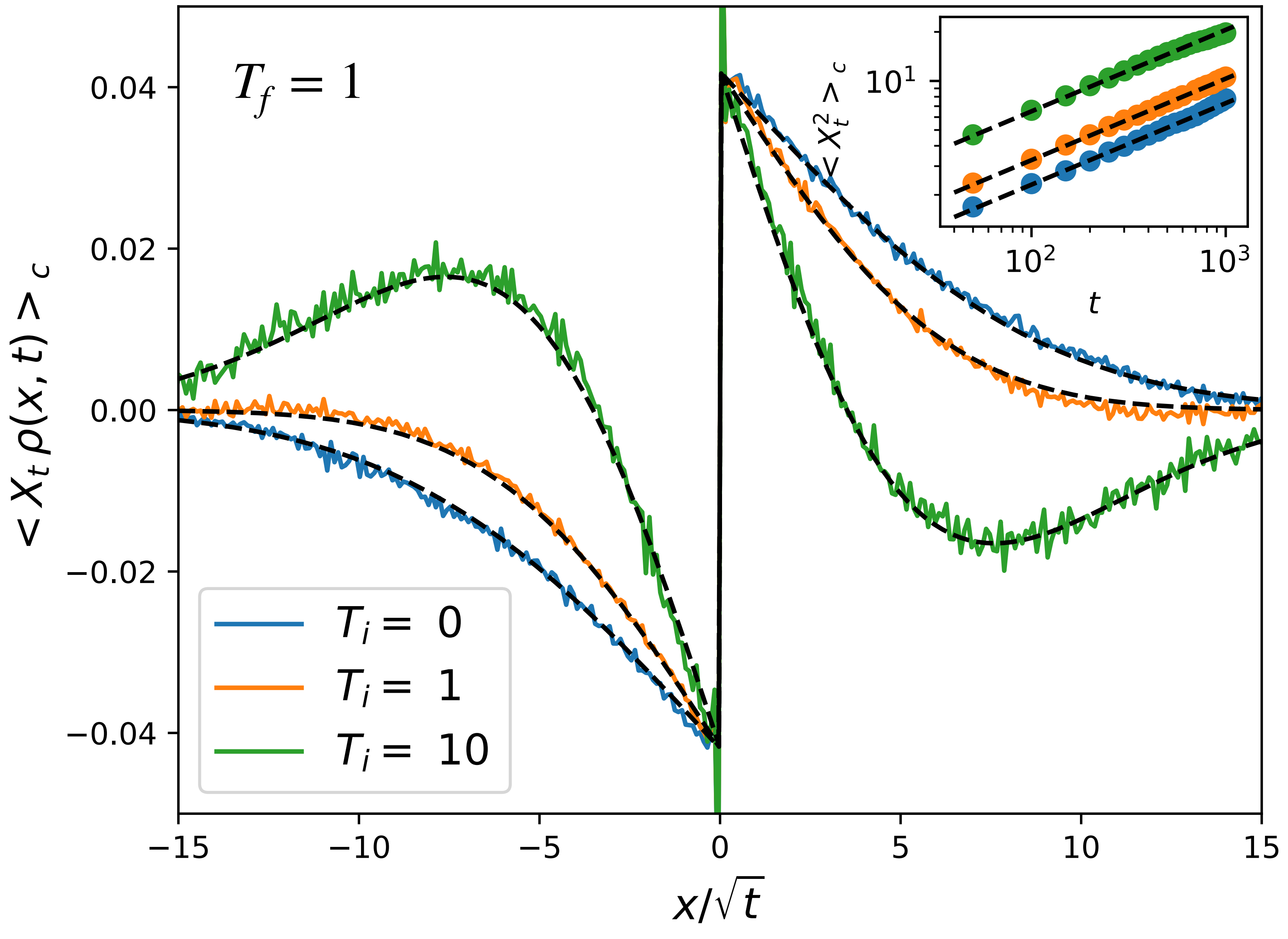
for interacting Brownian particles

$$\partial_t \rho(x,t) = \partial_x[D(\rho)\partial_x \rho + \sqrt{\sigma(\rho)}\,\eta(x,t)]$$



(2) Tracer-bath correlation profiles?

- In **single-file** systems, $X = X[\rho]$ (conservation # of particles)
- $\langle X \rho(X+x)\rangle$ from large-deviations approach
- Even after a temperature quench



Open problem: how do we couple X and ρ in $d > 1$?

A. Grabsch, D. Venturelli, O. Bénichou, arXiv:2504.08560 (to appear in Phys. Rev. Lett.)

To sum up

Spatial correlation profiles are worth checking out!

- ☑ $\left\langle Q_t^n \, \rho_r(t) \right\rangle$ give info on **response** of the bath in interacting particle systems (e.g. role of **loops**),
- ☑ and give access to **moments** of the current Q_t , e.g. $\left\langle Q_t^2 \right\rangle$ on infinite lattices in $d > 1$.
- ☑ Tracer-bath profiles $\left\langle X_t^n \, \rho_{X+r}(t) \right\rangle$ encode the **response** of the bath and control tracer statistics
- ☑ $\left\langle X_t \, \rho_{X+r}(t) \right\rangle \sim r^{1-d}$ **universal** wrt short-range 2-body interaction type
- ☐ Captured by **macroscopic fluctuation theory**?

J. Stat. Mech. (2024) 113208
arXiv:2411.09326 (to appear in Phys. Rev. Lett.)
arXiv:2504.08560 (to appear in Phys. Rev. Lett.)

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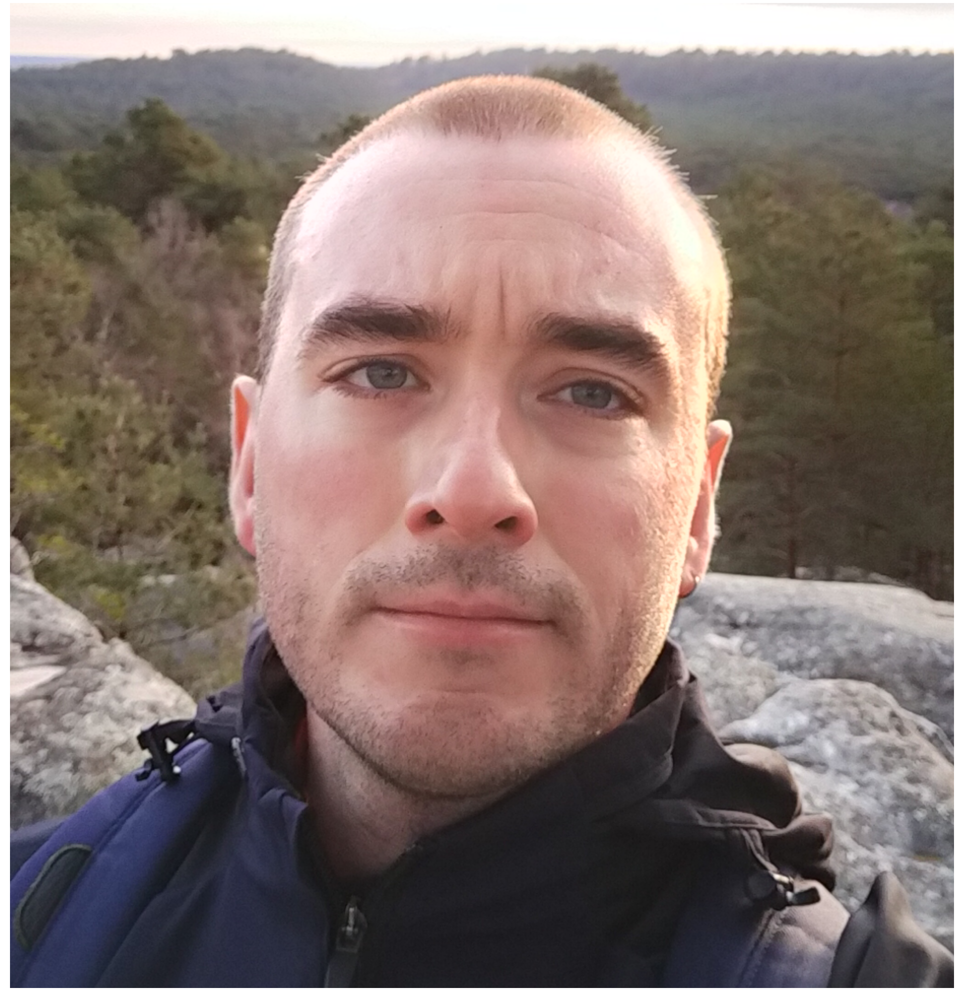
Thanks!



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